Manual ARIMA(1,1,1) Calculation for Forecasting

fire ARIMA(1,1,1) Example

We'll calculate an ARIMA(1,1,1) forecast step by step using the following data:

Month	Actual	Forecast	Error (Actual - Forecast)
Jan	100	–	–
Feb	120	110	+10
Mar	150	130	+20
Apr	?	181	?

Table 1: Passenger Data with Forecasts

Given Parameters:

- Using ARIMA(1,1,1):
- $p = 1$ (use 1 previous actual value □AR(1))
- $d = 1$ (data already differenced □working on differences)
- $q = 1$ (use 1 previous error □MA(1))

memo General Formula

For ARIMA(1,1,1), the differenced forecast is:

$$y'_t = c + \phi_1 y'_{t-1} + \theta_1 e_{t-1} + \epsilon_t$$

Where:

- y'_t: Forecasted difference for time t
- c: Constant (intercept)
- ϕ_1: Weight of AR(1) term
- y'_{t-1}: Previous differenced actual value
- θ_1: Weight of MA(1) term
- e_{t-1}: Previous forecast error
- ϵ_t: Random error (assumed 0 when forecasting)

numbers Assumed Parameters

Assume:

- $c = 5$
- $\phi_1 = 0.6$
- $\theta_1 = 0.4$

ruler Step 1: Find Differenced Data (y_t')

Since $d = 1$:

$$y_t' = y_t - y_{t-1}$$

Month	Actual (y_t)	Difference (y_t')
Jan	100	–
Feb	120	$120 - 100 = 20$
Mar	150	$150 - 120 = 30$

Table 2: Differenced Data

ruler Step 2: Previous Values for AR & MA

For April:

- Previous differenced actual (y_{t-1}'): 30 (from Mar)
- Previous error (e_{t-1}): +20 (from Mar)

ruler Step 3: Forecast Differenced Value (y_t')

Apply the formula:

$$y_t' = c + \phi_1 y_{t-1}' + \theta_1 e_{t-1}$$

Substitute:

$$y_t' = 5 + (0.6)(30) + (0.4)(20)$$

$$y_t' = 5 + 18 + 8 = 31$$

This is the forecasted difference.

ruler Step 4: Convert Back to Original Scale

For April's actual value (y_t):

$$y_t = y_{t-1} + y_t'$$

$$y_{\text{Apr}} = 150 + 31 = 181$$

check-mark-button Final Result

The forecast for April is 181 passengers.

light-bulb Intuition Recap

The forecast is built from:

- **AR(1)**: Previous differenced actual value (trend)
- **MA(1)**: Previous forecast error (residual correction)
- **c**: Intercept (baseline value)