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CSE 105 HW 4

1. If ri = rj for some I and j where 0 <= i < j <= n; Then the language of M is infinite because there is a repeatable sequence in the language of M. Let's say that the language of M is 1012101 and that the second "1" is i and the third "1" is j, then the second "1" and the third "1" are the same state, therefore containing a loop. Another accepted string in the language of M would be 101212101 as it just goes through the loop another time and then follows the original string. Let's say that the string entered into M doesn't go through the loop, then the shortest accepting string would be 10101. It follows that 101((21)*)01 is the language of M and thus, the language of M is infinite.
2. 
   a. If x is nonempty, the smallest, largest string accepted by N is a string that goes to every state. The states would be in series and only the last one would be an accepting state, therefore X has to be of length at most K. Any NFA containing a loop can be passed through without going through the loop. Say we use the example from question 1, the machine accepting 101((21)*)01 would accept the minimal string 10101. This accepted string of five letters is less than the number of states k, therefore X Contains some string of length at most K
   b. If you take the complement of X, the length of |X'| would be at most 2^k. However many states you have, taking the complement would mean to switch the accepting and non-accepting states. Since X is nonempty and there exists some string in X that is at most length 2, then there exists a string in X' that is of length at most 2^k. The shortest longest string in X has to be one where every state is used once before the string is accepted. A complement of this string would result in the accepting state to come much sooner in the machine, therefore the length of X' would be less than the length of X and therefore much less than 2^k. The shortest string in X would be a string with one letter. If that string is complemented, then the second state would have to be accepted. 2^1 is 2, therefore the length is still in the expected range.
3. Regular Languages Help
   a. Say X is the language of the empty string. The empty string is a regular language and is a subset of all languages. The empty string is therefore a subset of all non-regular languages over sigma.
   b. Say B is the nonregular language {0^n | n is prime} and A is the regular language {0, 00, 0000000}. A is a subset of B, and A is regular and B is nonregular.
   c. Say C is the nonregular language {0^n 1^n | n >= 0} and D is the regular language {0^n 1^m | n, m >= 0}. C is a subset of D, and D is regular and C is nonregular. We proved in class that C is nonregular using the pumping lemma.
4. Pumping Proof
   a. Say X = {wtw^R | w ∈ {0, 1}*, t ∈ {1}* , and both w, t are nonempty} is a regular language. Because X is a regular language, it has a pumping length P for X such that for

any string s in X where |s| >= p, s = xyz subject to |y| > 0, |xy| <= p, and x y^I z where I >= 0. Let's say s = (01)^p1(01)^p. Using the pumping lemma, we need to show that |y| > 0, |xy| <= p. There is no way that this string can be devided into wtw^r such that it can be broken into xyz. Therefore the assumption that X is regular is false and therefore X is not regular.

b. A CFG proving that this language is context-free is ({S, T, A},{0,1},R,S) where R = {S -> (0 U 1) T (0 U 1) | (0 U 1) S (0 U 1) ; T -> 1T | 1A ; A -> ε }
   This CFG works to show that the language is context free because it covers every possible set of strings {wtw^R | w ∈ {0, 1} ∗ , t ∈ {1} ∗ , and both w, t are nonempty} S represents how many time w will be repeated and T represents t. There is another variable A to lead to the empty string. This CFG covers all strings in the language.