$$\mathbb{P}(A_i \text{ a.e}) = \mathbb{P}\left(\bigcup_{n=1}^{\infty} \bigcap_{i \geq n} A_i\right) \uparrow = \lim_{n \to \infty} \mathbb{P}\left(\bigcap_{i \geq n} A_i\right)$$

$$1 - \mathbb{P}(A_i^c \text{ i.o})$$

$$\mathbb{P}\left(\liminf_{n \to \infty} A_n\right) \leq \liminf_{n \to \infty} \mathbb{P}(A_n)$$

(Fatou)

$$1 - \mathbb{P}(A_i^c \text{ a.e})$$

$$\mathbb{P}(A_i \text{ i.o}) = \mathbb{P}\left(\bigcap_{n=1}^{\infty} \bigcup_{i \geq n} A_i\right) \downarrow = \lim_{n \to \infty} \mathbb{P}\left(\bigcup_{i \geq n} A_i\right)$$

$$\mathbb{P}\left(\limsup_{n \to \infty} A_n\right) \geq \limsup_{n \to \infty} \mathbb{P}(A_n)$$

(Fatou)

$A_{n,\varepsilon} = \{|X_n - X| \geq \varepsilon \mid n \in \mathbb{N}\}$

$X_n \xrightarrow{a.s} X \iff \mathbb{P}(A_{n,\varepsilon} \text{ a.e}) = \mathbb{P}\left(\bigcup_{n=1}^{\infty} \bigcap_{n \geq i} A_{n,\varepsilon}\right) \overset{\text{רציפות}}{=} \lim_{n \to \infty} \mathbb{P}\left(\bigcap_{n \geq i} A_{n,\varepsilon}\right)$

$1 - \mathbb{P}(A_{n,\varepsilon}^{c} \text{ i.o})$

$\forall \varepsilon > 0 \quad \mathbb{P}(\liminf A_{n,\varepsilon}) = 1$

$\forall \varepsilon > 0 \quad \mathbb{P}(\limsup A_{n,\varepsilon}) = 0$

$X_n \xrightarrow{P} X \iff X_n \xrightarrow{a.s} X$

$\forall \varepsilon > 0 \quad \liminf \mathbb{P}(A_{n,\varepsilon}) = 1$

$\forall \varepsilon > 0 \quad \limsup \mathbb{P}(A_{n,\varepsilon}) = 0$

X_n ב"ת X_N

כך ש: $X_{n_k} \subset X_n$

$X_{n_k} \xrightarrow{a.s} X$

X קבוע