Advanced Models B 52805 (Midterm quiz, 2022)

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- Exam duration is 90 min.
- All questions have the same weight.
- Any printed/handwritten material can be used.
- All means of communications are strictly prohibited, incl. calculators, etc.
- Answer the questions in a clear way, dubious solutions will not be given credit.

Problem

Consider the first centeral absolute moment functional

$$T(F)=\int_0^1 |x-\mu(F)|dF(x)$$

on the subset of continuous distributions on the interval $[0,1]$, where

$$\mu(F)=\int_0^1 x dF(x)$$

is the mean functional.

Note: In questions (3)-(5) you may find useful the integration by parts formula for the Lebesgue–Stieltjes integrals

$$\int_a^b h(x)dg(x)=h(x)g(x)\Big|_a^b-\int_a^b g(x)dh(x).$$

(1) Specify the plug-in estimator $T(\widehat{F}_n)$ and evaluate it at the sample $\{\frac12,\frac14,\frac34\}$.

(2) Prove that $T(\widehat{F}_n)$ is consistent.

Hint: use the following instance of the triangular inequality

$$\big||x-a|-|x-b|\big|\le |b-a|,\quad \forall x,a,b\in\mathbb{R}.$$

(3) Show that the functional in question satisfies

$$T(F)=2\int_0^{\mu(F)}F(x)dx.$$

(4) Prove that T is Hadamard differentiable with the derivative

$$\dot{T}_F(G-F)=2\int_0^{\mu(F)}\big(G(u)-F(u)\big)du+2F(\mu(F))\int_0^1 ud(G(u)-F(u))$$

for any distribution function G.

(5) Show that the influence function is $L_F(x)=\psi_F(x)-\mathbb{E}_F\psi_F$ with

$$\psi_F(x)=2\big(\mu(F)-x\big)\mathbf{1}_{\{x\le\mu(F)\}}+2F\big(\mu(F)\big)x.$$

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