Financial Econometrics

- Week 1, Lecture 1 -

Introduction

VU Econometrics and Data Science
2024-2025

Paolo Gorgi

Useful info (i)

- **Course:** Financial Econometrics
- **Teacher:** Paolo Gorgi
- **Email:** p.gorgi@vu.nl
- **Office Hours:** Send me an e-mail!
- **Info:** Download *Course Information* from Canvas

Useful info (ii)

Course material:

1. Course slides
2. Lecture notes
3. Exercises
4. R exercises
5. Additional reading material? (see Course Information on Canvas)

Exercises:

- Exercise tutorials will take place on Wednesday.
- It is useful that you also practice yourself with the exercises. Solutions will be available on Canvas!

Useful info (iii)

R exercises:

- R code with explanations is provided (R files and Lecture Notes)
- You can practice with the R exercises

Software R:

- Powerful software for statistical analysis!
- Excellent documentation and large community of users!
- Download R and install it on your laptop.
  `https://cran.r-project.org/`
- You can also install RStudio (requires R to be installed).
  `https://www.rstudio.com/products/rstudio/download/`

Useful info (iv)

Grading Policy:

- Assignment (2 parts) + Exam.
- The assignment is worth 30% of the final grade.
- The assignment is mandatory.
- Minimum exam grade of 5.0 is required to pass the course.

$$\text{Final Grade} = 0.7 \times \text{Exam Grade} + 0.3 \times \text{Assignment Grade}.$$

Introduction (i)

Question: What is this course about?

Answer: This course is devoted to:

1. modeling financial data (stock returns);
2. specifying time-varying parameter models;
3. conducting inference on unknown parameters;
4. developing probabilistic analysis and predicting risk;

Important: Time-varying parameter models can be divided into two main classes:

- *observation-driven* models;
- *parameter-driven* models.

Introduction (ii)

Similarities: Both classes of models are capable of describing complex time-series dynamics:

1. time-varying conditional volatilities, tail probabilities, regression coefficients, conditional moments of higher-order, and much more!

Differences: these models approach the data in very different ways and require distinct statistical tools and techniques!

Course content is divided in main 3 parts:

1. Observation-driven models for volatility (Weeks 1-3)
2. Parameter-driven models for volatility (Week 4)
3. Extensions (Week 5)

Course content (i)

Part I: Observation-driven models

- Introduction (week 1)
- ARCH and GARCH models (week 1)

- ML Estimation for ARCH and GARCH (week 2)
- Economic and Financial Analysis (week 2)

- Multivariate GARCH models (week 3)
- ML Estimation for MV GARCH (week 3)
- Economic and Financial Analysis of MV GARCH (week 3)

Course content (ii)

Part II: Parameter Driven Models and extensions

- Stochastic Volatility Model (week 4)
- Indirect Inference Estimation (week 4)
- Extensions (week 5)

Today's class (i)

Plan for today: use time-series of *financial returns* as a motivation for the use of time-varying parameter models.

financial return data: clarifies the need to go beyond models of the conditional mean:

- Linear regression
- ARMA models

Note: we need models that can describe time-variation in conditional volatilities.

Today's class (ii)

1. Models for the conditional mean
    - Linear regression
    - AR(1) model
2. Properties of financial returns
    - Random walk of stock prices
    - Volatility clustering

Models for conditional mean

Linear regression

Introductory Econometrics: linear regression model

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

- y_t is the *dependent* or *endogenous* variable;
- x_t is the *independent* or *explanatory* variable;
- ε_t is the *error term* or *innovation*;
- α and β are the fixed *unknown parameters*;
- α is typically called *the intercept*;
- β is typically called *the slope.*

Conditional mean

Recall: The regression model is a model of the conditional expectation of y_t given x_t.

Important: Under the assumption of exogenous regressors, $\mathbb{E}(\varepsilon_t|x_t) = 0$, the conditional mean of y_t is the regression line:

$$\begin{aligned}\mathbb{E}(y_t|x_t) &= \mathbb{E}(\alpha + \beta x_t + \varepsilon_t|x_t) \\ &= \alpha + \beta \underbrace{\mathbb{E}(x_t|x_t)}_{=x_t} + \underbrace{\mathbb{E}(\varepsilon_t|x_t)}_{=0} \\ &= \alpha + \beta x_t.\end{aligned}$$

AR(1) model

Introductory Time Series: *autoregressive model of order 1*, or AR(1) model,

$$x_t = \phi x_{t-1} + \varepsilon_t \qquad \forall \ t \in \mathbb{Z}$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is a ***white noise*** sequence such that $\mathbb{E}(\varepsilon_t | x_{t-1}) = 0$.

White noise: serially uncorrelated $\mathbb{C}ov(\epsilon_t, \epsilon_{t-j}) = 0$ for $j \neq 0$, with mean zero $\mathbb{E}(\epsilon_t) = 0$, and finite unconditional variance $\mathbb{V}\text{ar}(\epsilon_t) = \sigma^2$.

Conditional mean and distribution

Conditional mean: the conditional mean of the AR(1) model is $\mathbb{E}(x_t|x_{t-1}) = \phi x_{t-1}$.

Conditional distribution: If the error term is iid Normal $\epsilon_t \sim N(0, \sigma^2)$, the AR(1) model gives us a description of the conditional distribution of x_t given its past

$$x_t|x_{t-1} \sim N(\phi x_{t-1}, \sigma^2).$$

Recall: ARMA models are useful for modeling temporal dependence in economic and financial time-series.

Weak stationarity (i)

- **Recall:** a time series $\{x_t\}_{t\in\mathbb{Z}}$ is **weakly stationary** if its *unconditional* mean $\mathbb{E}(x_t)$, variance $\mathbb{V}ar(x_t)$ and autocovariances $\mathbb{C}ov(x_t, x_{t-j})$, $j \neq 0$, are invariant in time.
- The AR(1) model is stationary if the coefficient is smaller than 1, i.e. $|\phi| < 1$.
- This means that there is not too much persistence in the series and therefore the process is "mean reverting".
- **Remark:** in general a model with time varying conditional mean can have a constant unconditional mean.

Weak stationarity (ii)

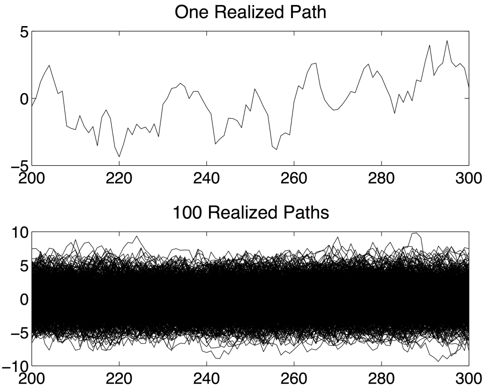


Figure: Single path [above] shows time-varying conditional mean but "mean reverting" behavior. Multiple paths [below] show invariance of the distribution (mean and variance are clearly constant over time).

Properties of financial returns

Random walk of stock prices (i)

In the following, we will argue that a time-series $\{p_t\}_{t\in\mathbb{Z}}$ of stock prices behaves essentially like a *random walk*.

Recall: We shall say that $\{p_t\}_{t\in\mathbb{Z}}$ follow a ***random walk*** if

$$p_t = p_{t-1} + \epsilon_t$$

where $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is a ***white noise*** sequence with $\mathbb{E}(\epsilon_t|p_{t-1}) = 0$.

Random walk of stock prices (ii)

In practice: the best forecast $\hat{p}_{t+1}$ is given by

$$\begin{aligned}\hat{p}_{t+1} &= \mathbb{E}(p_{t+1}|p_t) = \mathbb{E}(p_t + \epsilon_{t+1}|p_t)\\ &= \mathbb{E}(p_t|p_t) + \mathbb{E}(\epsilon_{t+1}|p_t) = p_t + 0 = p_t.\end{aligned}$$

If stock prices behave like *random walks*, **then** we should find evidence in the data that:

1. Prices are unit-root non-stationary
2. Price variations (returns or log-returns) are not only stationary but *white noise*.

Stock prices

Are stock prices unit-root non-stationary?

Figure: Daily prices of Apple and Intel stocks from 2006 to 2016

- There is no "mean-reverting" as we have seen for the AR(1).
- Prices seem non-stationary since their level changes over time.

Stock prices: unit-root test

We can use an augmented Dickey-Fuller (ADF) test to test the null hypothesis of unit root for Apple and Intel prices.

Table: p-values of ADF test for Apple and Intel stock prices

	daily	weekly	monthly
Apple	0.239	0.188	0.230
Intel	0.313	0.356	0.115

Conclusion: We can consider stock prices as unit root processes. The assumption of unit root is not rejected for all frequencies (daily, weekly and monthly).

Log-returns (i)

- We now focus our attention on returns (or log returns), i.e. price variations.
- As discussed before, if stock prices are *random walks*, then we should find that returns (or log-returns) are *white noise* (stationary and uncorrelated).
- Furthermore, studying the properties of returns (or log-returns) is of great interest because we are typically more interested in the price variation more than the price level itself.

Log-returns (ii)

- **Log-returns** $\{y_t\}_{t\in\mathbb{Z}}$ are defined as first differences of log-prices:

$$y_t = \log(p_t) - \log(p_{t-1}) = \log\left(\frac{p_t}{p_{t-1}}\right)$$

- We work with first differences of log-prices instead of prices because they have some appealing properties.
  - For instance, log-returns are a good approximation for returns rates
$$y_t = \log\left(\frac{p_t}{p_{t-1}}\right) \approx \frac{p_t - p_{t-1}}{p_{t-1}}$$
  - Therefore, if $y_t = 0.01$ we can say that the price from time $t-1$ to t increased of about 1%

Log-returns: plots

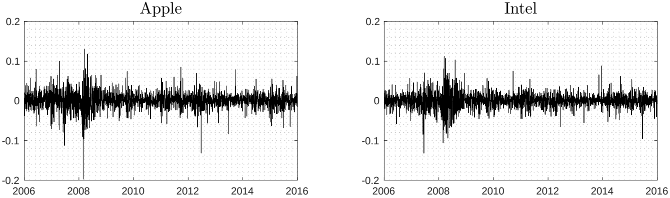


Figure: Daily log-returns of Apple and Intel from 2006 to 2016

- The mean of log-returns is constant over time and around zero
- Log-returns seem to be stationary

Log-returns: unit-root test

We perform the ADF test on log-returns.

Table: p-values of ADF test for Apple and Intel log-returns

	daily	weekly	monthly
Apple	0.001	0.001	0.001
Intel	0.001	0.001	0.001

Conclusion: log-returns are stationary. The null hypothesis of unit root is rejected for all frequencies.

Log-returns: autocorrelation (i)

- **Is there autocorrelation in log-returns?**

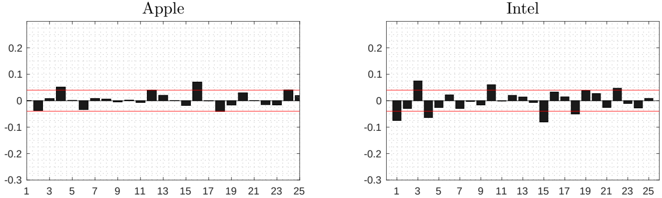


Figure: Sample ACF for daily log-returns of Apple and Intel.

- **Daily returns** show some evidence of autocorrelation (significant and negative fist lag autocorrelation). However, **autocorrelation is weak** and basically irrelevant in practice.
- Furthermore, any evidence of autocorrelation disappears with lower frequencies

Log-returns: autocorrelation (ii)

- **Weekly returns** show no evidence of autocorrelation for Apple and Intel stocks

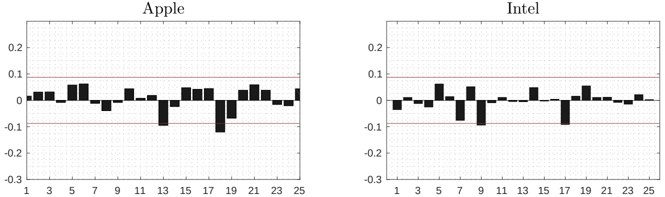


Figure: Sample ACF for weekly log-returns of Apple and Intel.

- We can clearly consider weekly returns as *white noise*.

Log-returns: autocorrelation (iii)

- We can estimate MA and AR models and see if the coefficients are significant.

Table: Estimates of MA(1) and AR(1) coefficients for Apple and Intel log-returns

	MA(1) daily	AR(1) daily	MA(1) weekly	AR(1) weekly	MA(1) monthly	AR(1) monthly
Apple	-0.026	-0.026	0.040	0.037	0.035	0.036
Intel	-0.044	-0.042	-0.040	-0.038	-0.038	-0.049

- Coefficients are significant only at daily frequency. This confirms previous findings.

Log-returns: autocorrelation (iv)

Overall:

- Temporal dependence in stock returns is very weak and often insignificant
- We can consider returns as *white noise* and stock prices as *random walks*
- Models for the conditional mean are not useful for modeling stock returns and prices

Question: does this mean that log-returns cannot be predicted?

Answer: no, the mean seems unpredictable but we can predict the variance (or volatility).

Forecasting white noise

Intro to Time Series:

white noise $\approx$ *no structure* $\approx$ *irrelevant for forecasting*

Financial Econometrics:

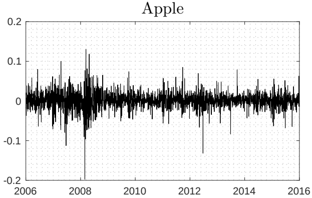
white noise $\approx$ *structured data* $\approx$ *relevant for forecasting*

Key ingredient: different focus!

conditional mean **vs** *conditional variance*

Volatility clustering

Volatility clustering: *periods of high volatility and periods of low volatility* (for instance, high volatility in 2008).



- Past returns can be used to predict volatility of future returns
- Predicting volatility is of key importance to access financial risk

Squared log-returns

- We now analyze squared log-returns $\{y_t^2\}_{t\in\mathbb{Z}}$
- **Idea:** returns have a mean of about zero, so squared returns offer a natural indicator of scale

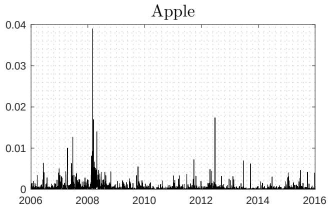


Figure: Daily squared log-returns of Apple and Intel.

- Clusters of volatility should reveal themselves as autocorrelation in squared log-returns (high y_{t-1}^2 followed by an high y_t^2).

Squared log-returns: autocorrelation (i)

- The **ACF** gives evidence of **autocorrelation in squared returns**.

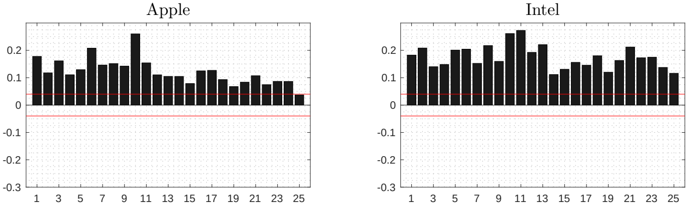


Figure: Sample ACF for the squared log-returns of Apple and Intel.

- There is information in past squared returns to **predict volatility of future returns**.

Distribution of log-returns

- We can further investigate the properties of stock returns by reporting their **sample moments and normality test**.

Table: Estimated moments and p-value of the Jarque-Bera test for Intel and Apple daily stock returns

Stock	Mean	Var	Skew	Kurt	JB
Apple	0.006	0.036	-4.979	47.712	0.001
Intel	-0.003	0.017	-1.873	10.394	0.001

- Jarque-Bera test suggests that stock returns are not Normally distributed.
- We can see that **Kurtosis is much larger than 3** (heavy-tailed distribution).

Modeling stock returns

Question: can we use a linear-Gaussian ARMA model to describe clusters of volatility and fat tails in stock returns?

Answer: no!

Example: AR(1) model

Let

$x_t = \phi x_{t-1} + \epsilon_t, \quad \text{with} \quad \{\epsilon_t\} \sim \text{NID}(0, \sigma^2)$

Then, the conditional mean is time-varying

$\mathbb{E}(x_t | x_{t-1}) = \phi x_{t-1}$

But the conditional variance is not...

$\mathbb{V}\text{ar}(x_t | x_{t-1}) = \mathbb{V}\text{ar}(\phi x_{t-1} + \epsilon_t | x_{t-1}) = 0 + \mathbb{V}\text{ar}(\epsilon_t) = \sigma^2$

And x_t is Gaussian... $x_t = \sum_{j=0}^{\infty} \phi^j \epsilon_{t-j}$

(x_t is a linear combination of Gaussian random variables)

Concluding summary

Findings:

1. Stock prices exhibit *random walk* behavior.
2. Log-returns behave like *white noise*.
3. *Volatility clustering* of log-returns (autocorrelation in y_t^2).
4. Log-returns are not normally distributed (heavy tails).

Conclusions:

1. We must go beyond models of the conditional expectation.
2. We must go beyond linear Gaussian models.
3. We need models capable of describing fat tails and volatility clustering: **ARCH and GARCH models**.