Assignment: part 2

Financial Econometrics

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Rules and instructions

Before you start working on the assignment read carefully the following instructions.

1. The assignment is a group assignment to be made in groups of 3 or 4 students, no more no less.
2. The deadline for Part 2 of the assignment is **Friday, May 30 at 23:00**.
3. The assignment must be uploaded to Canvas before the deadline mentioned above. You are expected to upload a pdf file with the solution of the assignment and an R file with the R code you used for the assignment. Please write name, surname and student number of all the members of the group on the cover page of the pdf file.
4. The answers (given in the pdf file) should be correct, clear and complete, with explanations of the methods you have implemented. Deliver a professional econometric report with appropriate justifications and insightful comments and remarks. Please consider all the theory that you know in analyzing the empirical results. If you are not sure how to proceed, just make your own decisions and justify them in the answer. Often there is not only one correct way to proceed.
5. The R code has to run without errors. It has to be well written and commented. You must write yourself the likelihood functions and the code. You cannot use built-in R functions to estimate models (except functions that have been used in the R code of the course such as `optim()`). You can use the R code provided in the course material.
6. To be fair to everyone, I will not answer any question directly concerning the assignment.

Part 2a: volatility modeling with SV and indirect inference

We are interested in estimating the parameters of a Stochastic Volatility model as an alternative modeling approach to GARCH models. Consider the US market index S&P 500 that is contained in the file `market.txt` to answer the following questions.

1. Estimate by indirect inference an SV model for the log-returns of the S&P 500 index[1]. In particular, use the sample variance of y_t, the sample kurtosis of y_t and first-order autocorrelation of absolute log-returns $|y_t|$ as auxiliary statistics and consider $H = 30T$. Report and comment the results.[2]

2. For the model estimated in Question 1, obtain and plot the filtered volatility. Compare the filtered volatility obtained from the SV model with the conditional variance σ_t^2 estimated from a GARCH(1,1) model. Comment on the results.

3. A colleague of yours suggests a different set of auxiliary statistics. In particular, the following statistics are proposed: sample mean of $|y_t|$, i.e. $\hat{s} = T^{-1} \sum_{t=1}^{T} |y_t|$ and 15 lags of the autocovariance function[3] of $|y_t|$, i.e. $T^{-1} \sum_{t=1}^{T} (|y_t| - \hat{s})(|y_{t-l}| - \hat{s})$ for $l = 0, 1, \ldots, 15$. Estimate the SV model by indirect inference using this set of auxiliary statistics. Report and comment the results.

4. Obtain and compare the filtered volatility obtained from the estimates in Question 1 with the one in Question 3. Comment on the results.

5. Suppose that you are interested in exploring additional volatility dynamics by allowing the transition equation of your SV model to be given by an AR(2) process. Would you consider the auxiliary statistics proposed in Question 1 or the auxiliary statistics proposed in Question 3? Justify your answer.

6. Estimate an SV-AR(2) model for the log-returns of the S&P 500 index. Use the auxiliary statistics you find more appropriate. Report and comment the results[4].

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[1] You can work with log-returns multiplied by 100, $r_t = 100 \times \log(p_t/p_{t-1})$, which give an approximation to relative price variations expressed as a percentage.

[2] You can implement the option control = list(maxit = 10000) within the optim function to obtain convergence.

[3] In R you can obtain 15 lags of the autocovariance function of a data vector `x` with the following code `acv_15 <- acf(xa, lag.max = 15, type = "covariance", plot = F)$acf`.

[4] You do not need to obtain the filtered path of σ_t^2.

Part 2b: dynamic regression and CAPM

Three risky assets have been identified as potential investment: Microsoft (MSFT), Bank of America (BAC), and Exxon Mobil (XOM).

You are expected to study the exposition of these three assets to the market risk. The weekly prices of the financial assets are in the files `MSFT.txt`, `BAC.txt` and `XOM.txt`. The file `market.txt` contains the S&P 500 market index. Furthermore, you can assume that the risk free rate is equal to zero $r^f = 0$. Use these datasets to answer the following questions.

1. Plot the log-returns of each asset as well as the market log-returns. Estimate the betas of the CAPM model for each of the three assets using OLS. Report and comment the results.

2. A colleague of yours suggests to invest in XOM by arguing the following: *"The current portfolio of the bank has an high exposition to the market risk. Therefore we should be careful and invest on the asset with lowest exposition to the market risk"*. Comment on this statement.

3. Estimate by ML a CAPM model with an *observation-driven* dynamic coefficient β_t for each of the assets. In particular, consider the following specification for β_t

$$\beta_t = \omega + \phi\beta_{t-1} + \alpha(y_{t-1} - \beta_{t-1}x_{t-1})x_{t-1}.$$

   Report the estimation results [5]. Plot the graph of the estimated β_t for each asset. Comment on the results.

4. Obtain the exposition to the market of each asset at time $T+1$, namely β_{T+1}. Comment on the consultant claim in Question 2 in light of the new evidence.

5. For each asset, obtain the time-varying beta β_t based on the bivariate CCC model. Plot the estimated dynamic betas and compare them with the ones obtained in Question 3. Comment on the results.

6. We now want to ensure that the dynamic coefficient β_t is always positive. In particular, we consider a *parameter-driven* dynamic regression model with an exponential link function for β_t, namely

$$\beta_t = \exp(f_t), \quad f_t = \alpha_0 + \alpha_1 f_{t-1} + \eta_t, \quad \{\eta_t\} \sim NID(0, \sigma_\eta^2).$$

   Estimate by indirect inference a dynamic CAPM model for each of the three assets considering this new specification for β_t. Report the parameter estimate and comment the results[6].

Good Luck!

[5] You can implement the option control = list(maxit = 10000) within the optim function to obtain convergence.

[6] You do not need to obtain the filtered path of β_t.