Modelling Collective System of Self Propelled Particles

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1 Introduction

Collective motion is observed in multiple natural processes, like bird flocks and fish schools and even in cells. In 1995, Vicsek et al. proposed a simple model of groups of noisy self-propelled particles interacting locally. In the model, at each time step, a given particle driven with a constant velocity assumes the average direction of motion of the particles in its neighbourhood of a predetermined interaction radius with some random noise added in aligning with the average direction.

The model proposed the initialization of the self-propelled particles in a square box of size L with periodic domains. A unit interaction radius is taken (r=1) and the time interval between successive alignment with average local direction is unit interval too ($\Delta t = 1$). The initialization takes places with simple initial conditions at time $t = 0$ with particles distributed across the square box with random starting directions and positions with a constant velocity.

2 Vicsek Model & The Modification

The Vicsek model restricts the particles in the two-dimensional space. The new direction of motion of a particle j with present angle θ_j^t after a time step $\Delta t = 1$ is determined by all k particles within the vicinity of its interaction radius. The relation is given by:

$$\theta_j^{t+1} = arg\left[\Sigma e^{i\theta_k^t}\right] + \eta\xi_j^t, \qquad (1)$$

where ξ_j^t is a white noise ($\xi \in [-\pi, \pi]$). For very high noise ($\eta > 1$), the particles essentially follow random paths while in absence of any noise ($\eta = 0$), particles align with each other perfectly in order.

The transition that lies between these two states of extremities is often described by the instantaneous order parameter:

$$\varphi^{t} = N^{-1} \left| \Sigma_{j=1}^{N} e^{i\theta_{k}^{t}} \right| \quad (2)$$

Where N is the total number of particles in the system.

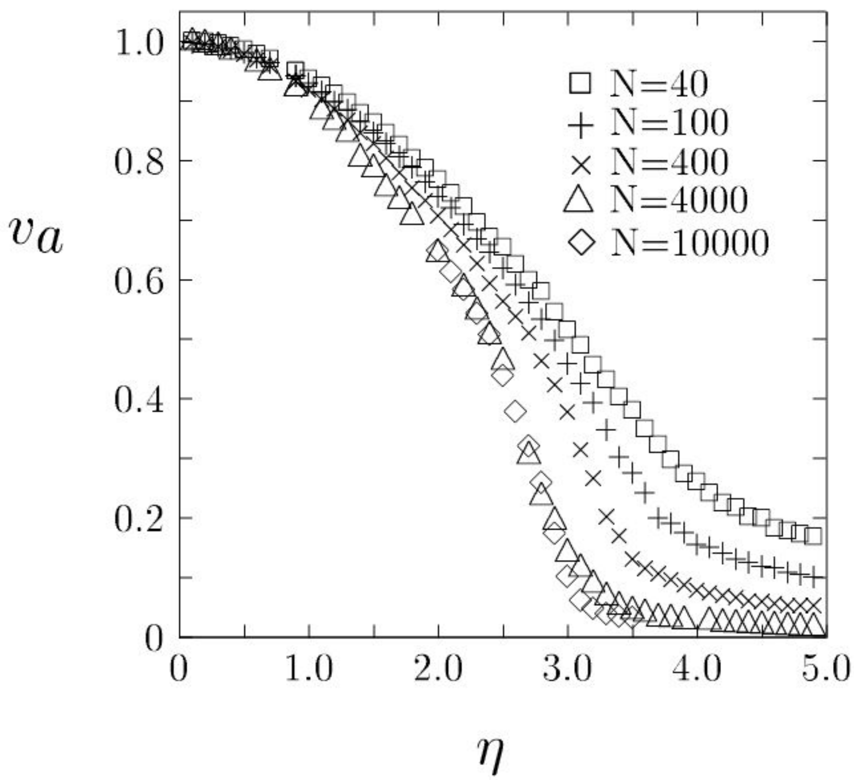


Fig 1. v_a is the order parameter and η is the noise (From Vicsek et al. 1995)
[(N=40,L=3.1),(N=100,L=5),(N=400,L=10),(N=4000,L=31.6),(N=10000,L=50)]

By tuning the particle noise η or the particle density ($\rho = N / L^2$) using the N & L parameters in square boxes with periodic domains, Vicsek et al. found that $\langle \phi \rangle$ varied continuously across the spectrum of parameters suggesting the presence of a critical point where the transition from a polar order to a non polar order took place.

However, in 2004, Grégoire and Chaté modified the simple model given by Vicsek et al. By taking into account, an “extrinsic” noise i.e. particles make errors when estimating the local interactions instead of the “intrinsic” noise of the Vicsek model where particles make the error when aligning with the new errorless calculated average velocity based on local interactions. The Eq. (1) then becomes:

$$\theta_j^{t+1} = arg \left[\Sigma_{k\sim j} e^{i\theta_k^t} + \eta n_j^t \xi_j^t \right] \tag{3}$$

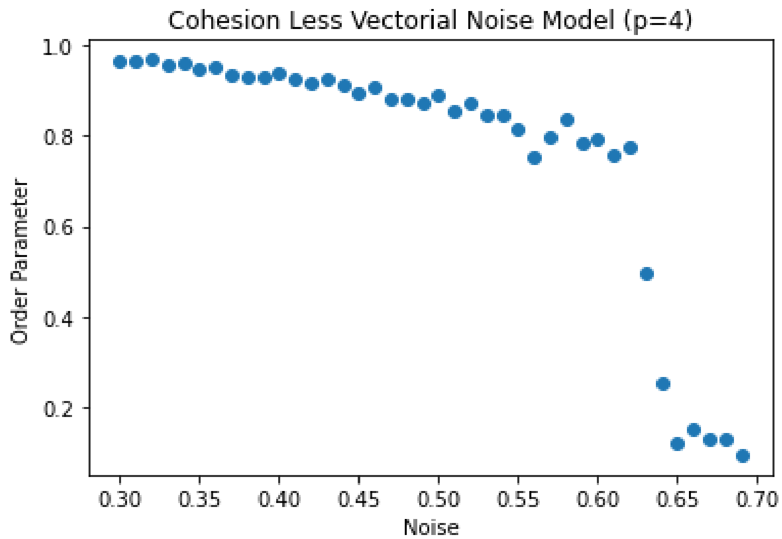
3 Coding The Model

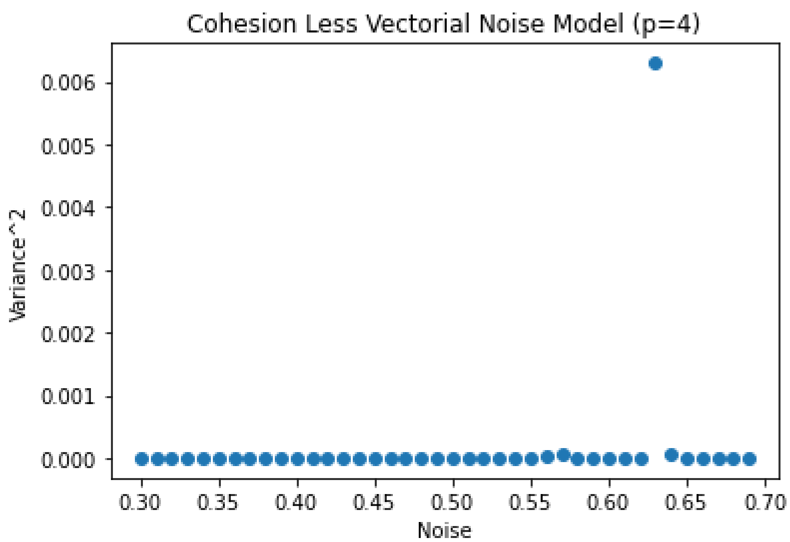
The code for the model was made with the help of Python. It is available at https://github.com/Amlan-Nayak/Vicsek-Model-Extrinsic-Noise

4 Evaluating The Model

Variance calculated for 5 values of order parameter at a specific noise
Order parameter plotted is average of the 5 values
Value of critical noise was determined by choosing the peak of the variance^2 vs noise graph

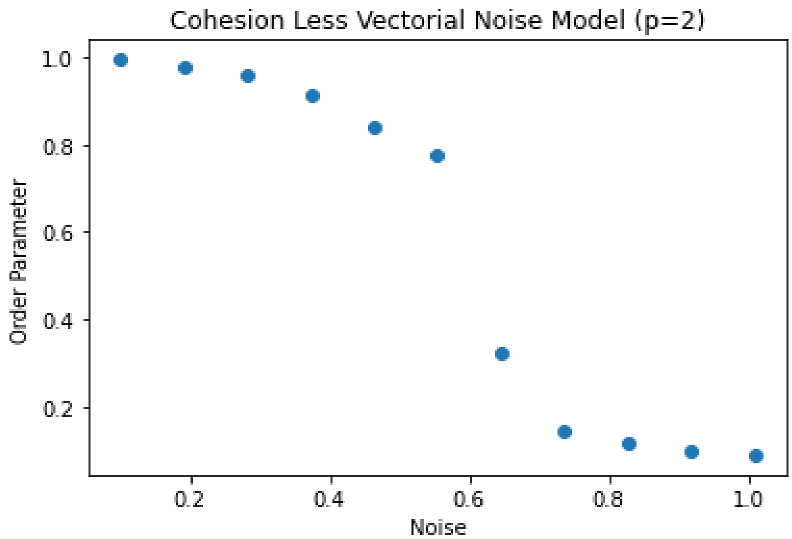
1. **v=0.06, r=1 and L = 10**

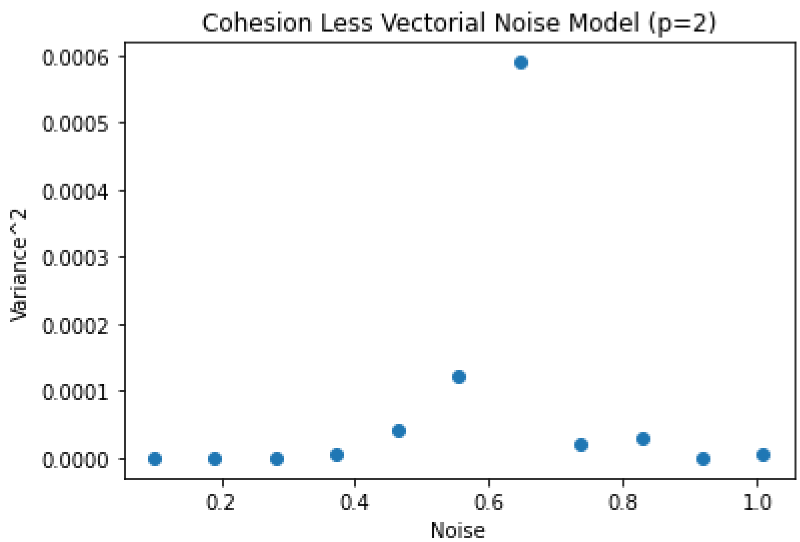




Critical Noise : 0.629

2. v=0.03, r=1 and L = 10





Critical Noise : 0.65

5 Inferences

The Vicsek model predicted a continuously varying $\langle \phi \rangle$ over the parameters and therefore a phase transition of continuous nature. However, by using an "extrinsic noise", Grégoire and Chaté showed that the phase transition is discontinuous. This result is apparent from the simulations as the values of $\langle \phi \rangle$ drop suddenly. Furthermore, at a particular value of noise, we see the most variation among values of order parameters on multiple iterations of the same system. The variation is high as a sudden change takes place at this particular value. This variation does not happen at other values. The presence of this specific point shows that the nature of the phase transition from ordered to disordered is discontinuous. This particular point is called the critical noise as it causes the discontinuous upheaval of the system.

References

[1] Grégoire and Chaté (2004), “Onset of collective and cohesive motion”, *Physical Review Letters*, https://doi.org/10.1103/PhysRevLett.92.025702

[2] Vicsek et al. (1995), “Novel type of phase transition in a system of self-driven particles”, *Physical Review Letters*, https://doi.org/10.1103/PhysRevLett.75.1226