CS 534 Artificial Intelligence
Assignment 2

Group 10

March 17, 2018

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1 Gibbs Sampling

2 Kalman Filter

The transition model is constructed using the GDP and the gdp growth rate from 1970 to 2017, data source is from world bank website [1]. Unit is trillion dollar.

There are 2 sensor models, in the first model our group use the Export and Import data from 1970 to 2017 [2], the unit is value in million dollars, to measure the GDP only, which is use the 2 sensors to measure the GDP.

The second model is use 3 sensors, the 2 sensors are still import and export, the additional sensor is the employment rate 3, to measure the GDP growth rate. The data source is from Office of Management and Budget.

2.1 Methodology

2.1.1 Kalman Filter Graph Model and Equations

The graph model of the Kalman filter is on the following:

The parameters of the Kalman filter and their equations are on the following [3].

For Kalman filter, its parameters are listed in the table 1:

Table 1: Kalman Filter Parameters Notation

Parameter	Notation
x_t	The state vector containing the terms of interest for the system (e.g., position, velocity, heading) at a time
F_t	The state transition matrix which applies the effect of each system state at time t. (e.g., the position and velocity at time t-1 both affect the position at time t)
w_t	the vector which contains the noise for each parameter in the state vector x_t. A zero mean multivariate Gaussian distribution with covariance matrix Q_t is used to describe the process noise vector.
z_t	The measurements vector.
H_t	the transformation matrix which can interpret the state vector x_t into measurement domain of z_t.
v_t	the measurement noise vector for each observed value in the measurement vector z_t. The model that to establish the measurement noise is also drawn from a zero mean multivariate normal distribution with covariance matrix R_t.
R_t	The covariance matrix for the observation noise.
Q_t	The covariance matrix for the process noise.

In the Kalman filter, the state matrix can be described as 2.1.1, which is transition model.

$$x_t = F_t x_{t-1} + w_t \tag{1}$$

At the time t an observation z_t of the true state x_t is made according to 2.1.1, which represents the observation model.

$$z_t = H_t x_t + v_t \tag{2}$$

The state of the filter is represented by 2 variables:

$\widehat{x}_{t\|t}$	a posterior state estimate given the observations up to t
$\widehat{P}_{t\|t}$	a posterior error covariance estimate given the observations up to t

The predict process of the Kalman filter can be described using equation 2.1.1 and 2.1.1.

$$\widehat{t}_{t|t-1} = F_t \widehat{t}_{t-1|t-1} \tag{3}$$

$$P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q_t \tag{4}$$

The update process of the Kalman filter is on the following, $\widetilde{y}_t$ is the residual covariance, K_t is the Kalman gain matrix:

$$\widetilde{y}_t = z_t - H_t \widehat{x}_{t|t-1} \tag{5}$$

$$S_t = R_t + H_t P_{t|t-1} H_t^T \tag{6}$$

$$K_t = P_{t|t-1} H_t^T S_t^{-1} \tag{7}$$

$$\widehat{x}_{t|t} = \widehat{x}_{t|t-1} + K_t \widetilde{y}_t \tag{8}$$

$$P_{t|t} = (I - K_t H_t) P_{t|t-1} (I - K_t H_T)^T + K_t \tag{9}$$

$$\widetilde{y}_{t|t} = z_t - H_t \widehat{x}_{t|t} \tag{10}$$

2.1.2 Transition Model

In the transition model, a state is defined by the GDP and GDP growth rate. For a state x_k, $x_k = \begin{bmatrix} x_g \\ x_{gr} \end{bmatrix}$. x_g represents the state of the GDP and x_{gr} represents the state of the GDP growth rate. The transition matrix F_t is calculated based on the matrix Q_t.

2.1.3 Sensor Model

Export and import data will be 2 sensors to measure the GDP data and the employment rate will be the sensor to measure the GDP growth rate as the extra credit part.

2.2 Results

2.2.1 Kalman Estimate Result: Sensor for GDP only

The Kalman filter estimate for the GDP data is illustrated in figure 1.

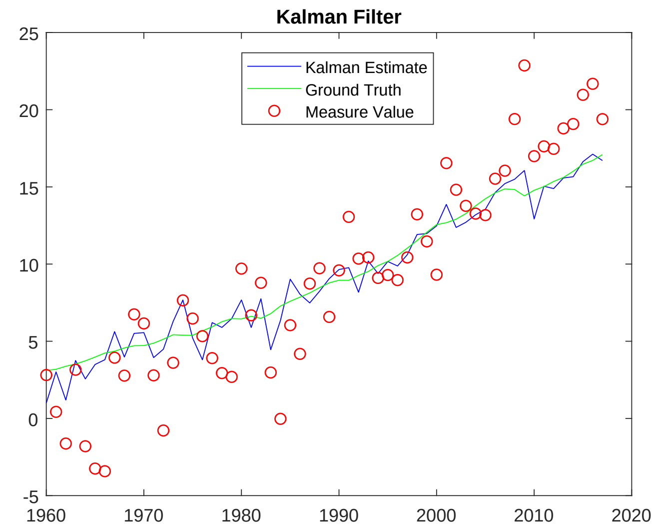

Figure 1: Kalman Filter Estimate Result

The probability distribution is illustrated in 2.

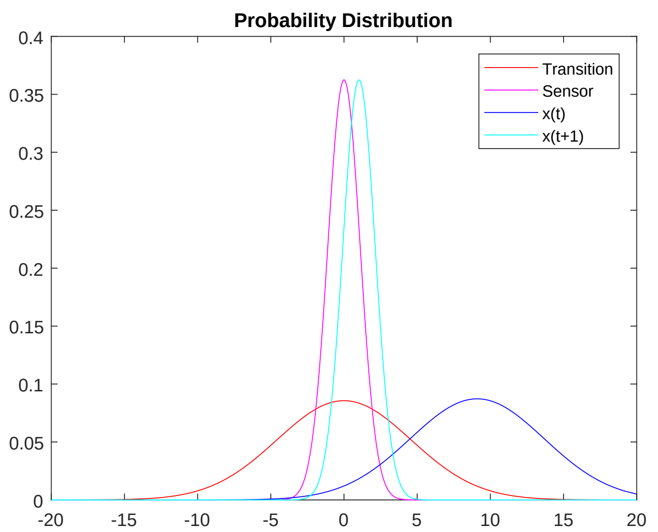

Figure 2: Kalman Filter Related Probability Distribution

2.2.2 Extra Part: Additional Sensor for GDP growth rate

The Kalman filter estimate for the GDP data is illustrated in figure 3.

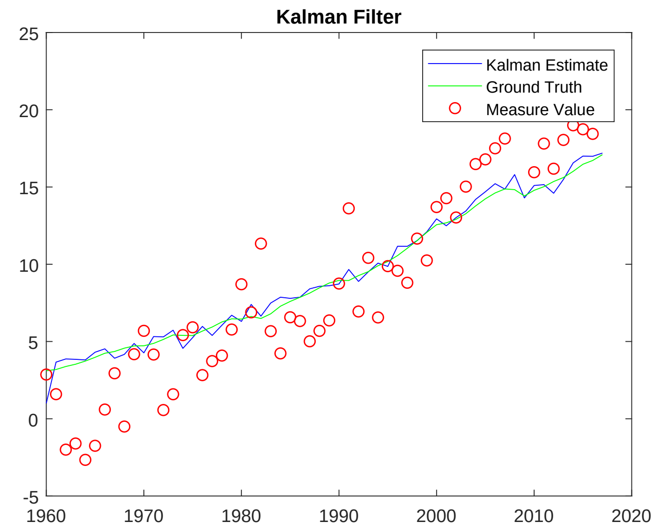

Figure 3: Kalman Filter Estimate Result

The probability distribution is illustrated in 4.

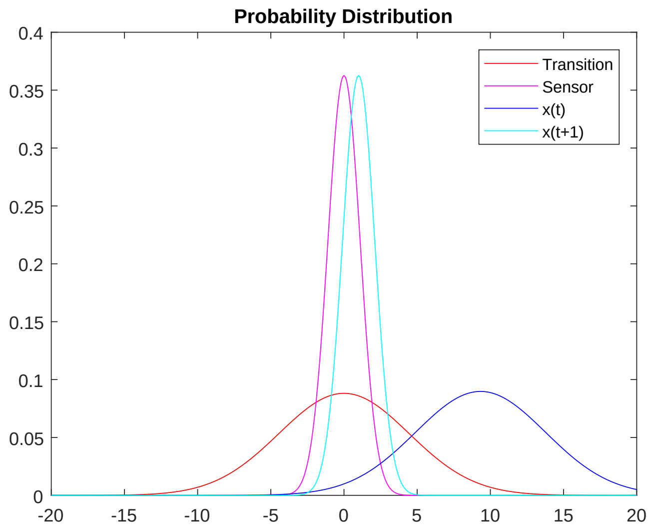

Figure 4: Kalman Filter Related Probability Distribution

2.3 Discussion

In the result part, the Kalman estimate result for the sensor model with 2 sensors and 3 sensors are given. In this part, we will use the Pearson's cofficient to compare the fitting result for the 2 sensors and 3 sensors.

The Pearson's coefficient table. The higher value means that the better fitting result.

Table 2: Comparison

Category	Value
GDP Ground Truth Variance	18.6672
2 Sensors Fitting Variance	19.3499
3 Sensors Fitting Variance	18.3853

Table 3: Pearson's Coefficient

Category	Value
2 Sensors Fitting Variance	0.9742
3 Sensors Fitting Variance	0.9919

From table 2 and 3, the 3 sensor model illustrated a better fitting result, the variance deviation is smaller and the Pearson's coefficieint is higher.

References

[1] W. B. Website, "United states gdp and gdp growth rate," https://data.worldbank.org/indicator/NY.GDP.MKTP.CD.

[2] F. Trade, "Guide to foreign trade statistics," https://www.census.gov/foreign-trade/guide/sec2.html#bop.

[3] R. Faragher, "Understanding the basis of the kalman filter via a simple and intuitive derivation, ieee signal processing magazine," https://www.cl.cam.ac.uk/~rmf25/papers/Understanding%20the%20Basis%20of%20the%20Kalman%20Filter.pdf, Cambridge University, England, Cambridge.