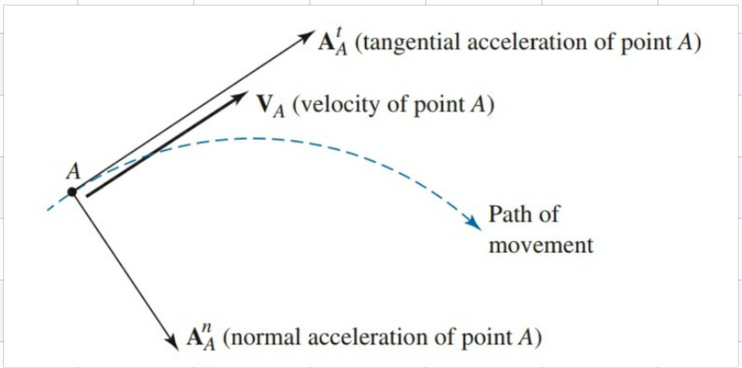
Acceleration Analysis of mechanics

Veloncity of moving body is Vector quantity having magnitude and direction.
a change in Veloncity over time result in:

- Change in magnitude only
- Change in direction only
- Change in magnitude and direction

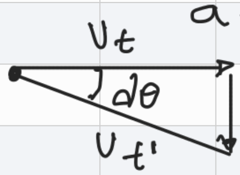
The rate of change of velocity with respect to time is known as acceleration.

Acceleration on circular path using normal and tangential component



$$v = V u_t$$

$$a = \frac{dv}{dt} = \underbrace{\frac{dv}{dt} \cdot u_t}_{a_t} + \underbrace{\frac{du_t}{dt} \cdot V}_{a_n} \quad v/\rho$$



$$du_t = u_t \, d\theta$$

$$\frac{du_t}{dt} = \frac{d\theta}{dt} = \frac{v}{\rho}$$

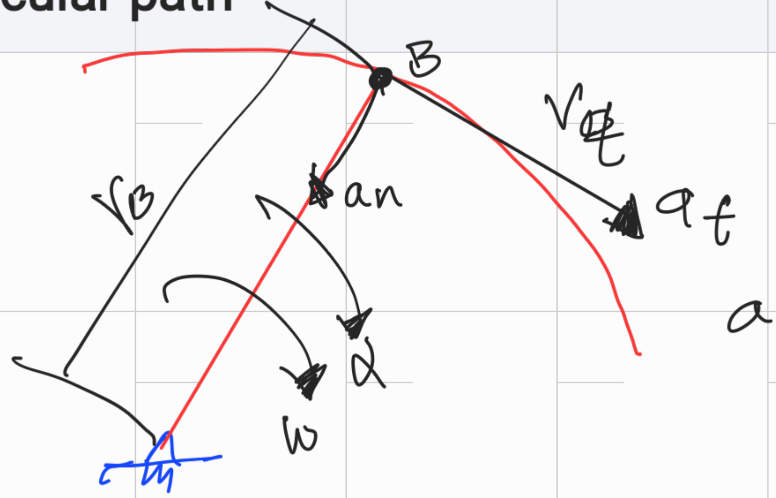
- The normal component is created as a result of a change in the direction of the velocity vector.
- The tangential component is formed as a result of a change in the magnitude of the velocity vector.

$$a_t = \frac{dv}{dt}$$

$$a_n = \frac{v^2}{\rho}$$

Note that : if the motion is straight line motion (acceleration on straight path) , there is only tagential acceleration which is due to change in the magnitude of velocity.

Acceleration of apoint on rigid body or link moving on circular path



a_n - normal acceleration
a_t = tangential >>

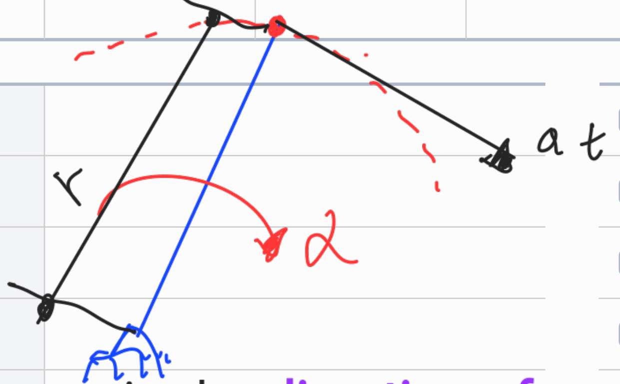
$\vec{a} = a_t u_t + a_n u_n$

n - measure acceleration due to Change in direction
t - >> >> >> >> >> >> >> Magnitude

Tangential Acceleration of apoint on link

- The magnitude of the tangential acceleration of point A on a rotating link 2 can be expressed as;

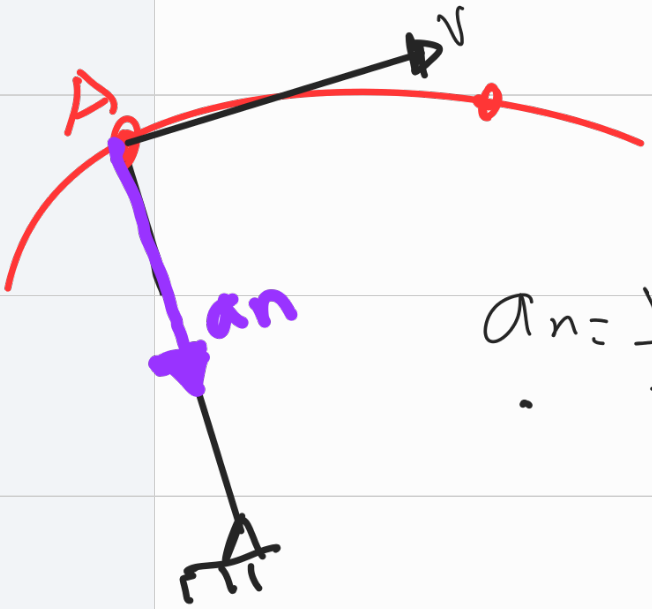
$$a_t = \frac{dv}{dt} = \frac{d(\omega \times r)}{dt} = \frac{d\omega}{dt}\cdot r = \underline{\underline{\alpha . r}}$$



- It acts in the direction of motion when the velocity increases or the point accelerates.
- Conversely, it acts in the opposite direction of motion when the velocity decreases or the point decelerates.

Normal Acceleration

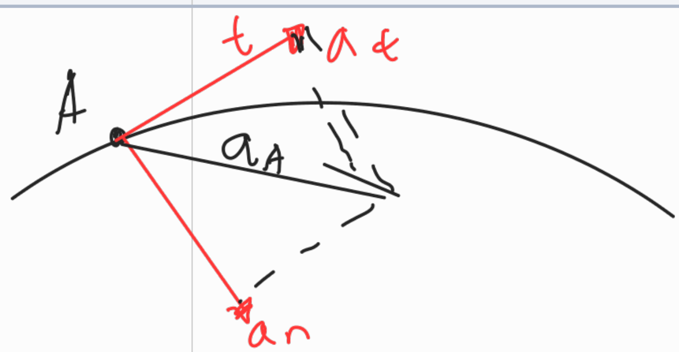
- Any change in velocity direction creates normal acceleration, which is always directed toward the center of rotation.
- To understand and drive the general equation for normal acceleration let as the link below :



$$a_n = \frac{V^2}{\rho} = \frac{V^2}{r} = \frac{\omega^2 \times r^2}{r} = \omega^2 \cdot r$$

Total Acceleration

- acceleration analysis is important because inertial forces result from accelerations.
- Inertial forces are proportional to the total acceleration of a body.
- The total acceleration, A, is the vector resultant of the tangential and normal components. Mathematically,



$$|a_A| = \sqrt{a_t^2 + a_n^2}$$

Important points

- If the body is moving with *constant angular veloncity* the tangential acceleration, at=0
- If the body is *moving on straight line* then the normal acceleration an=0
- The ***normal acceleration is always directed*** towards the center *point of rotation* at a given instants (parallel to the given link)
- The ***tangential acceleration*** is perpendicular to normal acceleration and directed in the direction of velocity or angular veloncity or acceleration

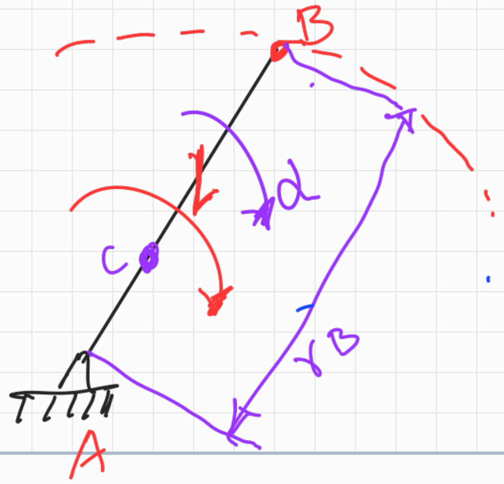
Method of Acceleration analysis of links or mechanisms

- Two methods are available namely analytical and graphical method.
- We focus on graphical method of solving acceleration of mechanisms.

Graphical method of finding acceleration of points on rigid links

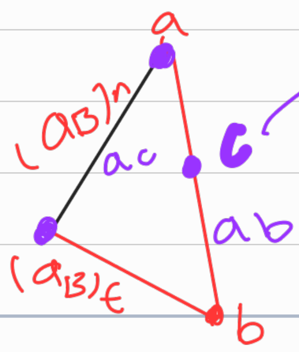
- Uses acceleration diagram and constructed using drawing instrument with suitable scales
- The solutions are obtained by measurement 1cm = 10m/s

Acceleration diagram of simple link



$a_n = \frac{v^2}{r} = 50\ m/s^2$

$a_t = \alpha \times r = 30$

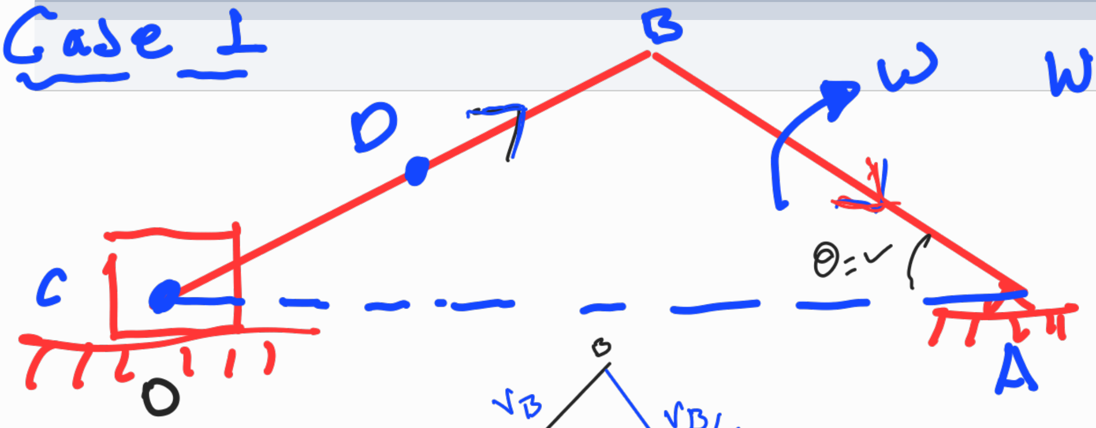


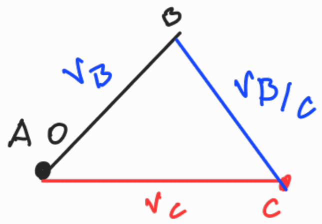
$\frac{AB}{AC} = \frac{ab}{ac}$

Acceleration diagram of slider crank mechanism

Case 1

$W = \text{Constant}$





$(a_B)_n = ✓$

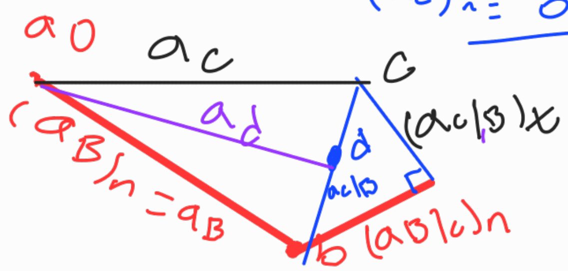
$(a_B)_t = 0$

$(a_{C/B})_n = ✓$

$(a_{C/B})_t = ?$

$(a_C)_t = ?$

$(a_C)_n = 0$



$$\frac{CD}{CB} = \frac{cd}{cb}$$

Case 2

$W \neq \text{constant}$

α ✓

$(a_B)_n = ✓$

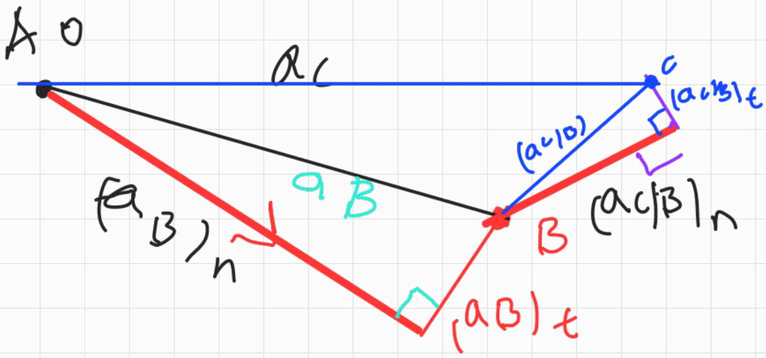
$(a_B)_t = ✓$

$(a_{C/B})_n = ✓$

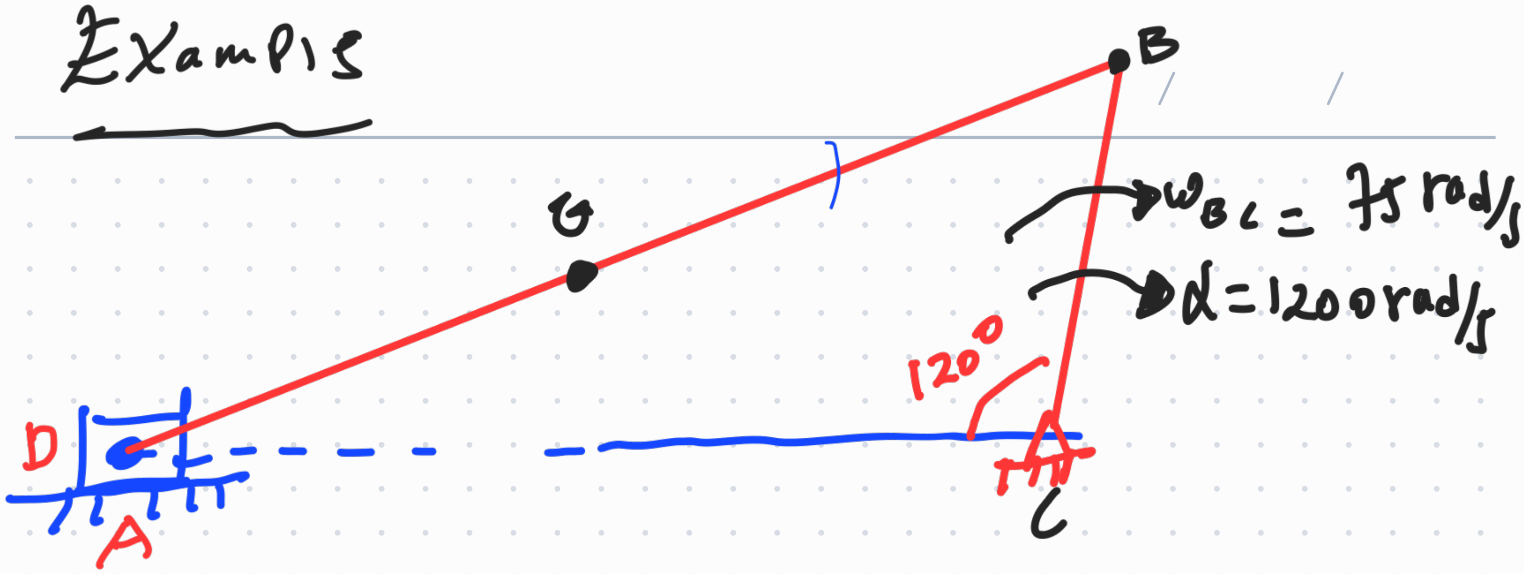
$(a_{C/B})_t = ?$

$(a_C)_t = ?$

$(a_C)_n = 0$



Examples



$AG = 20cm$

$GB = 10cm$

$BC = 10$ cm

In the position shown DIn

$V_B = ?$

$V_D = ?$

$V_{D/B} = ?$, $\omega_{D/B}$

$V_G = ?$

$a_G = ?$

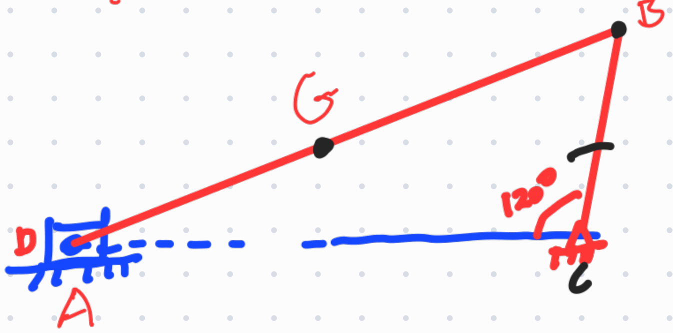
$a_{B/D} = ?$

$\alpha_{BD} = ?$

Step 2 Velocity diagram

Config Scale

1:10

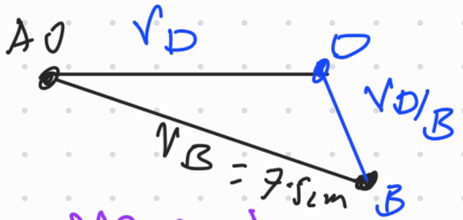


Velocity diagram scale

$V_B = 75 \times 10$

$= 750\ cm/s$

$1\ cm = 100\ cm/s$



Velocity polygon

By measurement and Scaleing up.

$V_{D/B} = 400\ cm/s$

$V_B = 750\ cm/s$

$V_D = 520\ cm/s$

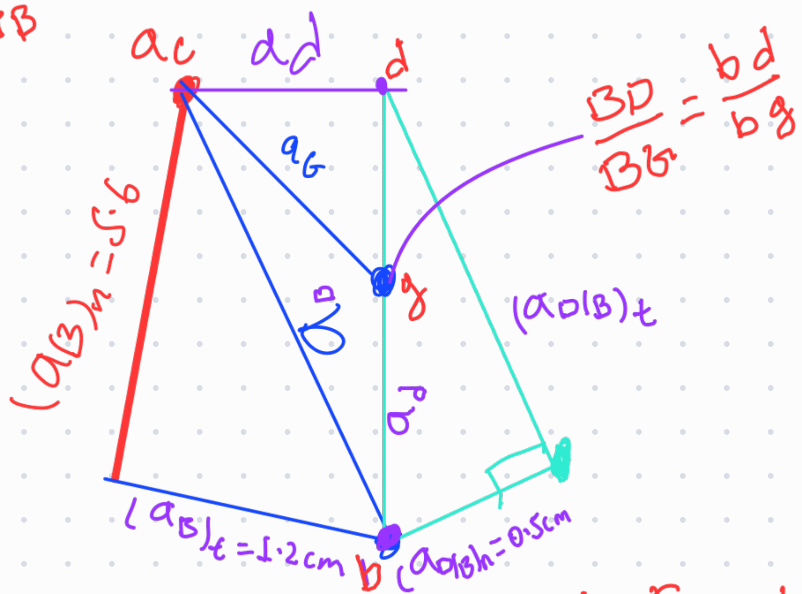
- Acceleration digram

a_B:
- $t = \alpha \times r = 1200 * 10 = 12000\ cm/s^2$
  $\perp n$, α- direction
- $n = \frac{V_B^2}{r} = \frac{(750)^2}{10} = 56250\ cm/s^2$ ↙C

a_D:
- $n = 0$
- $t = ?$ ——— (direction)

$a_{D/B}$:
- $n = \frac{V_{D/B}^2}{r} = 5333.3\ cm/s^2$ ↗B
- $t = ?$ $\perp n$

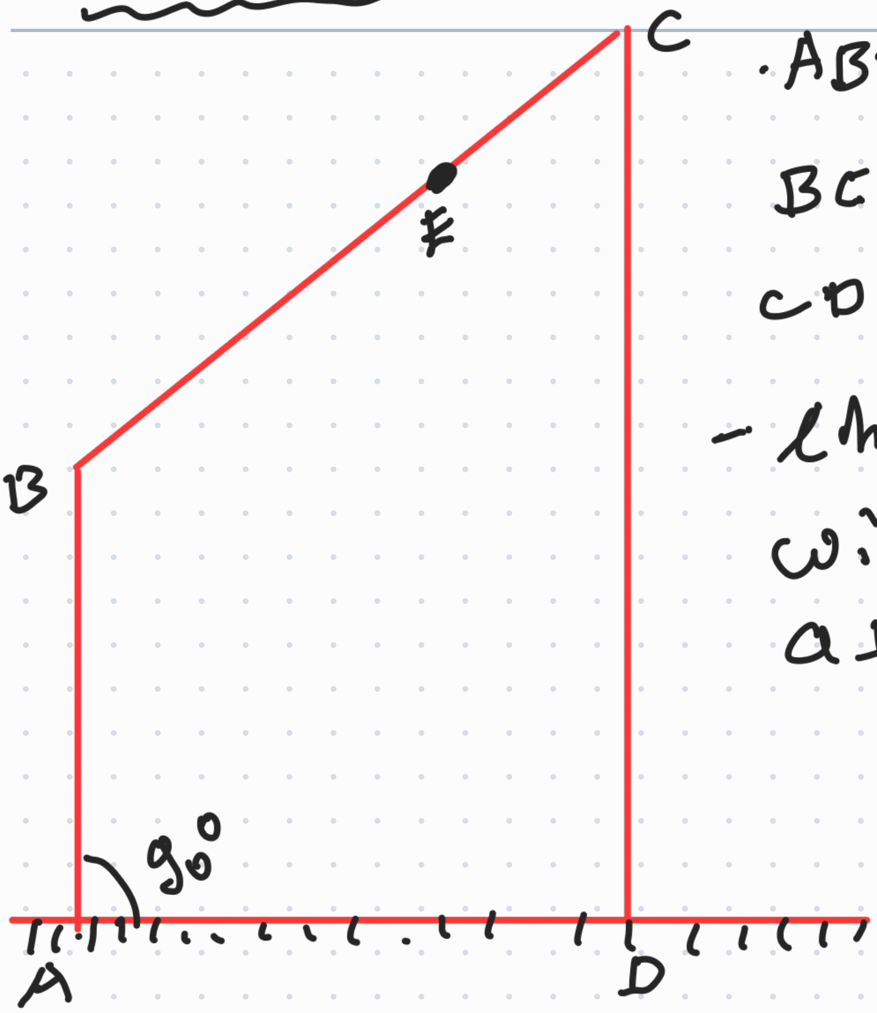
Acc, diagram Scale → $1\ cm = 10000\ cm/s^2$



i.e By measurement the values of $(a_{D/B})_t$, $(a_{D/B})$, a_d Can be Found.

- the angular acceleration of α_{BD} is Found using $\alpha_{BD} = \frac{(a_{B/D})_t}{r_{BD}}$

Example



- AB = 7.5 cm
- BC = 17.5 cm , EC = 5 cm
- CD = 15 cm , AD = 10 cm
- link AB rotates with uniformly at 120 r.p.m CW

Find – α_{BC} & α_{CD}

– a_E

Exercise 3

The mechanism shown in Figure 7.5 is used in a distribution center to push boxes along a platform and to a loading area. The input link is driven by an electric motor, which, at the instant shown, has a velocity of 25 rad/s and accelerates at a rate of 500 rad/s^2. Knowing that the input link has a length of 250 mm, determine t'

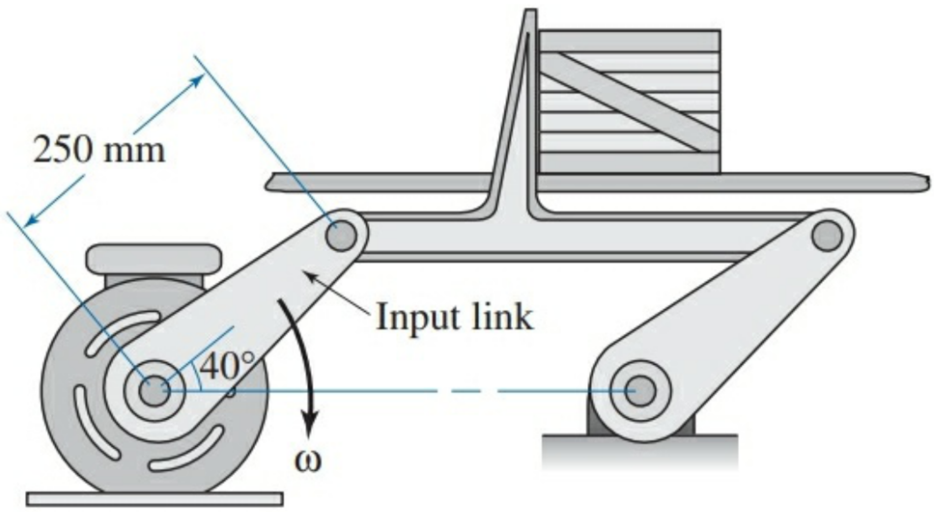


FIGURE 7.5 Transfer mechanism for Example Problem 7.4.

Exercise 4

The mechanism shown in Figure 7.13 is a common punch press designed to perform successive stamping operations. The machine has just been powered and at the instant shown is coming up to full speed. The driveshaft rotates clockwise with an angular velocity of 72 rad/s and accelerates at a rate of 250 rad/s^2. At the instant shown, determine the acceleration of the stamping die, which will strike the workpiece.