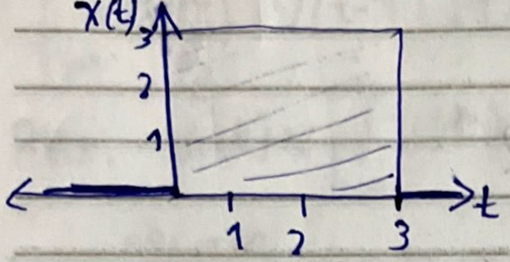
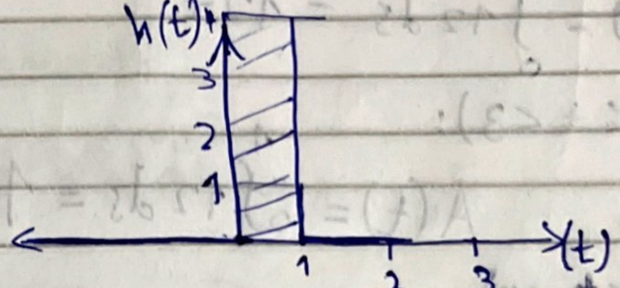
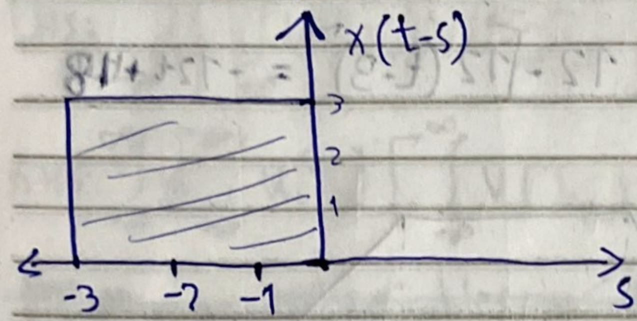
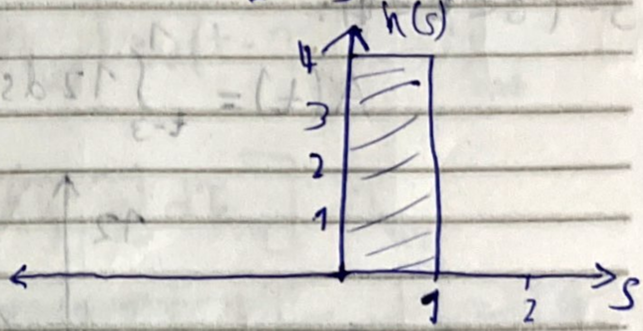
Analytical tut(1)

1. a) :

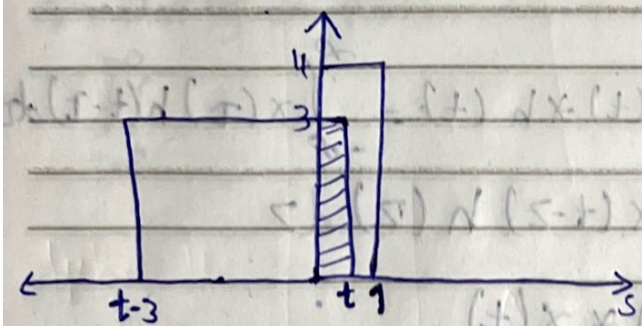


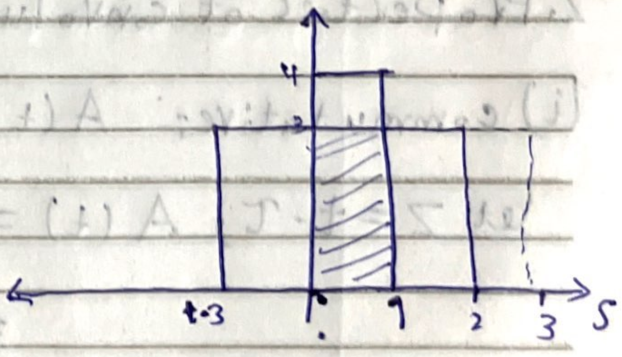


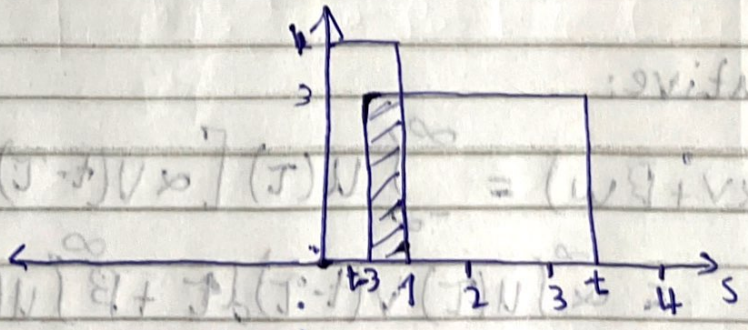




The different intervals: $(0<t<1)$, ~~$t<$~~ $(1<t<3)$ $(3<t<4)$. note that for $t<0$ or $t>4$ there is no overlap and the conv. result is 0.







b) 1- $(0<t<1)$:

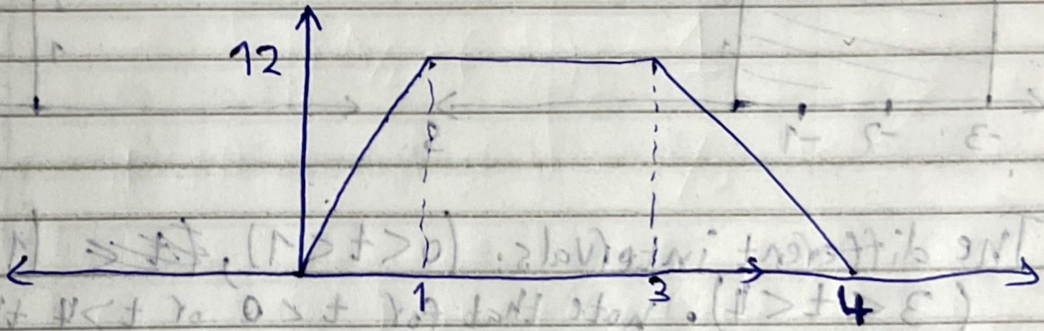
$$A(t) = \int_0^t 12\,ds = 12t$$

2- $(1<t<3)$:

$$A(t) = \int_0^1 12\,ds = 12$$

3- $(3<t<4)$:

$$A(t) = \int_{t-3}^{1} 12\,ds = 12 - [12(t-3)] = -12t+48$$



2. Properties of convolution:

(i) commutative: $A(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

let $z = t-\tau$, $A(t) = \int_{-\infty}^{\infty} x(t-z)\,h(z)\,dz$

$$= h(t) * x(t)$$

ii) Disriputive:

$$u * (\alpha v + \beta w) = \int_{-\infty}^{\infty} u(\tau)\,[\alpha v(t-\tau) + \beta w(t-\tau)]\,d\tau$$

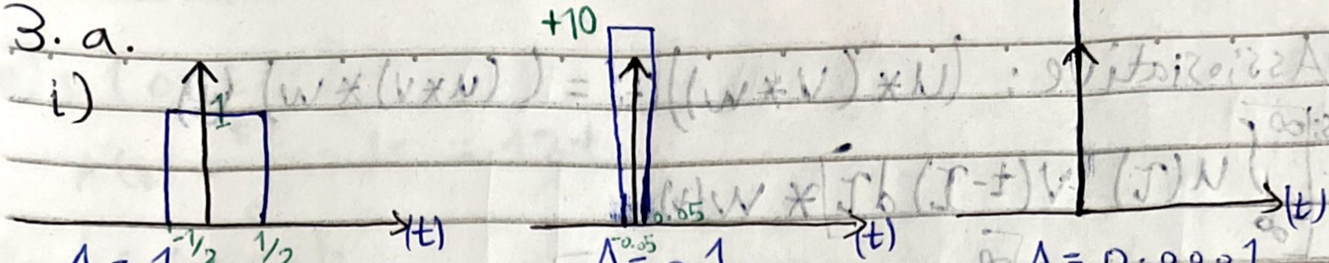
$$= \alpha \int_{-\infty}^{\infty} u(\tau)\,v(t-\tau)\,d\tau + \beta \int_{-\infty}^{\infty} u(\tau)\,w(t-\tau)\,d\tau$$

$$= \alpha (u * v)(t) + \beta (u * w)(t)$$

Date / / Subject

3. a.

i)



$A = 1$ $\qquad$ $A = 0.1$ $\qquad$ $A = 0.0001$

for smaller A, $R_A(t)$ gets narrower in width(t) and taller in amplitude.

ii) $$\int_{-\infty}^{\infty} R_A(t) = \underbrace{\int_{-\infty}^{-\frac{A}{2}} RA(t)\,dt}_{\text{zero}} + \underbrace{\int_{\frac{A}{2}}^{\infty} RA(t)\,dt}_{\text{zero}} + \int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{A}\,dt$$

$$= \frac{1}{A}\left[A/2 - (-A/2)\right] = 1$$

b) i) because it involves integrating $1/0$ which is not defined.

ii) ~~$\int 0\,dt = C$, while $\int$~~ We can view integration as the area under the function in the interval. Since $D(t) = 0$ for all $t \neq 0$, Then the area under it $= 0$.

2. ii) Associative: $\quad u * (v * w)(t) = ((u * v) * w)(t)$

$$\begin{cases} u * (v * w)(t) = \int u(\tau)\,(v * w)(t-\tau)\,d\tau \\ ((u * v) * w)(t) = \int (u * w)(\tau)\,w(t-\tau)\,d\tau \end{cases}$$

- $(u * (v * w)(t) = \int u(\tau) \int v(\sigma)\, w(t - \tau - \sigma)\,d\sigma\,d\tau$

$$\Rightarrow (u * (v * w))(t) = \int u(\tau) \int v(\sigma)\, w(t - \tau - \sigma)\,d\sigma\,dt$$
$$= \int v(\sigma) \int u(\tau)\, w(t - \tau - \sigma)\,d\tau\,d\sigma$$
$$= (u * v) * w)(t)$$

$$\Rightarrow ((u * v) * w)(t) = ((u * v) * w)(t)$$