# AAND - AT - Problemset 2

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$\boxed{1.a}$ :(

$\boxed{1.b}$ :(

$\boxed{1.c}$ :)

- A high cross correlation value at a certain time deley could mean :

  1. The two signals are similar but one of them is delayed (shows a similar but delayed response to the stimulus.

  2. One signal is driving the other, i.e. the activity of one neuron drives the activity of the other neuron with a certain del

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$\boxed{2}$

2. $\int_0^\infty A t e^{-\alpha t} \, dt$

$u = t \quad | \quad dv = e^{-\alpha t} dt$

$du = dt \quad | \quad v = \frac{1}{-\alpha} e^{-\alpha t}$

$$A \int_0^\infty t e^{-\alpha t} dt = \left[ \underbrace{\frac{-t}{\alpha} e^{-\alpha t} \Big|_0^\infty}_{\text{zero}} - \frac{1}{\alpha^2} e^{-\alpha t} \Big|_0^\infty \right] \times A$$

$$= \frac{A}{\alpha^2}$$

$$\frac{A}{\alpha^2} = 1 \rightarrow \boxed{A = \alpha^2}$$

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$\boxed{3.}$ Coefficient of variation of a poisson process (CV P)

$$P_{ISI}(\tau) = \begin{cases} re^{-r\tau} & , \tau \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

$\boxed{3.a}$ $$\int_{-\infty}^{\infty} p(\tau)\,d\tau = \underbrace{\int_{-\infty}^{0} p(\tau)\,d\tau}_{\to 0} + \int_{0}^{\infty} p(\tau)\,d\tau = \int_{0}^{\infty} p(\tau)\,d\tau \quad \ldots (1)$$

$$\Rightarrow \int_0^\infty p(\tau)\,d\tau = \int_0^\infty re^{-r\tau}\,d\tau = -e^{-r\tau}\Big|_0^\infty = -0-(-1) = \boxed{1}$$

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$\boxed{3.b}$ • $$\langle \tau \rangle = \int_{-\infty}^{\infty} p(\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} \tau r e^{-r\tau}\,d\tau$$

$$= r\int_{-\infty}^{\infty} \tau e^{-r\tau}\,d\tau$$

$$\stackrel{(1)}{=} r\int_{0}^{\infty} \tau e^{-r\tau}\,d\tau$$

$$= r\left(\tau \cdot \left(-\frac{e^{-r\tau}}{r}\right)\Big|_0^\infty - \int_0 -\frac{e^{-r\tau}}{r}\right)$$

$uv - \int u'v$

$u = \tau \;,\; v' = e^{-r\tau}$

$u' = 1 \;,\; v = -\frac{e^{-r\tau}}{r}$

$$= r\left(-\tau\frac{e^{-r\tau}}{r}\Big|_0^\infty - \quad + \frac{e^{-r\tau}}{r^2}\Big|_0^\infty\right)$$

$$= r\left(0 - \left(0 - \frac{1}{r^2}\right)\right)$$

$$= \boxed{\frac{1}{r}}$$

$$\bullet \quad \langle \tau^2 \rangle = \int_{-\infty}^{\infty} \tau^2 \, p(\tau) \, d\tau$$

$$= r \int_0^{\infty} \tau^2 \, e^{-r\tau} \, d\tau$$

$$= r\left( \tau^2 \cdot \left(-\frac{e^{-r\tau}}{r}\right)\Big|_0^{\infty} - \left( \int_0^{\infty} 2\tau \cdot \left(-\frac{e^{-r\tau}}{r}\right) \right)\right)$$

$$= r\left( \left(-\tau^2 \frac{e^{-r\tau}}{r}\right)\Big|_0^{\infty} - \left( 2\tau\left(\frac{e^{-r\tau}}{r^2}\right)\Big|_0^{\infty} - \int 2\frac{e^{-r\tau}}{r^2} \right)\right)$$

$$= r\left( \left(-\tau^2 \frac{e^{-r\tau}}{r}\right)\Big|_0^{\infty} - \left( 2\tau\left(\frac{e^{-r\tau}}{r^2}\right)\Big|_0^{\infty} - \left(-2\frac{e^{-r\tau}}{r^3}\right)\Big|_0^{\infty} \right)\right)$$

$$= r\left( -\tau^2 \frac{e^{-r\tau}}{r} - 2\tau \frac{e^{-r\tau}}{r^2} - 2\frac{e^{-r\tau}}{r^3} \right)\Big|_0^{\infty}$$

$$= \left( e^{-r\tau} \left( -\tau^2 - \frac{\tau}{r} - \frac{2}{r^2} \right) \right)\Big|_0^{\infty}$$

$$= 0 - 1\left(0 - 0 - \frac{2}{r^2}\right)$$

$$= \boxed{\frac{2}{r^2}}$$

$$uv - \int u'v$$

$$u = \tau^2, \quad v' = e^{-r\tau}$$
$$u' = 2\tau, \quad v = -\frac{e^{-r\tau}}{r}$$

$$u = 2\tau, \quad v' = -\frac{e^{-r\tau}}{r}$$
$$u' = 2, \quad v = \frac{e^{-r\tau}}{r^2}$$

$$\bullet \quad \sigma^2 = \langle \tau^2 \rangle - \langle \tau \rangle^2$$

$$= \frac{2}{r^2} - \left(\frac{1}{r}\right)^2$$

$$= \boxed{\frac{1}{r^2}}$$

3.c $\sigma = \boxed{\frac{1}{r}}$

$$CV = \frac{\sigma_\tau}{\langle \tau \rangle}$$

$$= \left(\frac{1}{r}\right) / \left(\frac{1}{r}\right)$$

$$= \boxed{1}$$

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4.) ~~mean = $\langle n \rangle$ = r~~ the moments can be derived from the moment generating function:

$$g(\alpha) = \sum_{n=0}^{\infty} \frac{(rT)^n \exp(\alpha n)}{n!} \exp(-rT)$$

to be:

$\langle n \rangle = rT$, $\sigma^2 = rT$