# Acquisition and Analysis of Neural Data


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**Problem Set 2** Deadline: Tuesday 2nd May, 2023 at 23:55

Please work in groups of ideally 2 students and submit a single joint solution per group. Include the names of all group members in the files you submit. Please specify how much time you needed to finish each part.
Analytical Part: scan and upload your solution as a single PDF. Make sure all text is clearly readable.
Numerical Part: submit both a Python code file (or notebook) and a PDF or HTML file including figures and code. Use clear, descriptive variable names and comments.

## Analytical Part

### Convolution and coefficient of variation

**1. Convolution and cross-correlation (3 points)**
The operations of convolution and cross-correlation are conceptually similar but have different definitions and interpretations.

The convolution between two real signals $x(t)$ and $y(t)$ is defined as

$$(x * y)(t) = \int_{-\infty}^{\infty} \mathrm{d}s\, x(s) y(t-s)\,.$$

If, for example, $x(t) = \rho(t)$ is the neural response function and $y(t) = w(t)$ is a normalized averaging window, then $(x * y)(t)$ is a time-averaged firing rate.

Similarly, the cross-correlation between two real signals $x(t)$ and $y(t)$ is defined as

$$(x \star y)(t) = \int_{-\infty}^{\infty} \mathrm{d}s\, x(s) y(t+s)\,.$$

The cross-correlation is used as a measure of similarity between two waveforms as a function of a relative time shift $t$ between them.

(a) How are convolution and cross-correlation related to each other? Write the cross-correlation $(x \star y)(t)$ as a convolution $(\tilde{x} * y)(t)$. How is the new function $\tilde{x}$ related to the original function $x$? When are the two operations equivalent? (Hint: use variable substitution).

(b) Consider the functions $x(t) := [(t/t_{\text{peak}})e^{-t/t_{\text{peak}}}]_+$ (alpha function) and $y(t) := \delta(t - t_\delta)$ (Dirac delta). Compute the convolution $(x * y)(t)$ and the cross-correlation $(x \star y)(t)$. Sketch the result.

(c) Suppose you find a pronounced peak in the cross-correlation between two spike trains. How would you interpret this finding?

**2. Normalization of a probability density (2 points)**
A probability density function $p(x)$ is normalized such that $\int_{-\infty}^{\infty} \mathrm{d}x\, p(x) = 1$. The alpha function is often used for filtering. Consider the alpha function $x(t) = [Ate^{-\alpha t}]_+$. Find the scaling factor $A$ such that $\int_{-\infty}^{\infty} \mathrm{d}t\, x(t) = 1$. Hint: Use integration by parts and l'Hôpital's rule.

**3. Coefficient of variation of a Poisson process (4 points)**
The Poisson process is a continuous-time counting process where events are statistically independent from each other. The Poisson process is used for example to model stochastic neuronal firing, the arrival of customers in a queue, or the number of photons hitting a photodetector. A Poisson process with rate $r$ has inter-event intervals $\tau$ that are exponentially distributed:

$$p_{\text{ISI}}(\tau) = \begin{cases} re^{-r\tau} & \text{for} \quad \tau \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a) Show that $p_{\text{ISI}}(\tau)$ is a normalized probability density.

(b) Compute the mean $\langle\tau\rangle$ and the variance $\sigma_\tau^2$ of the inter-event intervals $\tau$.

(c) Compute the coefficient of variation $CV$ of the Poisson process. Remember $CV = \sigma_\tau/\langle\tau\rangle$.

Hint: recall that the mean $\langle x\rangle$, the second moment $\langle x^2\rangle$, and the variance $\sigma_x^2$ of a random variable $x$ are defined as follows:

$$\langle x\rangle := \int_{-\infty}^{\infty} \mathrm{d}x\, x\, p(x) \quad ; \quad \langle x^2\rangle := \int_{-\infty}^{\infty} \mathrm{d}x\, x^2\, p(x) \quad ; \quad \sigma^2 := \langle x^2\rangle - \langle x\rangle^2 \,.$$

**4. Moments of the Poisson Distribution (1 point)**
Go through Appendix B of the first chapter in the Dayan and Abbott book. Make sure you understand the moment estimation of the Poisson distribution.

## Numerical Part

## Trial-Averaged Firing Rate and Spike-Triggered Average

**1. Trial-Averaged Firing Rates (7 points)**
Download SpikeTimes.dat from the Moodle webpage and load it with the numpy function loadtext. The file contains a matrix of spike times in milliseconds. The dimensions are [m x n, where m=100 is the maximum spike-time index and n=100 is the number of trials. Note: each trial has a different number of spikes, the remaining entries are filled with nan. The trial length is $T = 5.5$ s, the resolution is 0.1 ms. It is convenient to convert the data in units of seconds.

(a) Make a raster plot using the pyplot function eventplot. You may need to set the axes limits manually for correct visualization.

(b) Calculate and plot the trial-averaged firing rate using a sliding rectangular window with a width of 30, 90, and 200 ms. Compute the trial-averaged response function first and then convolve it with the rectangular window.

(c) Include different numbers of trials (10, 50, 100). Comment on your results.

**2. Spike-Triggered Average (3 points)**
Download STA_data.mat from the Moodle website and load it with the scipy.io function loadmat. The file contains the stimulus vector and the spikes times of a model neuron that was exposed to the stimulus for 100 trials. The trial length is $T = 1$ s, the time resolution is 0.1 ms.

Calculate the spike-triggered average

$$C(\tau) = \left\langle \frac{1}{n} \sum_{\text{i}=1}^{n} s(t_{\text{i}} - \tau) \right\rangle = \frac{1}{\langle n\rangle} \int_0^{\text{T}} \mathrm{d}t\, \langle \rho(t)\rangle s(t-\tau)$$

where $s(t)$ is the stimulus, $\rho(t)$ is the neural response function, and the angular brackets denote trial averages. Plot the spike-triggered average for $0 \le \tau \le 50$ ms. Ensure that the STA is plotted according to the convention: $\tau$ positive and decreasing from left to right. To compute the STA, you can either loop over the spike times $t_i$ (first expression) or use the numpy function correlate (second expression). Both procedures shall lead to approximately the same result. Which one is more efficient?


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