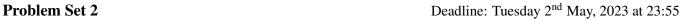
# **Acquisition and Analysis of Neural Data**

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Please work in groups of ideally 2 students and submit a single joint solution per group. Include the names of all group members in the files you submit. Please specify how much time you needed to finish each part.

Analytical Part: scan and upload your solution as a single PDF. Make sure all text is clearly readable.

Numerical Part: submit both a Python code file (or notebook) and a PDF or HTML file including figures and code. Use clear, descriptive variable names and comments.

# **Analytical Part**

### Convolution and coefficient of variation

### 1. Convolution and cross-correlation (3 points)

The operations of convolution and cross-correlation are conceptually similar but have different definitions and interpretations.

The convolution between two real signals x(t) and y(t) is defined as

$$(x*y)(t) = \int_{-\infty}^{\infty} ds \, x(s)y(t-s).$$

If, for example,  $x(t) = \rho(t)$  is the neural response function and y(t) = w(t) is a normalized averaging window, then (x \* y)(t) is a time-averaged firing rate.

Similarly, the cross-correlation between two real signals x(t) and y(t) is defined as

$$(x \star y)(t) = \int_{-\infty}^{\infty} \mathrm{d}s \, x(s) y(t+s) \,.$$

The cross-correlation is used as a measure of similarity between two waveforms as a function of a relative time shift *t* between them.

- (a) How are convolution and cross-correlation related to each other? Write the cross-correlation (x \* y)(t) as a convolution  $(\tilde{x} * y)(t)$ . How is the new function  $\tilde{x}$  related to the original function x? When are the two operations equivalent? (Hint: use variable substitution).
- (b) Consider the functions  $x(t) := [(t/t_{\text{peak}})e^{-t/t_{\text{peak}}}]_+$  (alpha function) and  $y(t) := \delta(t t_{\delta})$  (Dirac delta). Compute the convolution (x \* y)(t) and the cross-correlation (x \* y)(t). Sketch the result.
- (c) Suppose you find a pronounced peak in the cross-correlation between two spike trains. How would you interpret this finding?

### 2. Normalization of a probability density (2 points)

A probability density function p(x) is normalized such that  $\int_{-\infty}^{\infty} dx \ p(x) = 1$ . The alpha function is often used for filtering. Consider the alpha function  $x(t) = [Ate^{-\alpha t}]_+$ . Find the scaling factor A such that  $\int_{-\infty}^{\infty} dt \ x(t) = 1$ . Hint: Use integration by parts and l'Hôpital's rule.

## 3. Coefficient of variation of a Poisson process (4 points)

The Poisson process is a continuous-time counting process where events are statistically independent from each other. The Poisson process is used for example to model stochastic neuronal firing, the arrival of customers in a queue, or the number of photons hitting a photodetector. A Poisson process with rate r has inter-event intervals  $\tau$  that are exponentially distributed:

$$p_{\rm ISI}(\tau) = \begin{cases} re^{-r\tau} & \text{for } \tau \ge 0\\ 0 & \text{otherwise} \end{cases}$$
 (1)



- (a) Show that  $p_{ISI}(\tau)$  is a normalized probability density.
- (b) Compute the mean  $\langle \tau \rangle$  and the variance  $\sigma_{\tau}^2$  of the inter-event intervals  $\tau$ .
- (c) Compute the coefficient of variation CV of the Poisson process. Remember  $CV = \sigma_{\tau}/\langle \tau \rangle$ .

Hint: recall that the mean  $\langle x \rangle$ , the second moment  $\langle x^2 \rangle$ , and the variance  $\sigma_x^2$  of a random variable x are defined as follows:

 $\langle x \rangle := \int_{-\infty}^{\infty} \mathrm{d}x \, x \, p(x) \quad ; \quad \langle x^2 \rangle := \int_{-\infty}^{\infty} \mathrm{d}x \, x^2 \, p(x) \quad ; \quad \sigma^2 := \langle x^2 \rangle - \langle x \rangle^2 \, .$ 

# 4. Moments of the Poisson Distribution (1 point)

Go through Appendix B of the first chapter in the Dayan and Abbott book. Make sure you understand the moment estimation of the Poisson distribution.

#### **Numerical Part**

## Trial-Averaged Firing Rate and Spike-Triggered Average

# 1. Trial-Averaged Firing Rates (7 points)

Download SpikeTimes.dat from the Moodle webpage and load it with the numpy function loadtext. The file contains a matrix of spike times in milliseconds. The dimensions are  $[m \times n]$ , where m=100 is the maximum spike-time index and n=100 is the number of trials. Note: each trial has a different number of spikes, the remaining entries are filled with nan. The trial length is T=5.5 s, the resolution is 0.1 ms. It is convenient to convert the data in units of seconds.

- (a) Make a raster plot using the pyplot function eventplot. You may need to set the axes limits manually for correct visualization.
- (b) Calculate and plot the trial-averaged firing rate using a sliding rectangular window with a width of 30, 90, and 200 ms. Compute the trial-averaged response function first and then convolve it with the rectangular window.
- (c) Include different numbers of trials (10, 50, 100). Comment on your results.

### 2. Spike-Triggered Average (3 points)

Download STA\_data.mat from the Moodle website and load it with the scipy io function loadmat. The file contains the stimulus vector and the spikes times of a model neuron that was exposed to the stimulus for 100 trials. The trial length is T=1 s, the time resolution is 0.1 ms.

Calculate the spike-triggered average

$$C(\tau) = \left\langle \frac{1}{n} \sum_{i=1}^{n} s(t_i - \tau) \right\rangle = \frac{1}{\langle n \rangle} \int_{0}^{T} dt \ \langle \rho(t) \rangle s(t - \tau)$$

where s(t) is the stimulus,  $\rho(t)$  is the neural response function, and the angular brackets denote trial averages. Plot the spike-triggered average for  $0 \le \tau \le 50$  ms. Ensure that the STA is plotted according to the convention:  $\tau$  positive and decreasing from left to right. To compute the STA, you can either loop over the spike times  $t_i$  (first expression) or use the numpy function correlate (second expression). Both procedures shall lead to approximately the same result. Which one is more efficient?

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