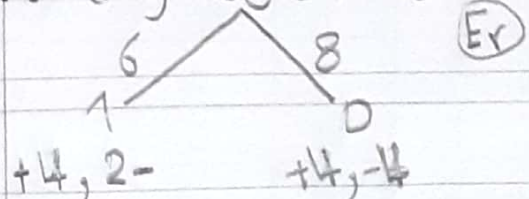
$$IG = Entropy(S) - \sum \frac{S_v}{S} Entropy(S_v)$$

Early registration (Er)

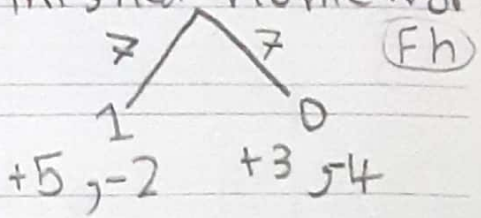


$E(Er-1) = \frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log\frac{2}{6}$

$= 0.92$

$E(Er-0) = 1$

Finished homework (Fh)



$E(fh-1) = 0.86$

$E(fh-0) = 0.98$

$E(S) = \frac{-8}{14}\log_2(8/14) - \frac{6}{14}\log_2(6/14) = 0.98$

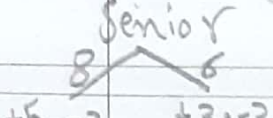
$IG(Er) = 0.98 - \frac{6}{14} \times 0.92 - \frac{8}{14} \times 1$

$= 0.014$

$IG(Fh) = 0.98 - \frac{7}{14} \times 0.86 - \frac{7}{14} \times 0.98$

$= 0.27$

Senior

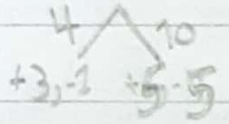


$E(Sen-1) = 0.95$

$E(Sen-0) = 1$

$IG(Senior) = 0.98 - \frac{8}{14} \times 0.95 - \frac{6}{14} \times 1 = 0.0085$

Likes coffe

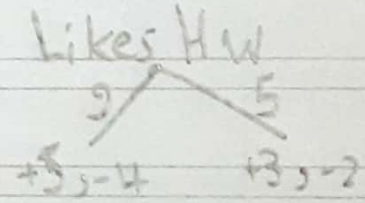


$E(LC-1) = 0.8$

$E(LC-0) = 1$

$IG(LC) = 0.98 - \frac{4}{14} \times 0.8 - \frac{10}{14} \times 1 = 0.037$

Likes HW



$E(LH-1) = 0.99$

$E(LH-0) = 0.97$

$IG(LH) = 0.98 - \frac{2}{14} \times 0.99 - \frac{5}{14} \times 0.97 = 0.0028$

Finished homework has highest IG

$\therefore$ Fh is chosen as our node

Depth 2

FH=1

Er	Senior	LC	LikesHw	A
1	0	0	1	1
1	1	0	1	1
0	1	0	1	0
0	1	0	0	1
0	0	1	1	1
1	1	0	0	1
0	1	1	1	0

Er: 3, 4 (+3 -0, +2 -2)

Sen: 5, 2 (3 -2, +2 -0)

LC: 2, 5 (+1 -1, +4 -1)

LikesHW: 5, 2 (+3 -2, 2 0)

$E(S) = 0.86$

$IG(Er) = 0.86 - \frac{3}{7}0 - \frac{4}{7}x1 = 0.2885$

$IG(sen) = 0.86 - \frac{5}{7}x0.97 - \frac{2}{7}x0 = 0.167$

$IG(LC) = 0.86 - \frac{2}{7}x1 - \frac{5}{7}x0.72 = 0.06$

$IG(LH) = 0.86 - \frac{5}{7}x0.97 - \frac{2}{7}x0 = 0.167$

We choose Er as internal node

FH=0

Er	Senior	LC	LikesHw	A
0	1	0	0	0
0	1	1	1	1
1	0	0	1	0
0	1	0	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1

Er: 3, 4 (+1 -2, 2 -2)

Sen: 3, 4 (2 -1, 1 -3)

LC: 2, 5 (2 0, 1 -4)

LH: 2, 5 (2 -2, 1 -2)

$E(S) = 0.98$

$IG(Er) = 0.98 - \frac{3}{7}x0.918 - \frac{4}{7}x1 = 0.015$

$IG(sen) = 0.98 - \frac{3}{7}x0.918 - \frac{4}{7}0.81 = 0.123$

$IG(LC) = 0.98 - \frac{2}{7}x0 - \frac{5}{7}x0.72 = 0.465$

$IG(LH) = 0.98 - \frac{4}{7} - \frac{3}{7}x0.918 = 0.015$

We choose LC as internal node

b) a CART Descision tree could have been shorter, as CART Algorithm applies pruning.