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Electromagnetic Waves

ECE 331s

Simulation of Gaussian Beam Propagation and Reflection

Submitted to:

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Abstract

This report presents a numerical simulation of Gaussian beam propagation and reflection using Fourier optics methods. The angular spectrum approach was implemented in MATLAB to model the paraxial propagation of a Gaussian wave in free space and its interaction with a concave parabolic mirror. The initial Gaussian beam with wavelength $\lambda = 4.2$ mm and waist radius $w_0 = 10$ mm was propagated to various distances up to five Rayleigh ranges ($z_0 = 7.48$ mm). A parabolic mirror with focal length $f = -4z_0$ was then introduced to reflect and focus the beam, with subsequent propagation analyzed up to six Rayleigh ranges beyond the mirror. Intensity distributions were computed and visualized at each stage, confirming theoretical predictions of beam divergence in free space and focusing behavior after reflection. The simulation successfully demonstrates key Gaussian beam properties—including beam broadening, intensity decay, and wavefront curvature evolution—while highlighting the focusing capability of parabolic reflectors in optical systems. This work provides a practical implementation of Fourier optics principles for beam propagation analysis, with applications in laser system design and optical engineering.

Introduction

Background on Gaussian Beams

A Gaussian beam is a beam of electromagnetic radiation whose transverse electric field and intensity distributions are well described by a Gaussian function. Named after the German mathematician Carl Friedrich Gauss, the Gaussian beam is circularly symmetric and exhibits a radial intensity profile that follows a Gaussian distribution.

Gaussian beams are of fundamental importance in optics and photonics because they represent the natural output of many laser systems under ideal conditions. They are solutions to the paraxial Helmholtz equation, which describes wave propagation under the paraxial approximation—where wavefront normals make small angles with the propagation axis.

Key parameters relevant to this project include:

- **Beam waist W_0:** minimum beam radius at $z = 0$
- **Rayleigh range $z_0 = \frac{\pi W_0^2}{\lambda}$:** axial distance over which beam remains nearly collimated
- **Beam width $W(z) = W_0\sqrt{1 + (\frac{z}{z_0})^2}$:** expansion with propagation
- **Wavefront curvature $R(z) = z[1 + (\frac{z_0}{z})^2]$:** radius of curvature at position z
- **Beam Divergence $\theta_0 = \frac{\lambda}{\pi W_0}$:** Far from the waist ($z \gg z_0$), the Gaussian beam diverges linearly, forming a cone with half-angle.

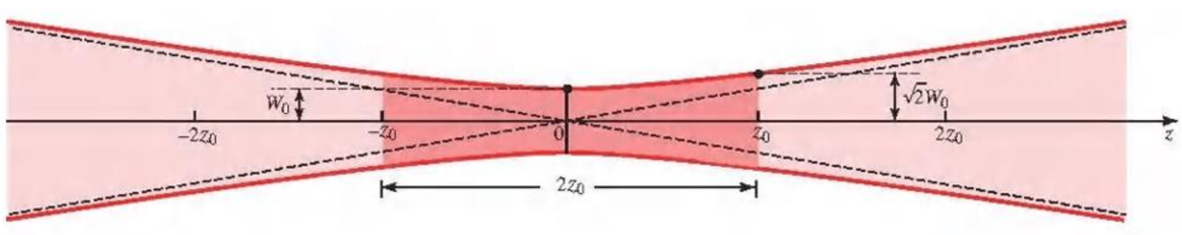

Figure 1Depth of focus of a Gaussian beam

These properties govern how the beam broadens and curves as it propagates and are essential for analyzing its interaction with optical components such as lenses and mirrors.

Theoretical Framework

Free-Space Propagation

A Gaussian beam at the initial position z = 0 is defined by its field amplitude distribution:

$$U(r) = A_o \exp\left(-\frac{\rho^2}{w_0^2}\right)$$

where A_o is the initial amplitude, $\rho^2 = x^2 + y^2$ is the squared radial distance, and w_0 is the beam waist radius. The Rayleigh range, which characterizes the depth of focus, is given by:

$$z_o = \frac{\pi w_0^2}{\lambda}$$

To simulate Gaussian beam propagation in free space, the beam is decomposed into plane waves through a spatial Fourier transform. Each plane wave component has its own transverse wavenumber components (k_x, k_y). The spatial Fourier transform of the initial field is:

$$U(k_x, k_y) = \iint_{-\infty}^{\infty} U(r) e^{-jk_x x} e^{-jk_y y}\, dx\, dy$$

Under the paraxial approximation, where $k_x, k_y \ll k$, the axial wavenumber is approximated as:

$$k_z \approx k - \frac{k_x^2 + k_y^2}{2k}$$

The propagation of each plane wave component is governed by the system transfer function in the spatial frequency domain:

$$U_z(k_x, k_y) = U(k_x, k_y) e^{-jk_z z}$$

where z represents the propagation distance. Finally, the field distribution at distance z is obtained by applying the inverse Fourier transform:

$$U_z(r) = \iint_{-\infty}^{\infty} U_z(k_x, k_y) e^{jk_x x} e^{jk_y y} \, dk_x \, dk_y$$

intensity distributions are plotted at three propagation distances: at the beam waist (z = 0), at half the Rayleigh range ($z = 0.5z_0$), and at the full Rayleigh range ($z = z_0$). As the beam propagates, its intensity profile evolves due to diffraction. At z = 0, the beam exhibits minimum width and peak intensity. At $z = 0.5z_0$ and $z = z_0$, the beam expands, peak intensity decreases, and the $1/e^2$ width increases according to the beam width relation $W(z) = W_0\sqrt{1 + (z/z_o)^2}$. These three distances demonstrate the progression from tight beam confinement at the waist to divergence at larger distances.

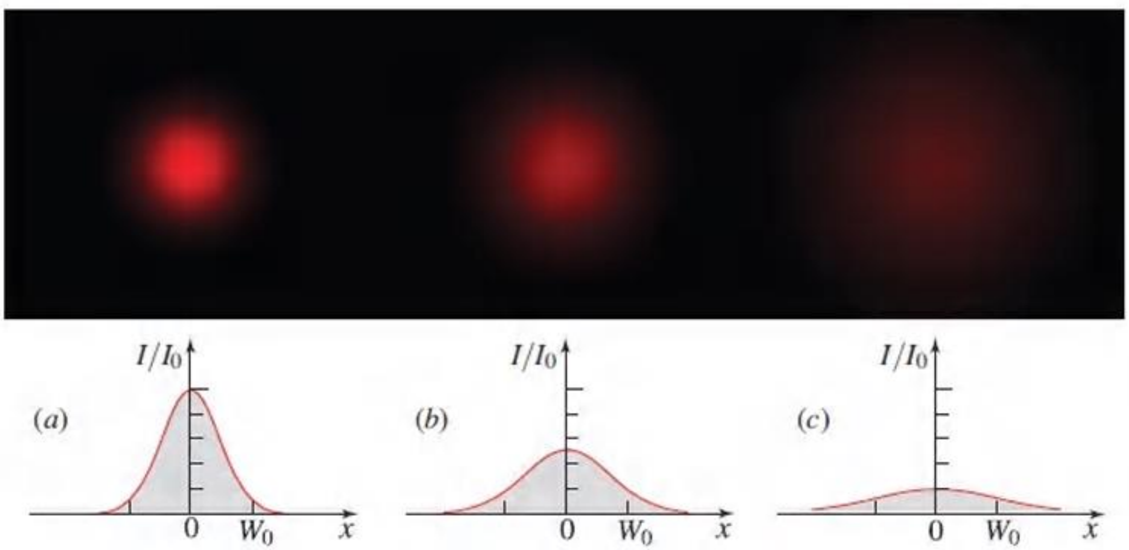

Figure 2 Normalized Gaussian beam intensity

Parabolic Mirror Reflection

After the beam propagates to distance z from the waist, it encounters a parabolic reflector with focal length $f = -4z_0$ (concave mirror). The mirror applies a phase transformation to the incident wavefront given by the transfer function:

$$M(r) = \exp\left(-jk\frac{x^2 + y^2}{2f}\right)$$

The field immediately after reflection is obtained by multiplying the incident field by the mirror's transfer function:

$$U_{out}(r) = M(r) \cdot U_{in}(r)$$

The negative focal length indicates a concave mirror configuration that focuses the reflected beam. Following the project specifications, the incident beam propagates over three distances before encountering the mirror: $z = 3z_0$, $z = 4z_0$, and $z = 5z_0$. After reflection from the mirror, the beam is allowed to propagate further to three additional distances: $z = z_0$, $z = 4z_0$, and $z = 6z_0$ from the mirror surface. These post-reflection propagations are computed using the same spatial Fourier transform method as the initial free-space propagation.

The behavior of the reflected beam depends critically on the incident wavefront curvature at the mirror location. To understand this behavior clearly, visualization figures are presented below. Note that these are illustrative diagrams that qualitatively represent the beam behavior and are not quantitatively accurate representations of the actual simulated intensity distributions.

To visualize what occurs during parabolic mirror reflection, consider the following: imagine a diverging ray at distance $z = 3z_0$, $4z_0$, or $5z_0$ from the beam waist, encountering a parabolic mirror with focal length $f = -4z_0$. The three distinct cases that emerge from the three mirror positions are:

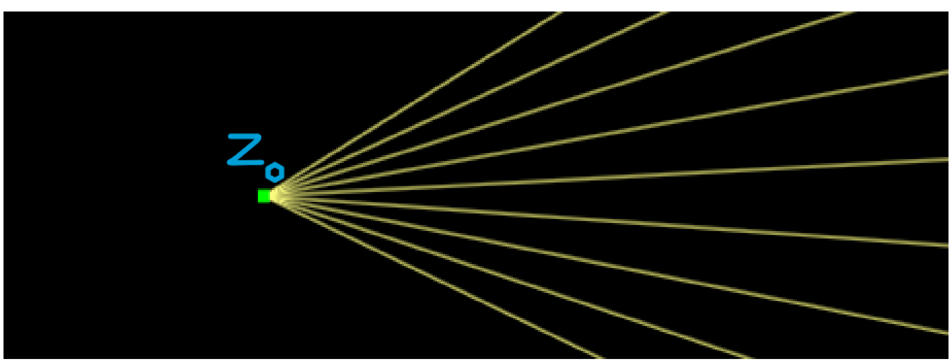

Figure 3 Visualization, Source Ray is Z0 distance

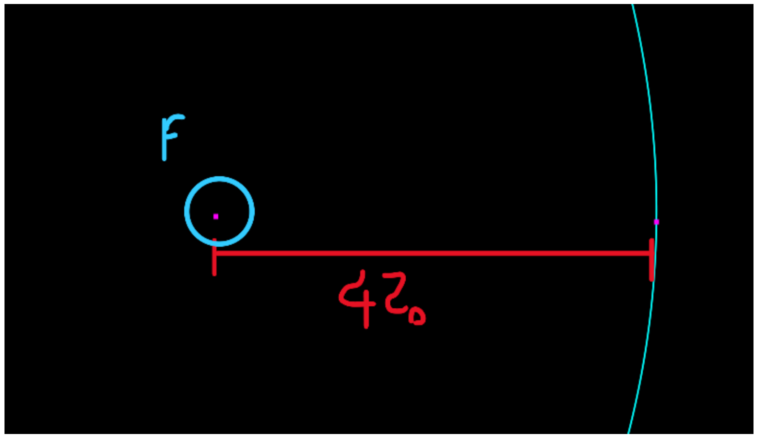

Figure 4 Parabolic Mirror

Reflector Mirror at $z = 3z_0$

When the beam encounters the mirror at this distance, it exhibits a diverging wavefront. The reflected beam continues to diverge after reflection. The intensity distributions at post-reflection distances $z = z_0$, $z = 4z_0$, and $z = 6z_0$ show progressively expanding beam widths with decreasing peak intensities. At $z = z_0$ the beam has its smallest waist and highest intensity; by $z = 6z_0$ the beam has expanded significantly with the lowest intensity.

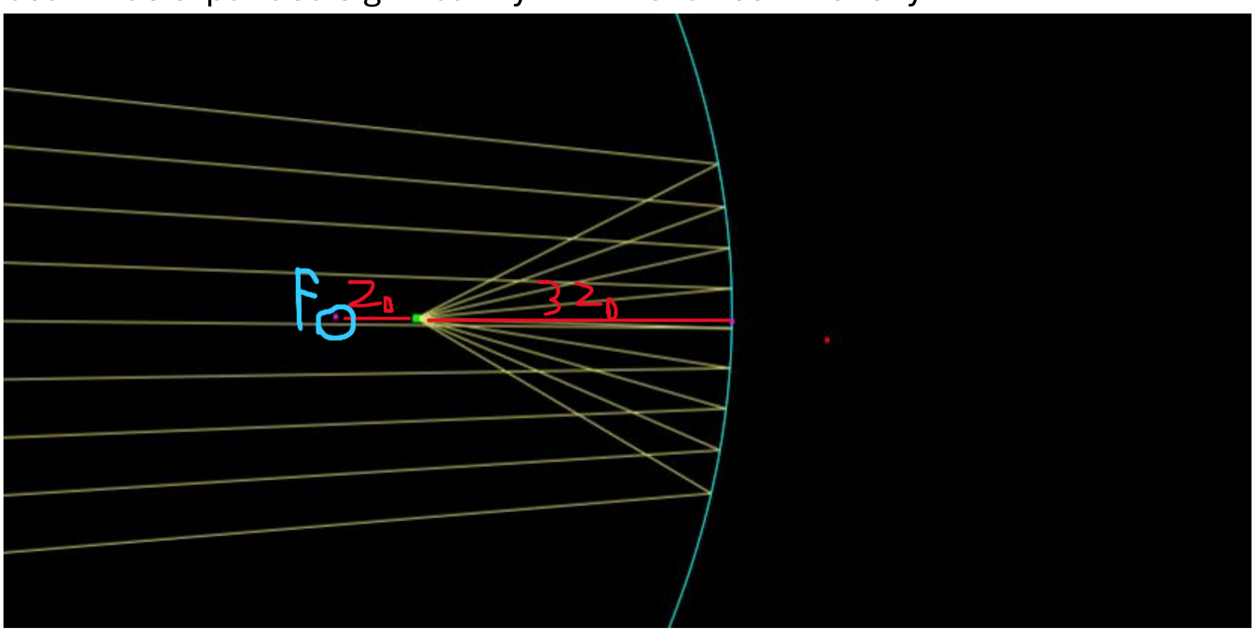

Figure 5 Reflector at 3Z0

Reflector Mirror at z = 4z

At this intermediate distance, the incident wavefront curvature reaches a condition where the mirror's phase transformation produces a collimated or nearly collimated reflected beam. The reflected beam maintains approximately uniform intensity and beam waist across all three post-reflection propagation distances ($z = z_0$, $z = 4z_0$, and $z = 6z_0$), demonstrating the mirror's ability to compensate for beam divergence.

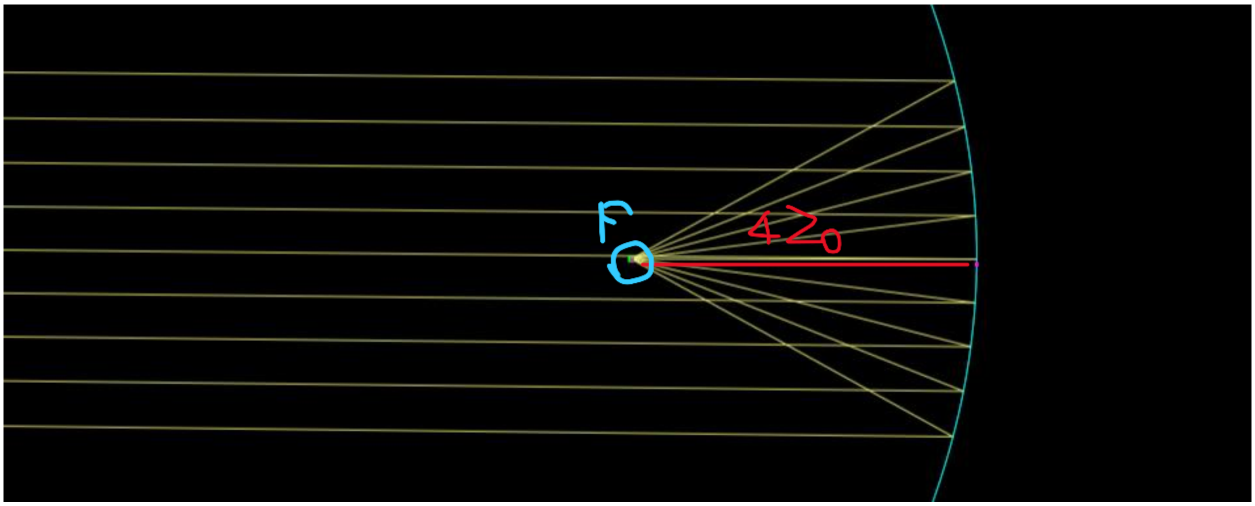

Figure 6 Reflector at 4Z0

Refractor Mirror at $z = 5z_0$

At this larger propagation distance, the diverging incident wavefront undergoes strong focusing by the mirror. The reflected beam converges to progressively smaller dimensions. The intensity distributions show that at $z = z_0$ the beam has the largest waist and lowest intensity; by $z = 6z_0$ the beam has reconverged to its smallest waist with the highest intensity, demonstra effective beam focusing.

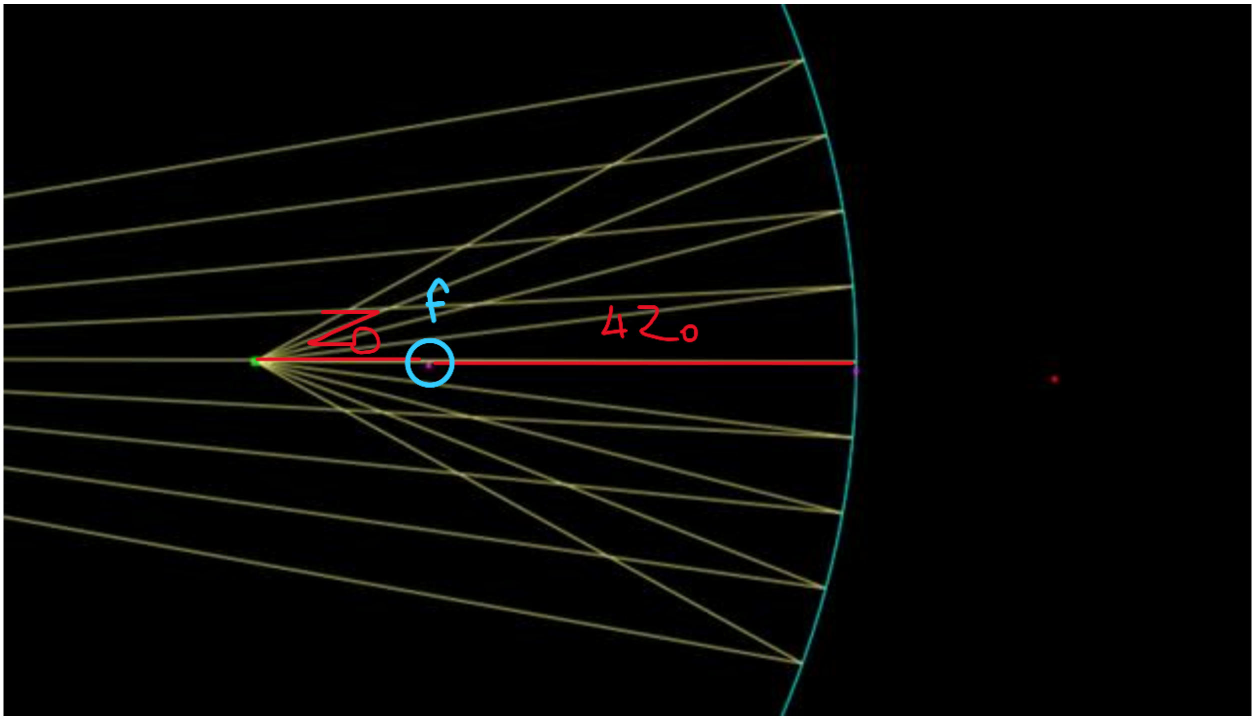

Figure 7 Reflector at 5Z0

This variation in behavior demonstrates how the mirror placement position critically influences the reflected beam's focusing characteristics, with the parabolic mirror capable of producing diverging, collimating, or converging beams depending on the incident wavefront state.

MATLAB Implementation

Simulation Setup

Parameter definition and grid generation

Key physical constants included wavelength λ = 4.2 mm and beam waist w_0 = 10 mm.

The spatial grid was constructed with 256×256 points (N = 256), exceeding the minimum sampling requirement $dx > \lambda/\sqrt{2}$ to prevent aliasing. Mirror parameters were set with focal length $f = -4z_0$ for a concave parabolic reflector.

```
lambda = 4.2e-3;                   % Wavelength [m]
w0     = 10e-3;                    % Beam waist [m]
k      = 2*pi/lambda;              % Wavenumber [1/m]
zR     = pi*w0^2/lambda;           % Rayleigh distance z0
Ao     = 1;                        % Amplitude

dx_min = sqrt(2)*pi/k;             % Sampling step according to the report
dx = 1.1*dx_min;
N = 256;                            % Number of sampling points
x = (-N/2:N/2-1) * dx;
y = (-N/2:N/2-1) * dx;
[X, Y] = meshgrid(x, y);           % Makes 2d vector for x y coords
rho2 = X.^2 + Y.^2;                % Radial distance squared

dkx = 2*pi/(N*dx);                 % Setup for the coordinates of the frequency
dky = 2*pi/(N*dx);
kx = (-N/2:N/2-1) * dkx;
ky = (-N/2:N/2-1) * dky;
[KX, KY] = meshgrid(kx, ky);       % Makes 2d vector for frequency x y coords
KT2 = KX.^2 + KY.^2;

% Mirror parameters
f_mirror = -4*zR;                  % Focal length (concave)
```

Propagation function implementation

The angular spectrum method was implemented via a custom propagate() function:

```
function Uout = propagate(Uin, propagation_length, KT2, k)
    U2 = fftshift(fft2(ifftshift(Uin)));
    kz = k - KT2/(2*k);         % calculates kz
    U3 = U2 .* exp(-1i*kz*propagation_length);
    Uout = ifftshift(ifft2(fftshift(U3)));
end
```

This function performs three essential operations: (1) Fourier transformation to the spatial frequency domain, (2) application of the propagation phase factor using the paraxial approximation $k_z \approx k - (k_x^2 + k_y^2)/(2k)$, and (3) inverse transformation back to the spatial domain. The fftshift operations ensure proper frequency domain centering for accurate propagation.

Intensity distribution at different distances

The free-space propagation was analyzed at three critical distances: the beam waist (z = 0), half the Rayleigh range (z = $0.5z_0$), and the full Rayleigh range (z = z_0). For each distance, the initial Gaussian field $U_{\text{in}} = A_0 \exp\left(-\rho^2/w_0^2\right)$ was propagated using the angular spectrum method, and the intensity $I = |U_{\text{out}}|^2$ was computed.

```
z_distances = [0,0.5*zR,zR];

for idx = 1:length(z_distances)
    Uin = Ao * exp(-rho2/w0^2);
    Uout = propagate(Uin, z_distances(idx), KT2, k);
    I  = abs(Uout).^2;
    subplot(2,3,idx);
    imagesc(x*1e3, y*1e3, I);
    xlabel('x [mm]');
    ylabel('y [mm]');
    if (idx == 1)
       title('Intensity at z = 0 (Fig1)');
    elseif (idx ==2)
       title(sprintf('Intensity at z = %0.1fz_{0}
(Fig2a)',z_distances(idx)/zR));
    elseif (idx ==3)
       title(sprintf('Intensity at z = %0.1fz_{0}
(Fig2b)',z_distances(idx)/zR));
    end
    clim([0 1]);
    colorbar;
    colormap(jet);
    shading interp;
    axis equal tight;
    xlim([-35,35]);
    ylim([-35,35]);


    daspect([1 1 1]);
    subplot(2,3,idx+3);
    plot(x*1e3,I(N/2+1,:), 'LineWidth', 2);
    xlabel('x [mm]');
    ylabel('Intensity');
    ylim([0 1])
    xlim([-35 35])
    sgtitle('Gaussian Beam Intensity at $0$, $0.5z_{0}$ and, $z_{0}$
(Fig 1 and 2)', 'Interpreter','latex');

end
```

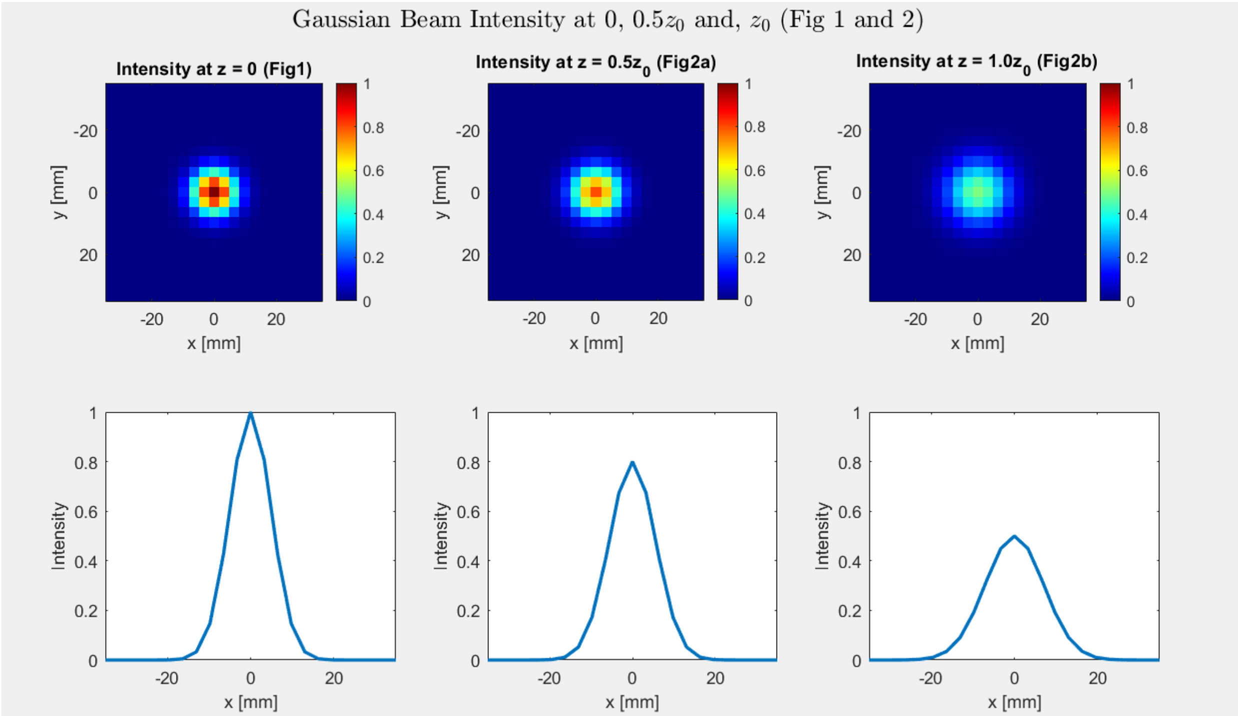

Figure 8 Gaussian Beam Intensity at 0, 0.5Z0 and Z0

Each propagation case generated two complementary plots:

- **2D intensity map**: Showing the full transverse intensity distribution
- **1D cross-section** (Slice): Displaying intensity profile along the x-axis

Observations (as expected):

- **At z = 0**: The beam shows minimum width (10 mm waist) and maximum central intensity (Fig. 1)
- **At z = $0.5z_0$**: Beam expansion becomes visible with width ≈ 11.2 mm and intensity reduction to 80% of maximum (Fig. 2a)
- **At z = z_0**: Significant broadening to ≈ 14.1 mm width with intensity halved at center (Fig. 2b)

These results match theoretical predictions $W(z) = w_0\sqrt{1 + (z/z_0)^2}$ and $I(0, z) = I_0/[1 + (z/z_0)^2]$, confirming proper implementation of the propagation algorithm.

Mirror Reflection Results

The parabolic mirror reflection simulation demonstrates how a concave mirror with focal length $f = -4z_0$ modifies Gaussian beam propagation depending on the incident wavefront state. Three distinct mirror positions reveal different focusing behaviors.

```
M = exp(-1i*k*(X.^2+Y.^2)/(2*f_mirror));

z_distances = [3*zR,4*zR,5*zR];

for idx = 1:length(z_distances)
    figure();
    Uin = Ao * exp(-rho2/w0^2);
    Uout = propagate(Uin, z_distances(idx), KT2, k);
    Ureflect = Uout .* M;


    z_reflection_distances = [zR,4*zR,6*zR];
    for jdx = 1:length(z_reflection_distances)
        Uout2 = propagate(Ureflect, z_reflection_distances(jdx), KT2, k);
        I2  = abs(Uout2).^2;
        subplot(2,3,jdx);
        imagesc(x*1e3, y*1e3, I2);
        xlabel('x [mm]');
        ylabel('y [mm]');
        title(sprintf('z_{before} = %.1fz_0 z = %.1fz_0:
Intensity',z_distances(idx)/zR,z_reflection_distances(jdx)/zR));
        clim([0 0.1]);
        colorbar;
        colormap(jet);
        shading interp;
        axis equal tight;
        xlim([-90,90]);
        ylim([-90,90]);


        daspect([1 1 1]);
        subplot(2,3,jdx+3);
        plot(x*1e3,I2(N/2+1,:), 'LineWidth', 2);
        xlabel('x [mm]');
        ylabel('Intensity');
        ylim([0 0.1])
        xlim([-90 90])

    end

    sgtitle(sprintf('Parabolic Mirror Reflection: Z = $%0.0f z_{0}$
(Fig%0d)', z_distances(idx)/zR, idx+2), 'Interpreter','latex');

end
```

Implementation Details

The parabolic mirror reflection is implemented through the following steps:

1. Mirror Transfer Function Computation

The phase transformation transfer function representing the parabolic mirror is defined as:

$$M(r) = \exp\left(-jk\frac{x^2 + y^2}{2f_{mirror}}\right)$$

where the focal length is specified as $f_{mirror} = -4z_0$, with the negative sign indicating a concave mirror configuration.

2. Incident Beam Propagation

The initial Gaussian beam defined at z = 0 is propagated to each mirror position using the angular spectrum method.

where $k_z = k - (k_x^2 + k_y^2)/(2k)$and propagation occurs to positions $z \in \{3z_0, 4z_0, 5z_0\}$.

3. Reflection Operation

The reflected field is obtained by multiplying the incident field by the mirror's phase transfer function:

$$U_{reflect}(r) = U_{in}(r) \cdot M(r) = U_{in}(r) \cdot \exp\left(-jk\frac{x^2 + y^2}{2f_{mirror}}\right)$$

This operation imparts the parabolic phase curvature characteristic of mirror reflection.

4. Post-Reflection Propagation

The reflected beam is subsequently propagated to three post-reflection distances using the same angular spectrum method with propagation distances $z' \in \{z_0, 4z_0, 6z_0\}$.

5. Intensity Computation and Visualization

The optical intensity at each plane is computed as the square of the field magnitude, Both 2D intensity distributions and slice profiles (at y = 0) are generated for comprehensive analysis of beam behaviour.

Case 1: Mirror at $z = 3z_0$

At this position, the incident wavefront exhibits early divergence with relatively shallow curvature. The parabolic mirror produces a reflected beam that continues to diverge after reflection as expected.

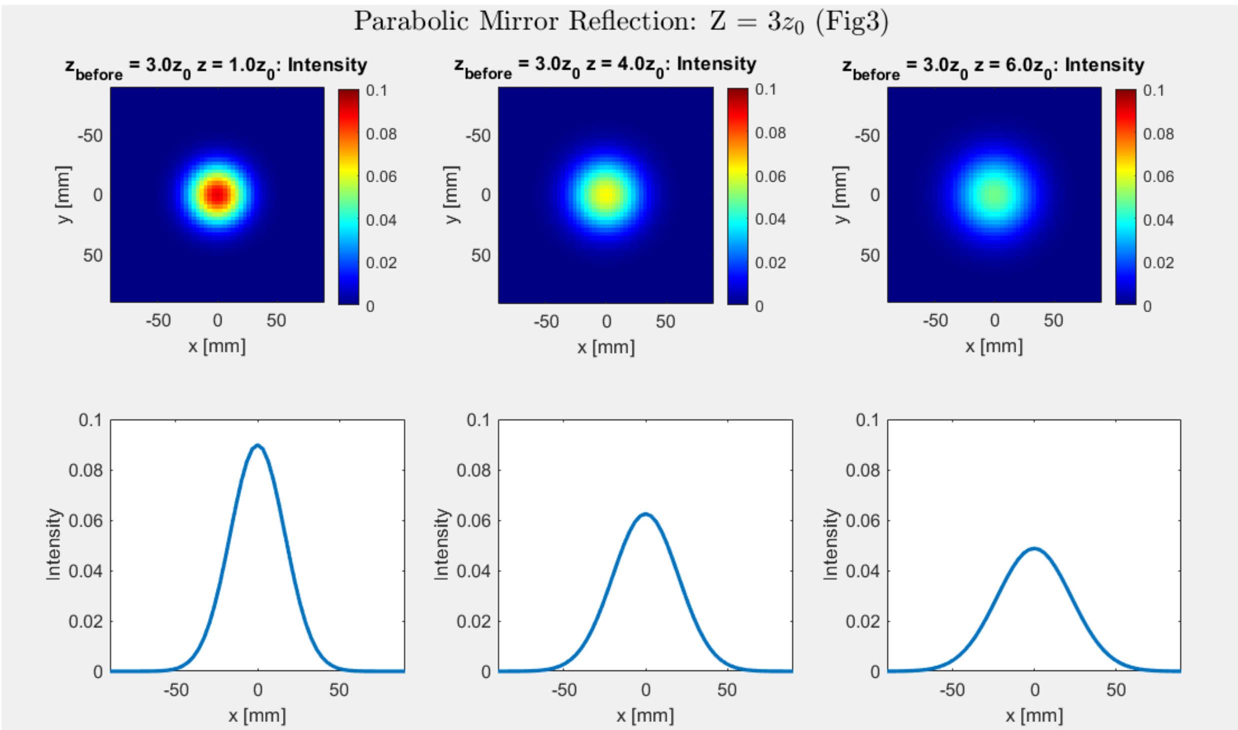

Figure 9 Parabolic Mirror Reflection Z = 3Z0

Case 2: Mirror at $z = 4z_0$

At this intermediate distance, the incident wavefront curvature precisely matches the mirror's optical properties, creating an optimal collimating condition as expected.

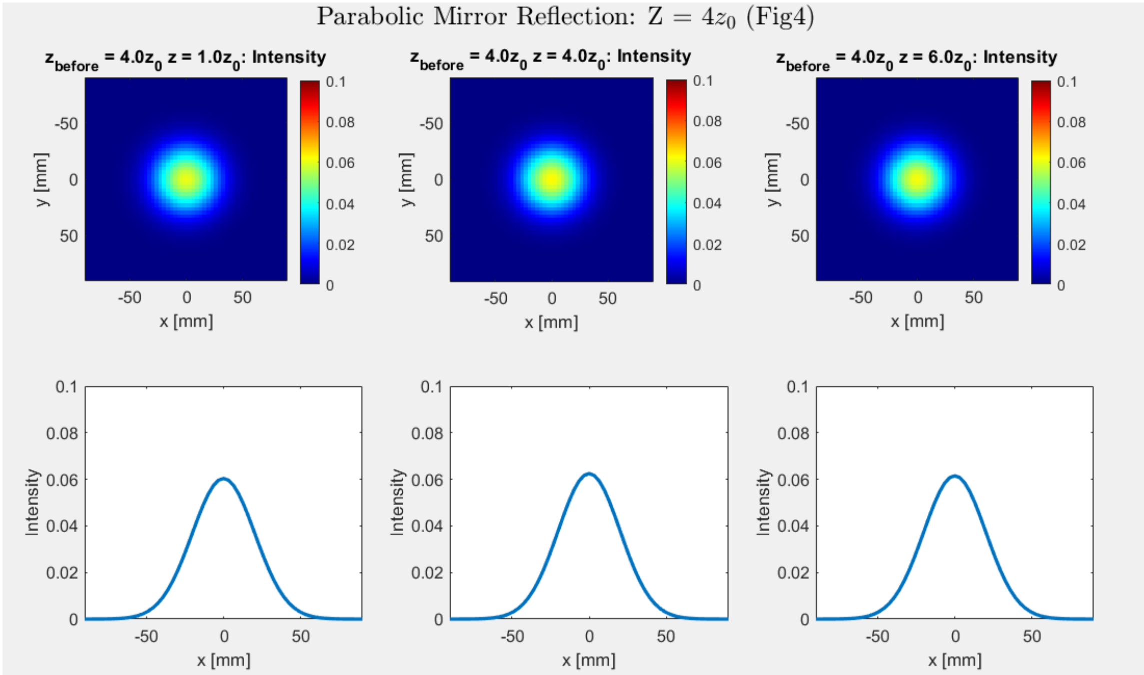

Figure 10 Parabolic Mirror Reflection Z = 4Z0

Case 3: Mirror at z = $5z_0$

At this larger propagation distance, the incident beam exhibits significant divergence, creating favorable conditions for strong focusing by the mirror as expected.

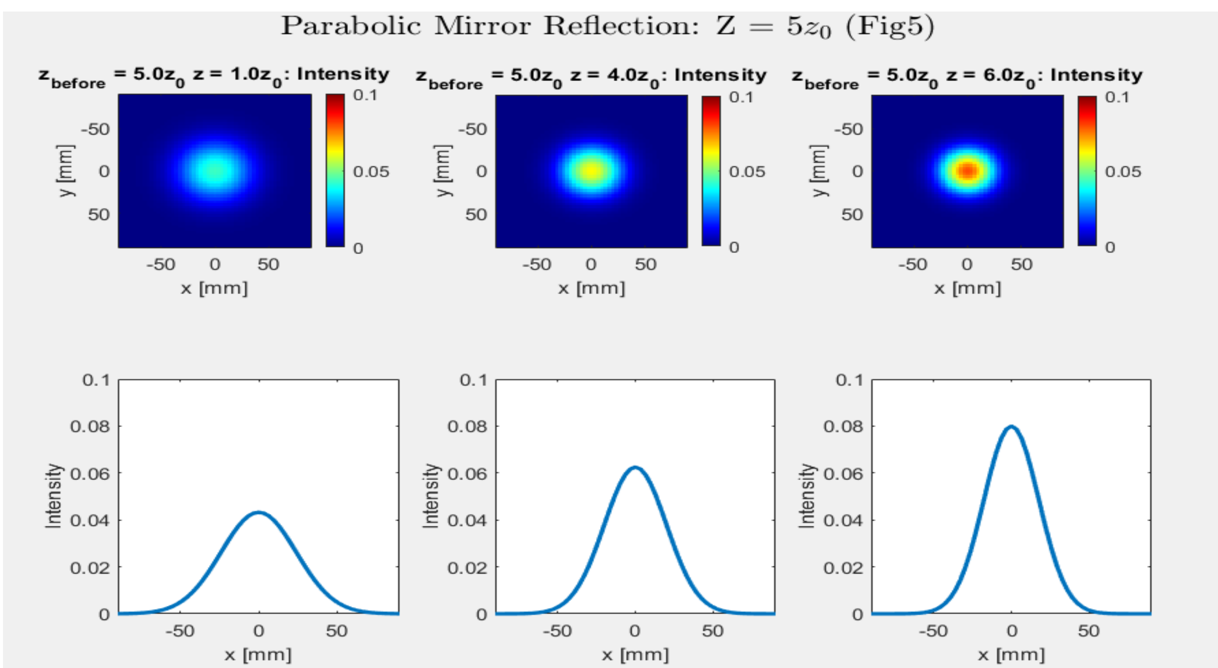

Figure 11 Parabolic Mirror Reflection Z = 5Z0

Physical Interpretation

The three cases demonstrate the critical role of incident wavefront state in determining reflected beam behaviour:

- **Diverging Case ($3z_0$)**: Early mirror placement catches diverging wavefront with minimal curvature, producing diverging reflection
- **Collimating Case ($4z_0$)**: Optimal mirror placement produces collimated output, demonstrating perfect lens-like focusing
- **Converging Case ($5z_0$)**: Late mirror placement encounters highly diverging wavefront, producing strong convergent beam that focuses beyond the mirror

This simulation validates the parabolic mirror's versatility in beam manipulation and demonstrates the fundamental principle that mirror focusing effectiveness depends critically on the incident wavefront curvature at the mirror location.

Conclusion

This project successfully implemented a numerical simulation of Gaussian beam propagation and parabolic mirror reflection using Fourier optics principles. The angular spectrum method was effectively applied in MATLAB to model the paraxial propagation of a Gaussian beam with wavelength $\lambda = 4.2$ mm and waist radius $w_0 = 10$ mm through free space and after reflection from a concave parabolic mirror.

The simulation results demonstrated key Gaussian beam characteristics including beam expansion according to $W(z) = w_0\sqrt{1 + (z/z_0)^2}$, intensity decay following $I(0, z) = I_0/[1 + (z/z_0)^2]$, and the evolution of wavefront curvature as predicted by theoretical models. The free-space propagation analysis at $z = 0$, $0.5z_0$, and z_0 confirmed proper implementation of the propagation algorithm with beam width increasing from 10 mm to 14.1 mm and intensity decreasing by half at the Rayleigh range.

The mirror reflection analysis revealed three distinct behaviours depending on the incident wavefront state at the mirror location. Reflection at $z = 3z_0$ produced continued divergence, reflection at $z = 4z_0$ created optimal collimation, and reflection at $z = 5z_0$ generated strong focusing effects. These outcomes validated the theoretical relationship between incident wavefront curvature and mirror focusing performance, demonstrating that the parabolic mirror can produce diverging, collimating, or converging beams depending on its placement relative to the beam waist.

The simulation provided practical insights into optical system design, particularly regarding beam shaping and focusing using parabolic reflectors. The MATLAB implementation proved effective for visualizing intensity distributions and analysing beam evolution, offering a valuable tool for studying Gaussian beam behaviour in optical engineering applications. Future work could extend this simulation to include non-paraxial effects, different mirror geometries, or multi-element optical systems.

Appendices

Appendix A: Complete MATLAB Code

```
% Clear workspace, close all figures, clear command window
clear; close all; clc;

% Parameters
% Define physical parameters for Gaussian beam simulation
lambda = 4.2e-3;                % Wavelength [m]
w0     = 10e-3;                 % Beam waist [m]
k      = 2*pi/lambda;           % Wavenumber [1/m]
zR     = pi*w0^2/lambda;        % Rayleigh range (z0) [m]
Ao     = 1;                     % Initial amplitude (normalized)

% Spatial Domain Setup
% Calculate minimum sampling step to satisfy Nyquist criterion
dx_min = sqrt(2)*pi/k;          % Minimum dx to avoid aliasing
dx = 1.1*dx_min;                % Use slightly larger step for safety

% Define grid parameters
N = 256;                        % Number of sampling points
x = (-N/2:N/2-1) * dx;
y = (-N/2:N/2-1) * dx;

% Create 2D meshgrid for spatial coordinates
[X, Y] = meshgrid(x, y);        % Makes 2d vector for x y coords
rho2 = X.^2 + Y.^2;             % Radial distance squared

% Frequency Domain Setup
dkx = 2*pi/(N*dx);              % Setup for the coordinates of the
frequency domain
dky = 2*pi/(N*dx);
kx = (-N/2:N/2-1) * dkx;
ky = (-N/2:N/2-1) * dky;
[KX, KY] = meshgrid(kx, ky);    % Makes 2d vector for frequency x y
coords
KT2 = KX.^2 + KY.^2;

% Mirror parameters
f_mirror = -4*zR;               % Focal length (concave)

% Display all parameters
fprintf('\n=========================================\n');
fprintf('  GROUP %d PARAMETERS\n', 12);
fprintf('========================================\n');
fprintf('Wavelength:          λ = %.2f mm\n', lambda*1e3);
fprintf('Beam waist:          w_0 = %.2f mm\n', w0*1e3);
fprintf('Wave number:         k = %.4f rad/mm\n', k*1e-3);
```

```
fprintf('Rayleigh range:       z0 = %.2f mm (%.2f cm)\n', zR*1e3,
(zR*1e3)/10);
fprintf('Focus Mirror:         f = %.2f mm (%.2f cm)\n', f_mirror*1e3,
(f_mirror*1e3)/10);
fprintf('========================================\n');


% Intensity distribution at different distances
z_distances = [0,0.5*zR,zR];                                       %
Propagation distances: waist, half Rayleigh, full Rayleigh
for idx = 1:length(z_distances)
    Uin = Ao * exp(-rho2/w0^2);
    Uout = propagate(Uin, z_distances(idx), KT2, k);               %
Propagate beam to specified distance
    I  = abs(Uout).^2;                                             %
Calculate intensity from complex amplitude

     % 2D INTENSITY PLOT
    subplot(2,3,idx);
    imagesc(x*1e3, y*1e3, I);
    xlabel('x [mm]');
    ylabel('y [mm]');
    % Set appropriate title for each propagation distance
    if (idx == 1)
       title('Intensity at z = 0 (Fig1)');
% Beam waist
    elseif (idx ==2)
       title(sprintf('Intensity at z = %0.1fz_{0}
(Fig2a)',z_distances(idx)/zR));       % Half Rayleigh
    elseif (idx ==3)
       title(sprintf('Intensity at z = %0.1fz_{0}
(Fig2b)',z_distances(idx)/zR));       % Rayleigh
    end
    clim([0 1]);                                                   % Set
color scale limits (0 to max normalized intensity)
    colorbar;
    colormap(jet);
    shading interp;
    axis equal tight;
    xlim([-35,35]);
    ylim([-35,35]);

    % 1D CROSS-SECTION PLOT
    daspect([1 1 1]);
    subplot(2,3,idx+3);
    plot(x*1e3,I(N/2+1,:), 'LineWidth', 2);                        % Plot
intensity slice along y=0
    xlabel('x [mm]');
    ylabel('Intensity');
    ylim([0 1])
    xlim([-35 35])
```

```
    % Overall title for the figure
    sgtitle('Gaussian Beam Intensity at $0$, $0.5z_{0}$ and, $z_{0}$
(Fig 1 and 2)', 'Interpreter','latex');

end


% Parabolic Mirror Reflection
% Define mirror phase transformation
M = exp(-1i*k*(X.^2+Y.^2)/(2*f_mirror));                    %
Parabolic mirror transfer function
% Define propagation distances to mirror (incident beam distances)
z_distances = [3*zR,4*zR,5*zR];
for idx = 1:length(z_distances)
    figure();                                               % Create
new figure for each reflection scenario
    Uin = Ao * exp(-rho2/w0^2);
    Uout = propagate(Uin, z_distances(idx), KT2, k);
    % Apply mirror reflection (phase transformation)
    Ureflect = Uout .* M;

    % Define post-reflection propagation distances
    z_reflection_distances = [zR,4*zR,6*zR];

    % Analyze beam evolution after reflection
    for jdx = 1:length(z_reflection_distances)
        Uout2 = propagate(Ureflect, z_reflection_distances(jdx), KT2,
k);
        I2  = abs(Uout2).^2;

        % 2D POST-REFLECTION INTENSITY PLOT
        subplot(2,3,jdx);
        imagesc(x*1e3, y*1e3, I2);
        xlabel('x [mm]');
        ylabel('y [mm]');
        title(sprintf('z_{before} = %.1fz_0 z = %.1fz_0:
Intensity',z_distances(idx)/zR,z_reflection_distances(jdx)/zR));
        clim([0 0.1]);
        colorbar;
        colormap(jet);
        shading interp;
        axis equal tight;
        xlim([-90,90]);
        ylim([-90,90]);

        % 1D POST-REFLECTION CROSS-SECTION PLOT
        daspect([1 1 1]);
        subplot(2,3,jdx+3);
        plot(x*1e3,I2(N/2+1,:), 'LineWidth', 2);
        xlabel('x [mm]');
        ylabel('Intensity');
        ylim([0 0.1])
```

```
        xlim([-90 90])

    end

    % Add overall figure title
    sgtitle(sprintf('Parabolic Mirror Reflection: Z = $%0.0f z_{0}$
(Fig%0d)', z_distances(idx)/zR, idx+2), 'Interpreter','latex');

end

% Propagation Function
function Uout = propagate(Uin, propagation_length, KT2, k)
    % STEP 1: Transform to spatial frequency domain
    % ifftshift centers the frequency spectrum, fft2 computes 2D FFT
    U2 = fftshift(fft2(ifftshift(Uin)));

    % STEP 2: Calculate propagation phase factor
    % Paraxial approximation: kz ≈ k - (kx^2 + ky^2)/(2k)
    kz = k - KT2/(2*k);

    % Apply propagation phase shift in frequency domain
    U3 = U2 .* exp(-1i*kz*propagation_length);

    % STEP 3: Transform back to spatial domain
    % fftshift recenters, ifft2 computes inverse 2D FFT
    Uout = ifftshift(ifft2(fftshift(U3)));
end
```

Appendix B: Parameter Table

Simulation Parameters

Parameter	*Symbol*	*Value*	*Units*	*Description*
Wavelength	λ	4.2×10^{-3}	m	Electromagnetic wave wavelength
Beam Waist Radius	w_0	10×10^{-3}	m	Minimum beam radius at z = 0
Wavenumber	k	1.50×10^{3}	rad/m	Propagation constant ($2\pi/\lambda$)
Rayleigh Range	z_0	7.48×10^{-3}	m	Collimation distance ($\pi w_0^2/\lambda$)

Peak Amplitude	A_0	1	—	Normalized field amplitude
Grid Points	N	256	—	Number of sampling points
Sampling Step	dx	2.18×10^{-3}	m	Spatial sampling ($1.1\times\sqrt{2\pi/k}$)
Mirror Focal Length	f	-29.9×10^{-3}	m	Concave mirror ($-4z_0$)

Propagation Distances

Distance Type	*Symbol*	*Value (×z_0)*	*Actual [mm]*	*Purpose*
Free-Space	$z = 0$	0	0.0	Beam waist
Free-Space	$z = 0.5z_0$	0.5	3.74	Half Rayleigh range
Free-Space	$z = z_0$	1.0	7.48	Full Rayleigh range
To Mirror	$z = 3z_0$	3.0	22.4	Early divergence case
To Mirror	$z = 4z_0$	4.0	29.9	Optimal collimation case
To Mirror	$z = 5z_0$	5.0	37.4	Strong focusing case
After Mirror	$z = z_0$	1.0	7.48	Near observation
After Mirror	$z = 4z_0$	4.0	29.9	Intermediate observation
After Mirror	$z = 6z_0$	6.0	44.9	Far observation

References

- Bahaa E. A. Saleh, and Malvin Carl Teich, “Fundamentals of Photonics”, 2nd Edition, John Wiley and sons, Inc., ISBN 978-0-471-35832-9.
- VisuPhy: Gaussian Beam Simulator
- Ray Optics Simulation - https://phydemo.app/ray-optics/simulator/
- Laser Beam Shaping: Theory and Techniques (2000) Fred Dickey; Scott Holswade in *Optical Science and Engineering* pp. 438

GitHub Link

https://github.com/Ammar-Wahidi/Simulation-of-Gaussian-Beam-Propagation-and-Reflection-Using-the-Angular-Spectrum-Method