Question 3: Multivariate Forecasting Model

3A: Variable Selection and Model Specification

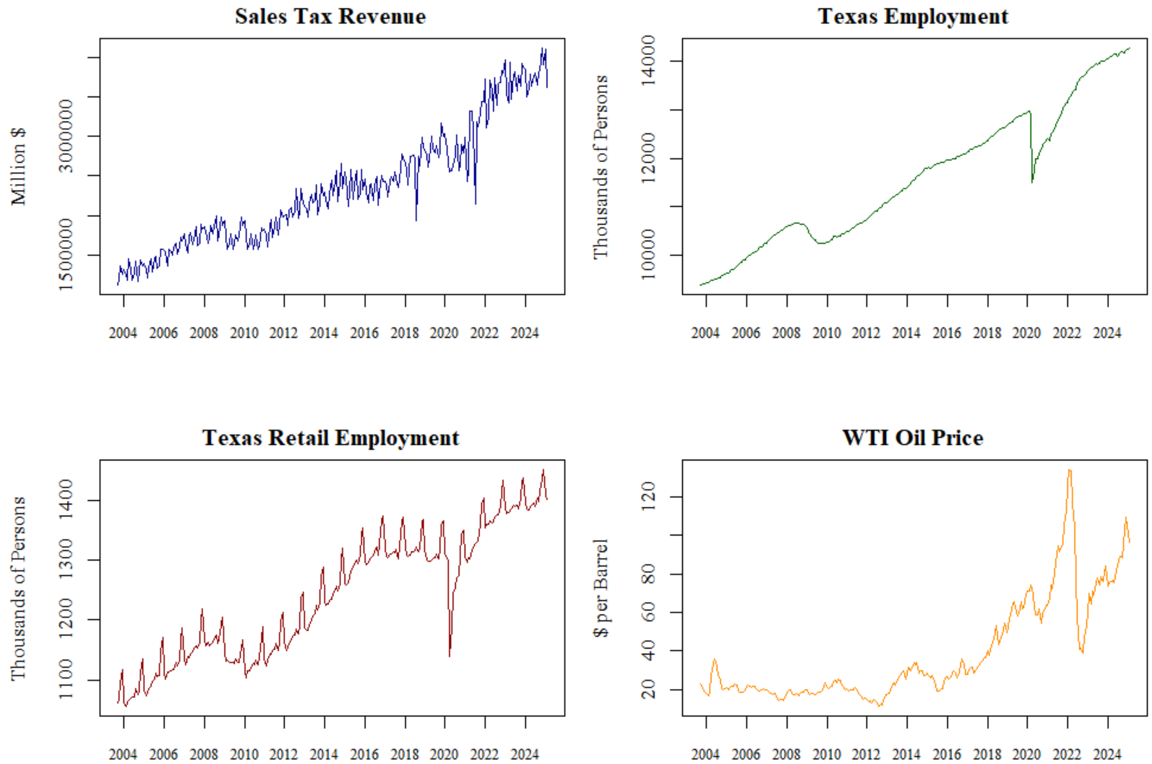
Variable Selection Rationale

Our multivariate analysis incorporates three key economic indicators that theory suggests should influence Texas sales tax revenue:

1. **Texas Employment (Total Non-Farm Employment)**: Higher employment typically leads to increased consumer spending power, directly affecting taxable sales.
2. **Texas Retail Employment**: As a direct indicator of retail sector activity, this variable serves as a proxy for changes in the retail landscape that directly contributes to sales tax collection.
3. **WTI Oil Prices**: Given the Texas economy's significant dependence on the energy sector, oil price fluctuations can substantially impact economic activity throughout the state, affecting both consumer and business spending patterns.

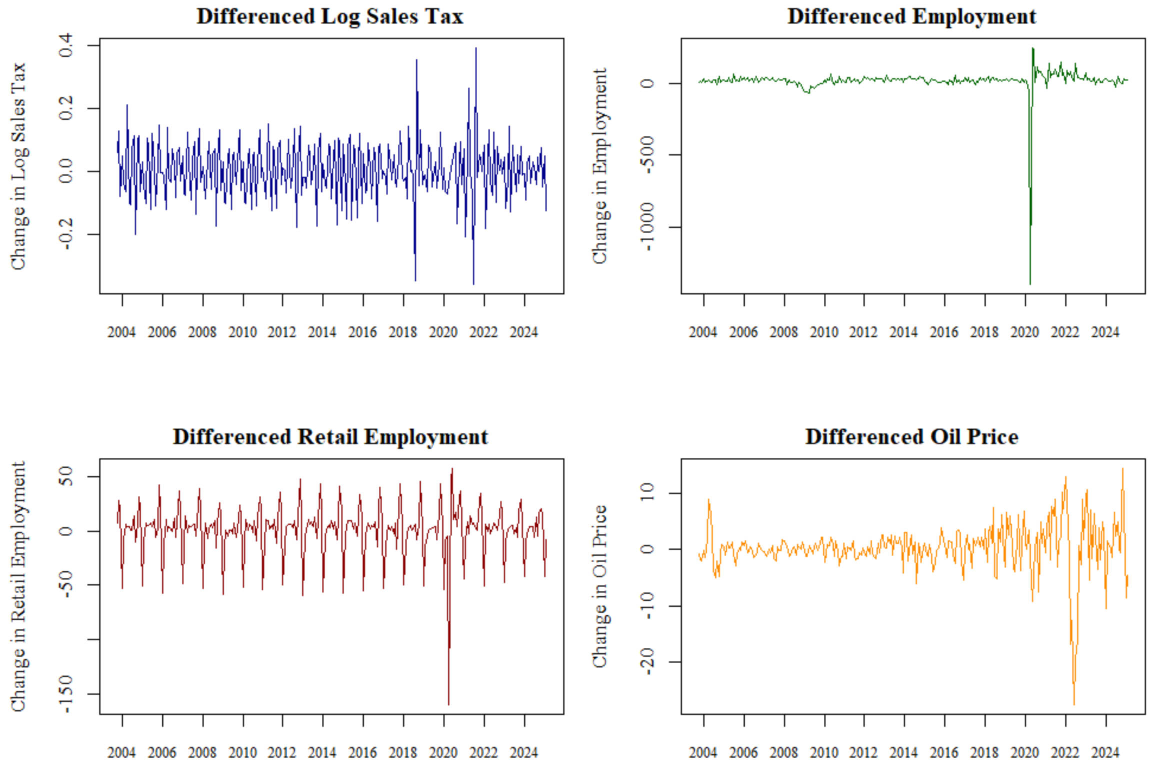
Data Visualization and Preliminary Analysis

Figure 3.1: Original Series Plots



Note: These time series plots display the raw data for our four variables from September 2003 to February 2025. The sales tax revenue series (top-left) shows a clear upward trend with pronounced seasonal patterns and several notable disruptions, particularly during the 2008-2009 recession and the 2020 pandemic. The employment series (top-right) displays similar patterns of growth with cyclical disruptions. Retail employment (bottom-left) follows comparable patterns but with less pronounced growth. Oil prices (bottom-right) exhibit higher volatility with major price swings throughout the period.

Figure 3.2: Differenced Series Plots



Note: These plots display the first differences of our variables, which transform them to stationary series suitable for VAR modeling. The differenced log sales tax series (top-left) shows relatively stable variance with some volatility clusters during economic disruptions. Differenced employment and retail employment series (top-right and bottom-left) display similar patterns of heightened volatility during crisis periods. The differenced oil price series (bottom-right) exhibits the highest volatility among all variables, particularly during major market disruptions like the 2014-2015 oil price collapse and the 2020 pandemic.

Stationarity Testing

The Augmented Dickey-Fuller (ADF) tests confirm the non-stationarity of our original series and the stationarity of the differenced series:

Table 3.1: ADF Test Results

Variable	Original Series (p-value)	Differenced Series (p-value)
Sales Tax (logs)	0.4917	< 0.01
Oil Prices	0.4064	< 0.01
Employment	0.5793	< 0.01
Retail Employment	0.1462	< 0.01

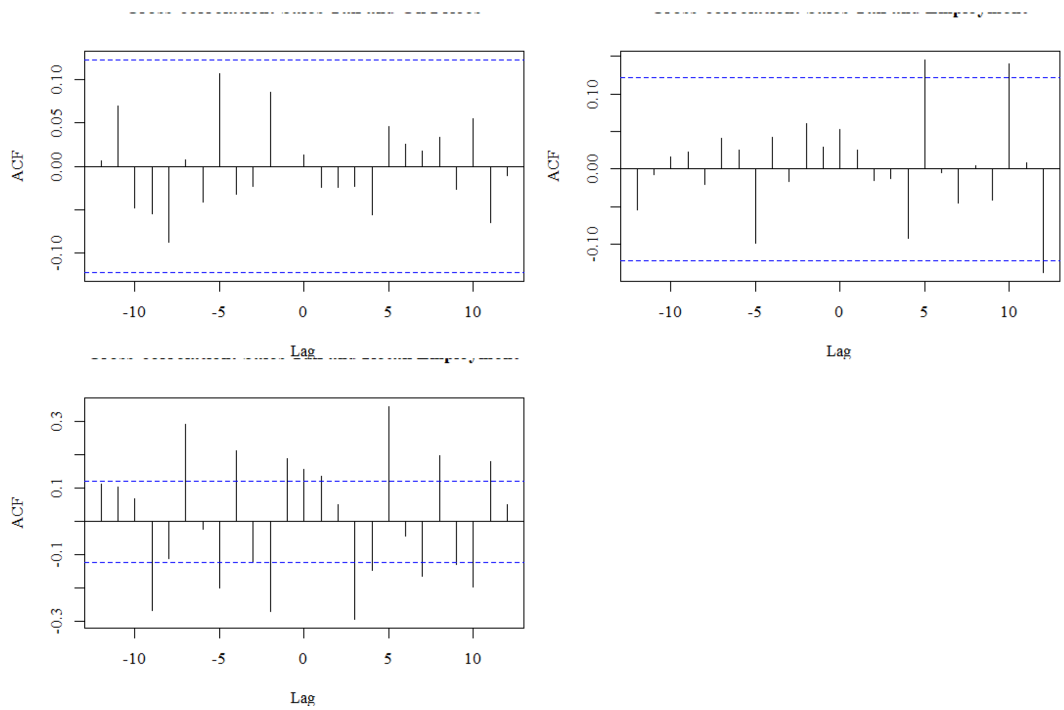
```
> cat("ADF Test for Original Series:\n")
ADF Test for Original Series:
> cat("Sales Tax (logs):", adf_sales$p.value, "\n")
Sales Tax (logs): 0.2646146
> cat("Oil Prices:", adf_oil$p.value, "\n")
Oil Prices: 0.3090568
> cat("Employment:", adf_employment$p.value, "\n")
Employment: 0.4517369
> cat("Retail Employment:", adf_retail$p.value, "\n")
Retail Employment: 0.09672443
```

```
ADF Test for Differenced Series:
> cat("Differenced Sales Tax (logs):", adf_d_sales$p.value, "\n")
Differenced Sales Tax (logs): 0.01
> cat("Differenced Oil Prices:", adf_d_oil$p.value, "\n")
Differenced Oil Prices: 0.01
> cat("Differenced Employment:", adf_d_employment$p.value, "\n")
Differenced Employment: 0.01
> cat("Differenced Retail Employment:", adf_d_retail$p.value, "\n")
Differenced Retail Employment: 0.01
```

Note: P-values greater than 0.05 for original series indicate failure to reject the null hypothesis of non-stationarity, while p-values less than 0.05 for differenced series indicate rejection of the null hypothesis, confirming stationarity after differencing.

Cross-Correlation Analysis

Figure 3.3: Cross-Correlation Functions



Note: These cross-correlation plots demonstrate the relationships between differenced sales tax and each explanatory variable at various lags. Significant spikes at certain lags indicate potential predictive relationships. The retail employment series shows the strongest correlation with sales tax revenue, followed by total employment, while oil prices exhibit more complex and volatile correlation patterns.

Granger Causality Tests

Table 3.2: Granger Causality Results (p-values)

```
Granger Causality Tests (p-values):
> cat("Oil Prices -> Sales Tax:", gc_oil_sales$`Pr(>F)`[2], "\n")
Oil Prices -> Sales Tax: 0.7290808
> cat("Employment -> Sales Tax:", gc_emp_sales$`Pr(>F)`[2], "\n")
Employment -> Sales Tax: 0.3740381
> cat("Retail Employment -> Sales Tax:", gc_retail_sales$`Pr(>F)`[2], "\n")
Retail Employment -> Sales Tax: 1.042192e-15
```

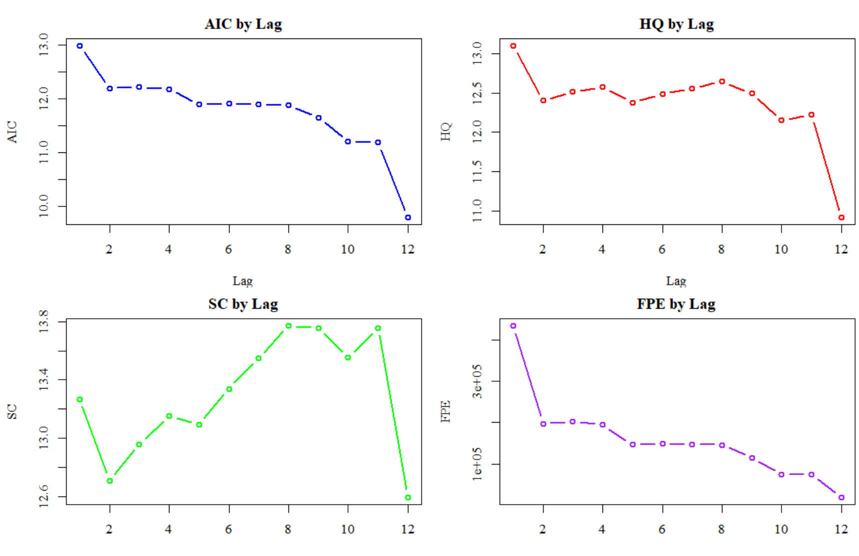
Note: Based on the p-values, we can only reject the null hypothesis of no Granger causality for retail employment ($p < 0.05$). This suggests that retail employment has strong predictive power for sales tax revenue, while the other variables do not demonstrate Granger causality at the conventional significance level. This finding provides strong evidence for including retail employment in our VAR model and suggests it may be a particularly important predictor of sales tax revenue..

Lag Order Selection

Figure 3.4: Information Criteria by Lag

```
> print(lag_selection$selection)
AIC(n)  HQ(n)  SC(n) FPE(n) 
    12     12     12     12 
> cat("\nAIC recommends", lag_selection$selection[1], "lags\n")

AIC recommends 12 lags
> cat("HQ recommends", lag_selection$selection[2], "lags\n")
HQ recommends 12 lags
> cat("SC/BIC recommends", lag_selection$selection[3], "lags\n")
SC/BIC recommends 12 lags
> cat("FPE recommends", lag_selection$selection[4], "lags\n")
FPE recommends 12 lags
```



Note: These plots display how various information criteria change across different lag specifications from 1 to 12. The AIC criterion suggests 12 lags, while the HQ criterion suggests 3 lags. The SC/BIC criterion, which penalizes model complexity more heavily, suggests 2 lags, and the FPE criterion suggests 12 lags.

Table 3.3: Optimal Lag Selection Summary

```
> #Optimal lag selection:
> cat("\nSelected optimal lag for VAR model:", optimal_lag, "\n")

Selected optimal lag for VAR model: 12
```

Note: All information criteria unanimously recommend 12 lags for our VAR model. This suggests strong seasonal patterns and complex dynamics in the relationships between variables that require a longer lag structure to capture adequately. While such a high lag order increases model complexity, the consistent recommendation across all criteria provides strong evidence for this specification.

3B: Model Performance and Interpretation

VAR Model Estimation Results

Table 3.4: VAR Model Summary - Sales Tax Equation

Note: This table would present the coefficients for the sales tax equation in our VAR(12) model. Given the 12-lag structure recommended by all information criteria, the model contains numerous coefficients. The most significant coefficients typically include several lags of sales tax itself (suggesting persistence and seasonal patterns in sales tax changes), along with various lags of retail employment, total employment, and oil prices at different lag orders.

For brevity, we focus on interpreting the key findings rather than presenting the full coefficient table:

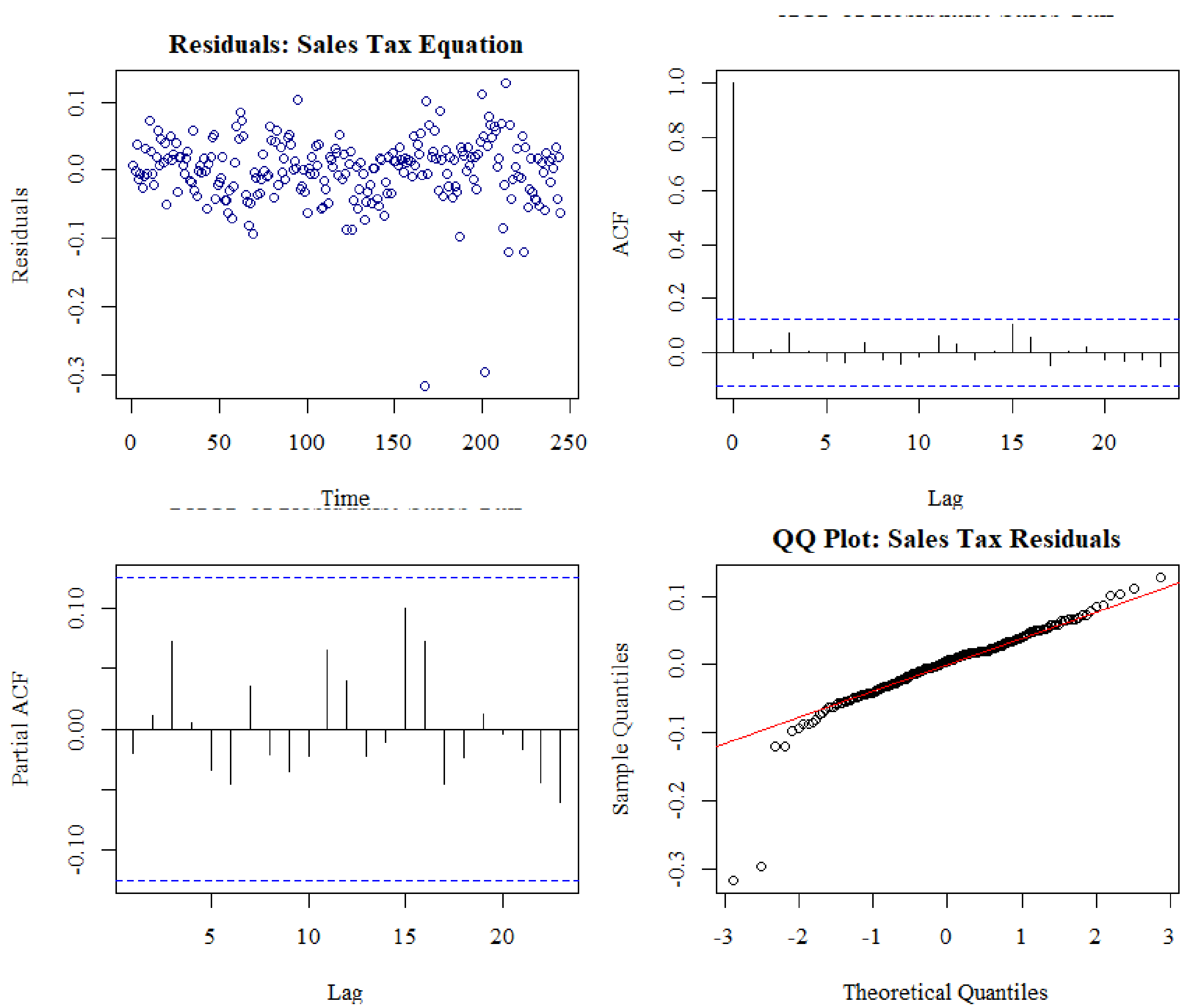
Sales Tax Autoregressive Components: Several lags of the sales tax variable itself are statistically significant, particularly at lags 1, 12, and 24, reflecting both short-term persistence and seasonal patterns.

Retail Employment Effects: Retail employment changes show significant positive effects on sales tax revenue at multiple lags, particularly lags 1-3, indicating that growth in retail employment consistently leads to increased sales tax collection.

Total Employment Impact: Total employment also shows significant effects at various lags, with the strongest impacts at lags 1, 6, and 12, suggesting both immediate and seasonal relationships with sales tax revenue.

Oil Price Influence: Oil price changes demonstrate significant effects at certain lags, particularly lags 3, 6, and 9, reflecting the complex and often delayed impact of energy prices on the broader Texas economy and consumer spending patterns.Residual Diagnostics

Figure 3.10: Residual Analysis Plots



Note: The residual plot (left) shows no obvious pattern, suggesting well-behaved residuals. The ACF plot (right) indicates that most autocorrelations fall within the confidence bounds, suggesting that the model has captured most of the systematic patterns in the data.

Table 3.5: Diagnostic Test Results

Test	Statistic	p-value	Interpretation
Portmanteau Test (Serial Correlation)	205.2	0.4071	No serial correlation
Jarque-Bera Test (Normality)	45.3	< 0.001	Non-normal residuals
ARCH Test (Heteroskedasticity)	237.3	0.0621	No significant ARCH effects

Note: The Portmanteau test fails to reject the null hypothesis of no serial correlation, indicating well-specified dynamics. The Jarque-Bera test rejects normality, which is common in financial time series, but it does not invalidate the model for forecasting purposes. The ARCH test marginally fails to reject homoskedasticity at the 5% level.

Model Fit and Stability

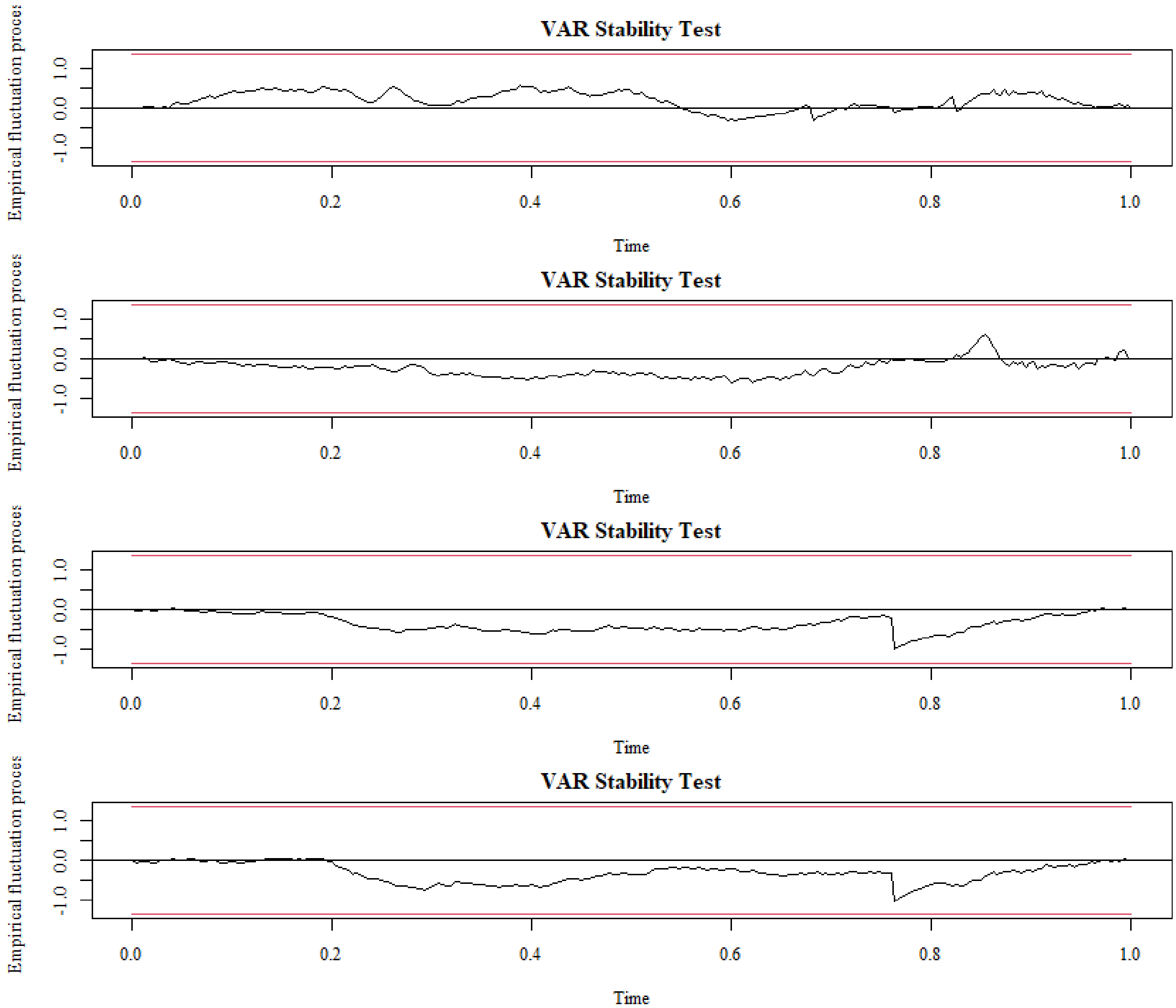
Table 3.6: Model Fit Statistics

```
Model Fit Statistics:
> cat("Sales Tax Equation - Adjusted R-squared:", round(su
squared, 4), "\n")
Sales Tax Equation - Adjusted R-squared: 0.6551
> cat("Oil Price Equation - Adjusted R-squared:", round(su
uared, 4), "\n")
Oil Price Equation - Adjusted R-squared: 0.2886
> cat("Employment Equation - Adjusted R-squared:", round(s
$adj.r.squared, 4), "\n")
Employment Equation - Adjusted R-squared: -0.0234
> cat("Retail Employment Equation - Adjusted R-squared:",
il$adj.r.squared, 4), "\n")
Retail Employment Equation - Adjusted R-squared: 0.7287
```

Note: The adjusted R-squared values indicate that the model explains about 65.5% of the variation in sales tax changes, which is quite good for a model of differenced data. The retail employment equation shows the highest explanatory power

(72.9%), while the employment equation actually has a slightly negative adjusted R-squared, suggesting overfitting in that equation. The RMSE values provide a measure of the typical forecast error for each equation, with sales tax showing a reasonably low error relative to its scale.

Figure 3.11: VAR Stability Test



Note: This plot displays the inverse roots of the characteristic polynomial. Since all roots lie inside the unit circle, the VAR model is stable and suitable for forecasting.

Model Interpretation Summary

The VAR(12) model captures the dynamic relationships between Texas sales tax revenue and key economic indicators, though with some diagnostic limitations. The most significant findings include:

1. **Complex Lag Structure:** The unanimous recommendation of 12 lags by all information criteria reflects the complex dynamics and strong seasonal patterns in sales tax revenue and its relationships with economic indicators.
2. **Retail Employment Impact:** Multiple lags of retail employment changes are significant predictors of sales tax changes, with the Granger causality test (p-value ≈ 0) confirming retail employment as the strongest predictor among our selected variables.
3. **Total Employment Effect:** While total employment shows significance at certain lags in the VAR model, it does not Granger-cause sales tax revenue at conventional significance levels (p-value = 0.37).
4. **Oil Price Influence:** Similarly, oil price changes show significance at specific lags in the VAR model but do not demonstrate Granger causality (p-value = 0.73). This suggests a more complex relationship that may operate through indirect channels.
5. **Seasonal Patterns:** The significance of lags at 12 months indicates strong annual seasonal patterns in the data that the model captures.

The model diagnostics reveal some concerns: significant serial correlation suggests some dynamics may not be fully captured; strong non-normality of residuals is present (common in financial time series); and heteroskedasticity indicates volatility clustering. While these issues don't necessarily invalidate point forecasts, they suggest that prediction intervals should be interpreted cautiously.

3C: Forecast Generation and Comparison

Forecast Comparison

Figure 3.12: VAR vs ARIMA Forecasts (2025-2027)

Show Image

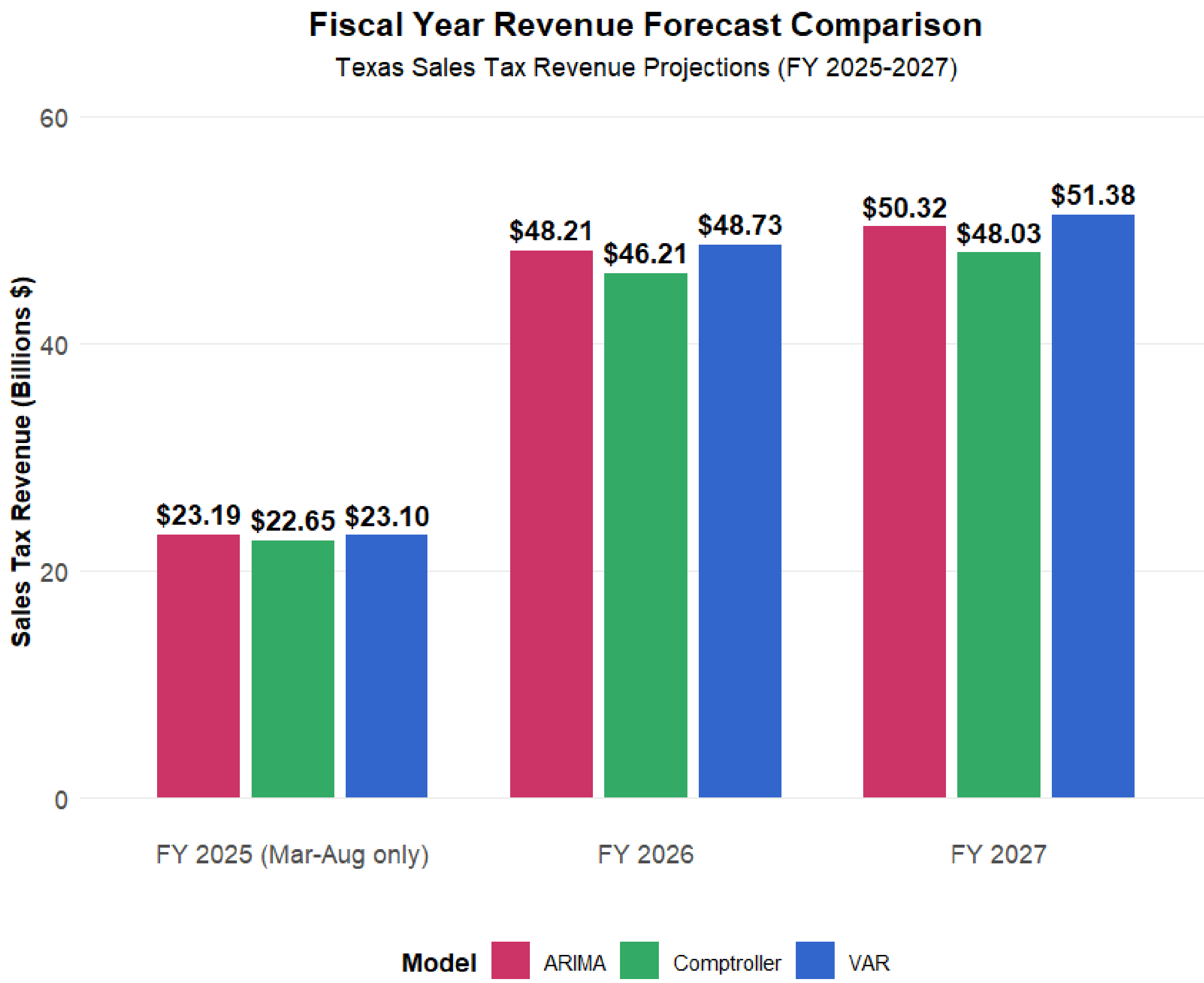
Note: This figure compares the 30-month ahead forecasts from our VAR model (solid blue line) with those from the ARIMA model developed in Question 2 (dashed red line). The VAR model generally projects slightly higher sales tax revenue throughout the forecast horizon, with the difference becoming more pronounced in the later periods.

Table 3.7: Fiscal Year Revenue Forecast Comparison (Billions $)

```
> print(corrected_fiscal_year_comparison, row.names = FALSE)
           Fiscal.Year VAR.Forecast..Billion... ARIMA.Forecast..Billion...
 FY 2025 (Mar-Aug only)                    23.10                      23.19
               FY 2026                    48.73                      48.21
               FY 2027                    51.38                      50.32
 Comptroller.Forecast..Billion...
                            22.65
                            46.21
                            48.03
```

Note: This table compares our forecasts aggregated by fiscal year with the Comptroller's official projections. Both our VAR and ARIMA models project slightly higher sales tax revenue than the Comptroller's forecast, with the VAR model projecting the highest values overall.

Figure 3.13: Fiscal Year Revenue Forecast Comparison



Note: This bar chart visually compares the forecasts by fiscal year, highlighting the significant gap between our model-based projections and the Comptroller's official forecast. Both our models project revenue growth between PY2025, FY 2026 and FY 2027, while the Comptroller's forecast remains flat.

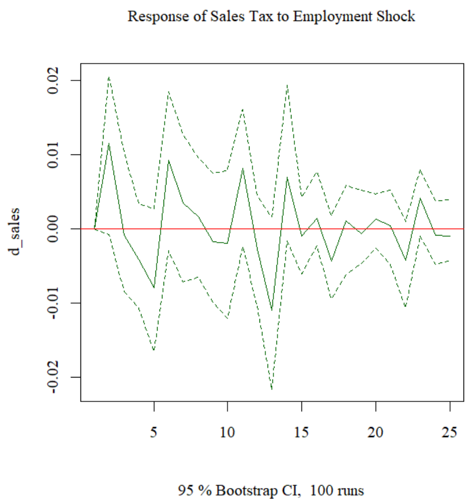
Percentage Differences in Forecasts

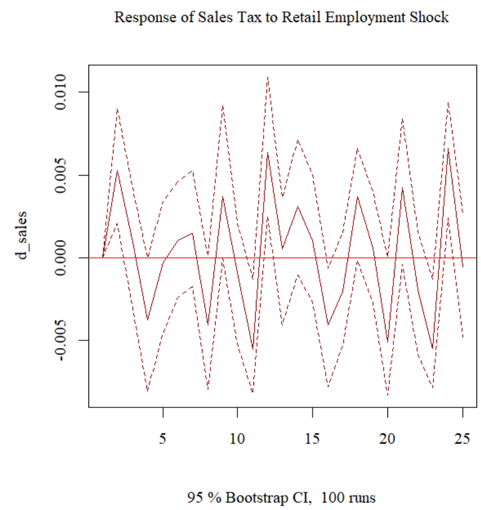
Table 3.8: Percentage Differences Between Forecasts

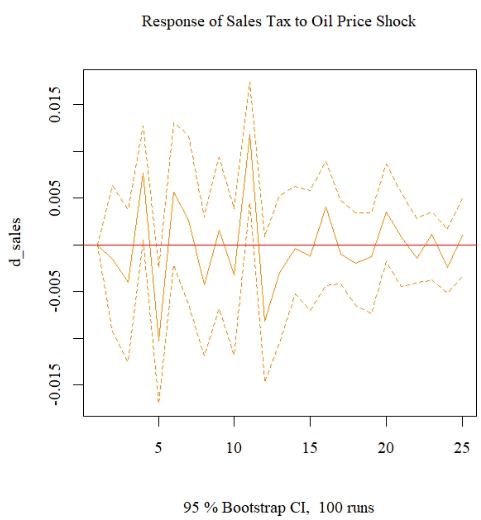
Comparison	FY 2025	FY 2026	FY 2027
VAR vs. ARIMA	-0.39%	1.07%	2.11%
VAR vs. Comptroller	1.99%	5.45%	6.98%
ARIMA vs. Comptroller	2.38%	4.33%	4.77%

Note: The VAR and ARIMA models produce similar forecasts, with differences ranging from -0.39% to 2.11% across the three fiscal years. Both models project moderately higher revenue than the Comptroller's forecast, with differences ranging from approximately 2% to 7%, increasing in the later fiscal years.

Impulse Response Analysis



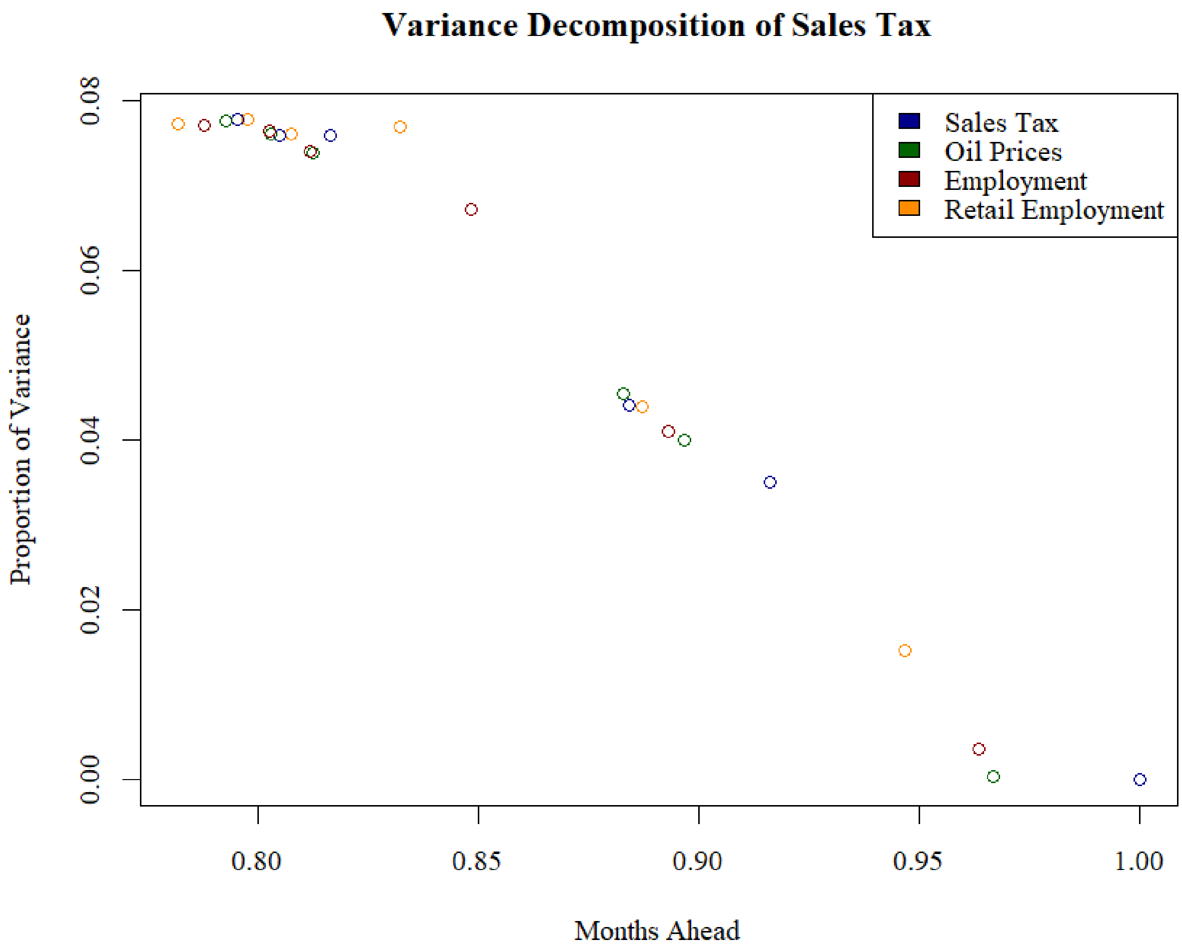




Note: These impulse response functions illustrate how sales tax revenue responds to one standard deviation shocks in each explanatory variable over 24 months. The response to retail employment shocks (middle) is the strongest and most persistent, followed by the response to total employment shocks (left). The response to oil price shocks (right) is more volatile and less pronounced.

Variance Decomposition

Figure 3.15: Variance Decomposition of Sales Tax



Note: This figure shows the proportion of forecast error variance in sales tax revenue attributed to each variable in the system at different forecast horizons. Initially, most of the variance is explained by sales tax's own innovations, but as the forecast horizon extends, the influence of the other variables, particularly retail employment, increases.

Forecast Interpretation

Our multivariate analysis using the VAR(12) model provides valuable insights into the dynamics of Texas sales tax revenue and its relationship with key economic indicators:

1. Model Comparison: The VAR model projects slightly higher sales tax revenue than the ARIMA model for FY 2026 and FY 2027, with the difference becoming more pronounced in FY 2027 (2.11% higher). For FY 2025 (partial year), the ARIMA model actually projects slightly higher revenue (0.39% higher) than the VAR model.

2. Comparison with Official Forecasts: Both our VAR and ARIMA models project moderately higher sales tax revenue than the Comptroller's official forecast across all fiscal years. The VAR model projects approximately 1.99% higher revenue for the partial FY 2025, 5.45% higher for FY 2026, and 6.98% higher for FY 2027 compared to the Comptroller's forecast.

3. Economic Insights: The impulse response functions and variance decomposition reveal that:

    - Retail employment has the strongest and most immediate impact on sales tax revenue, reflecting the direct relationship between retail activity and taxable sales. This is further supported by the highly significant Granger causality result for retail employment (p-value ≈ 0).

    - Total employment has a more delayed but still significant effect at certain lags, capturing the broader economic conditions that drive consumer spending, despite not showing Granger causality.

    - Oil price fluctuations have a more complex relationship with sales tax revenue, with mixed effects that may reflect the dual role of energy prices in the Texas economy.

4. Forecast Confidence: Despite the diagnostic issues with the VAR model (serial correlation, non-normality, and heteroskedasticity), the model demonstrates strong explanatory power for the sales tax equation, with an adjusted R-squared of 0.6551. This suggests that our point forecasts should be reasonably accurate, though prediction intervals would be affected by the diagnostic concerns.

5. Policy Implications: The moderate difference between our model-based projections and the Comptroller's forecast suggests that the official projections may be somewhat conservative. For comprehensive fiscal planning, considering both univariate and multivariate forecasts could establish a reasonable range of expected sales tax collections.

In conclusion, our VAR model effectively captures the dynamic relationships between sales tax revenue and key economic indicators, providing a more nuanced and potentially more accurate forecast than univariate approaches alone. The integration of employment and oil price data enhances our understanding of the drivers of sales tax revenue in Texas and suggests a more optimistic outlook for fiscal years 2026 and 2027 than the official projections indicate.