DBA3713 Group 3 Assignment 3

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Q1.

Below is the probabilistic outcome of X and Y

Regulation	**Prob, p**	**price_X, P_X**	**price_Y, P_Y**	**return_X, X**	**return_Y, Y**
strict	0.1	70	140	-0.3	0.16667
moderate	0.5	105	125	0.05	0.04167
mild	0.4	120	115	0.2	-0.0417

where $X_0 = 100, Y_0 = 120.$

Part (a)

$$E(X) = \sum p \cdot X = 0.07500$$

$$E(Y) = \sum p \cdot Y = 0.02083$$

$$\sigma_X^2 = \sum p \cdot \left(X - E(X)\right)^2 = 0.02063$$

$$\sigma_Y^2 = \sum p \cdot \left(Y - E(Y)\right)^2 = 0.00391$$

$$\sigma_{X,Y} = \sum p \cdot (X - E(X) \cdot (Y - E(Y) = -0.00885$$

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} = -0.98644$$

Part (b)

Expected return of portfolio = **0.04303**

Standard deviation of portfolio = **0.02327**

Sharpe ratio of portfolio = **1.63412**

Q2.

Part (a)

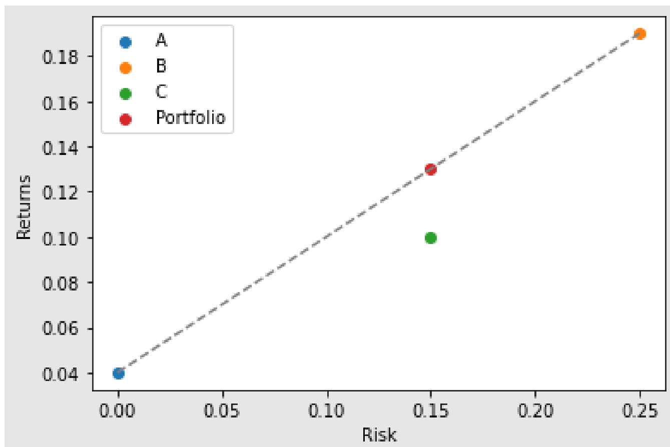
After taking the class, you suggest to him an alternative portfolio (consisting of a combination of A and B) that has the same standard deviation as portfolio C but higher expected return. Assume he has $20,000 to invest.

i. How much should he invest in A and how much in B?
ii. What is his expected return in this case?

$$\sigma_c{}^2 = w_a{}^2\sigma_a{}^2 + w_b{}^2\,\sigma_b{}^2 + 2\,\sigma_a{}^2\sigma_b{}^2 cov_{a,b}$$
$$Since\ \sigma_a = 0;\ cov_{a,b} = 0,$$
$$\therefore \sigma_c{}^2 = w_b{}^2\,\sigma_b{}^2$$

$$w_b = 0.6, w_a = 0.4$$
$$amount\ invested\ in\ A = \$20{,}000 * w_a = \$\mathbf{8{,}000}$$
$$amount\ invested\ in\ B = \$20{,}000 * w_b = \$\mathbf{12{,}000}$$
$$expected\ return = w_b * r_b + w_a * r_a = \mathbf{13}\%$$

Part (b)



Given the same risk ($\boldsymbol{\sigma} = \mathbf{15}\%$), the portfolio with A and B yields a higher return of 13%, compared to portfolio C's return of 10%. This improvement has been achieved as a result of the effects of diversification.

Q3.

For the period 1/1/1985 through 12/31/1992, calculate:

Part a(i)

The (in-sample) average excess return (i.e., the returns above the money market return) for each of the first four assets.

SB Non-US Bonds: 0.00838
MSCI EAFE: 0.01029
CRSP VW US common stocks: 0.00846
US Corp Bonds: 0.00488

Part a(ii)

The (in-sample) standard deviation for each of these excess returns:

SB Non-US Bonds: 0.036496
MSCI EAFE: 0.058388
CRSP VW US common stocks: 0.047121
US Corp Bonds: 0.015558

Part a(iii)

The (in-sample) covariance matrix (of the excess returns)

	SB Non-US Bonds	**MSCI EAFE**	**CRSP VW US common stocks**	**US Corp Bonds**
SB Non-US Bonds	0.001346	0.001230	-0.000179	0.000170
MSCI EAFE	0.001230	0.003445	0.001172	0.000177
CRSP VW US common stocks	-0.000179	0.001172	0.002244	0.000264
US Corp Bonds	0.000170	0.000177	0.000264	0.000245

Part b (i)

Calculate the weights of each of the four assets in the global minimum variance as well as the tangency portfolio.

Tangency:

SB Non-US Bonds: 0.26592
MSCI EAFE: - 0.03509
CRSP VW US common stocks: 0.14398
US Corp Bonds: 0.62519

GMV:

SB Non-US Bonds: 0.06087
MSCI EAFE: - 0.00088
CRSP VW US common stocks: 0.00208
US Corp Bonds: 0.93793

What does the weights being negative mean?

Negative weights indicate a short position on the respective stocks.

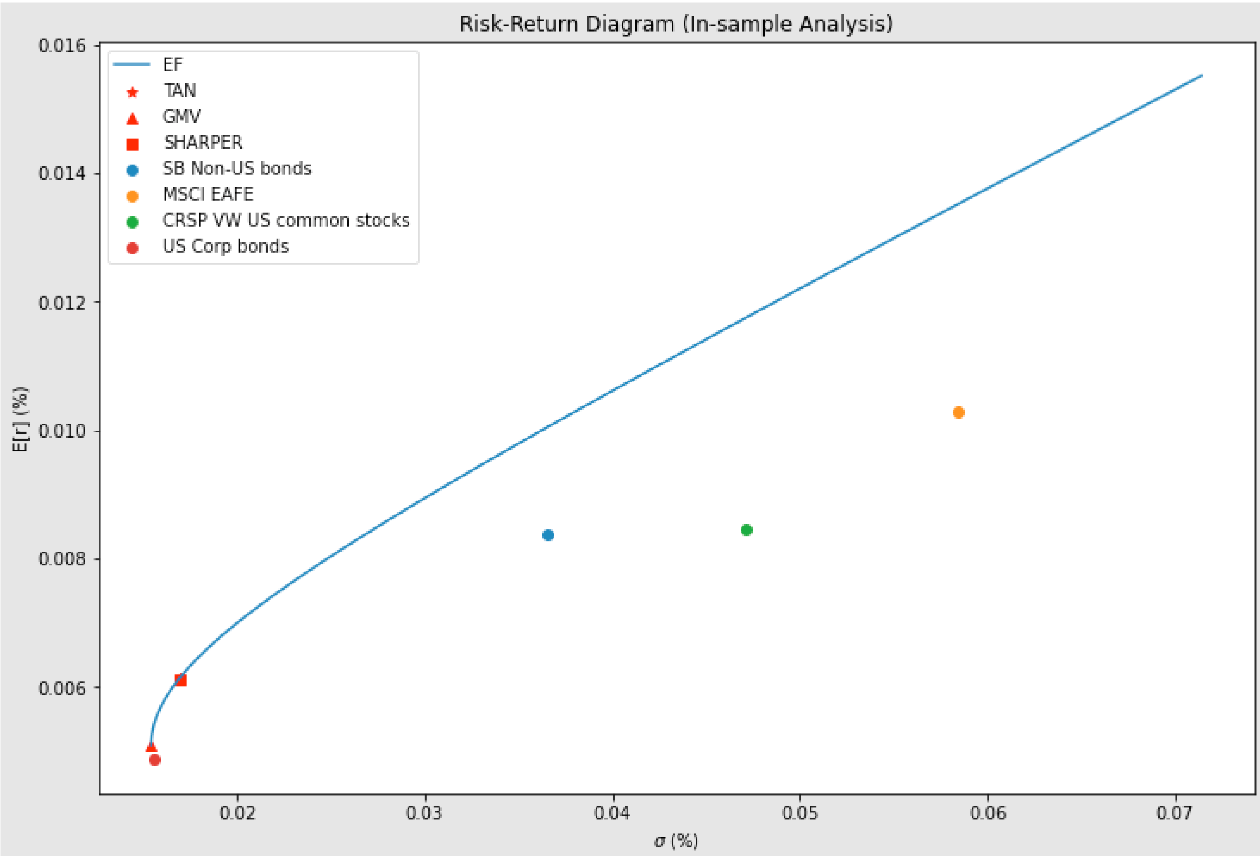
Part b (ii)

Weights of each of the four assets in a Sharpe ratio maximizing portfolio with no-shorting constraint.

SB Non-US Bonds: 0.23
MSCI EAFE: 0
CRSP VW US common stocks: 0.12
US Corp Bonds: 0.65

Part b (iii)

Plot the ex-post (i.e., in-sample) efficient frontier for this set of assets



Q4.

Covariance matrix of portfolio P

	Stock 1	Stock 2	Riskless
Stock 1	0.16	0.02	0
Stock 2	0.02	0.09	0
Riskless	0	0	0

Part (a)

What is the variance of portfolio P?

$$w = \begin{pmatrix} 0.4 \\ 0.4 \\ 0.2 \end{pmatrix}, V = \begin{pmatrix} 0.16 & 0.02 & 0 \\ 0.02 & 0.09 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sigma_P^2 = w^T V w = 0.04640$$

Part (b)

The betas of Stock 1 and Stock 2 can be obtained by taking the ratio of the stock's covariance with the market versus market variance.

$$\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2}$$

This yields the following asset betas:

Security	**Betas,** $\boldsymbol{\beta_i}$	**Weights,** $\boldsymbol{w_i}$
Stock 1	1.6	0.4
Stock 2	0.8	0.4
Riskless	0	0.2

The portfolio beta can thus be obtained by taking the weighted average of the above betas:

$$\beta_P = \sum w_i \cdot \beta_i = 0.96$$

Part b (i)

Assuming CAPM is correct, where the expected asset of any asset can be calculated from the following:

$$E[r_i] - r_f = \beta_i \cdot (E[r_m] - r_f)$$

Where $E[r_m]$ = 10% and r_f = 4%.

The expected return on portfolio P can be calculated:

$$E[r_P] = \beta_P \cdot (r_m - r_f) + r_f = 0.96(0.10 - 0.04) + 0.04$$

Part b (ii)

Find an efficient portfolio that consist of both the market portfolio, m and the riskless asset, r_f, condition on $\sigma_P^2 = \sigma_{\text{eff}}^2$:

$$\sigma_{eff}^2 = \sigma_m^2 \cdot w_m^2 + 2\sigma_{r_f,m} w_m w_{r_f} + \sigma_{r_f}^2 w_{r_f}^2 = \sigma_P^2$$

Since $\sigma_{r_f}^2 = 0$ and $\sigma_{r_f,m} = 0$, the efficient portfolio variance can be simplified to:

$$\sigma_{eff}^2 = \sigma_m^2 \cdot w_m^2$$

From here, we can back out $w_m$and calculate the expected efficient portfolio returns, $E\left[r_{eff}\right]$:

$$w_m = \sqrt{\frac{\sigma_{eff}^2}{\sigma_m^2}} = \sqrt{\frac{\sigma_P^2}{\sigma_m^2}}$$

$$E[r_{eff}] = w_m \cdot E[r_m] + (1 - w_m)r_f$$

The resulting efficient portfolio therefore involves taking a leverage position on the market portfolio and short position on the risk-free asset.

$w_m = 1.07703$

Expected portfolio returns = 0.10462

Q5.

Part a

Set up four different portfolios, finding monthly rates of return on each.

Our interpretation of "You may use the T-bill rate as the riskless rate and work with the excess returns (i.e., the additional returns compared to the riskless rate) throughout this question," is to calculate the excess returns for each stock from the start and work with those values directly. We do not set up portfolios using returns, then deduct the riskless rate.

Monthly rates of returns (first 5 months):

Portfolio_1	Portfolio_2	Portfolio_3	Portfolio_4
0.07765	0.058200	0.061425	0.061054
-0.02470	-0.002133	-0.014825	-0.030485
-0.07330	-0.040100	-0.031475	-0.015408
-0.04590	-0.020567	-0.010775	-0.012123
0.04515	0.057800	0.026625	0.023392

Part a (i)

Calculate the average monthly excess return and its variance on these four portfolios.

	Average excess return	Variance
Portfolio 1 (GM, IBM)	0.00493	0.00408
Portfolio 2 (GM, IBM, Anheuser Busch)	0.00727	0.00275
Portfolio 3 (GM, IBM, Anheuser Busch, Toyota)	0.00857	0.00210
Portfolio 4 (All stocks)	0.0122	0.00251

Part a (ii)

For each of these four portfolios, use the following two methods to estimate the (sample) beta versus the "World Market" portfolio:

• Run a linear regression on historical data and use the slope as beta

$$r_i - r_f = \alpha + \beta(r_{market} - r_f) + \varepsilon_i$$

• Use the covariance definition of beta directly. Confirm that these two methods lead to the same estimate.

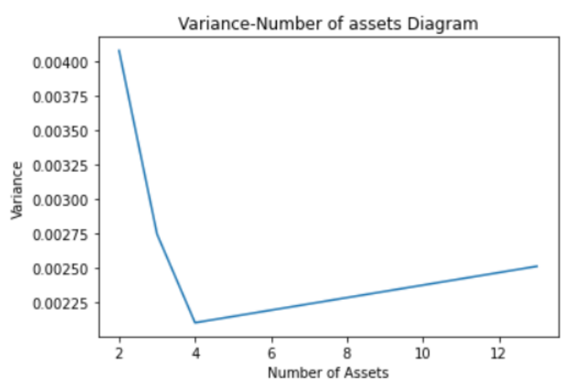
$$Beta = \frac{cov(r_{market}, r_i)}{var(r_{market})}$$

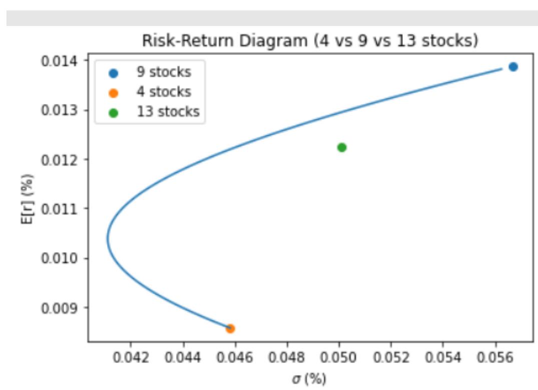
	β (Linear Regression)	Beta (Covariance)
Portfolio 1 (GM, IBM)	0.761	0.761
Portfolio 2 (GM, IBM, Anheuser Busch)	0.720	0.720
Portfolio 3 (GM, IBM, Anheuser Busch, Toyota)	0.778	0.778
Portfolio 4 (All stocks)	0.931	0.931

Hence, we see that using Linear Regression or the covariance definition of beta directly gives us the same values of Beta.

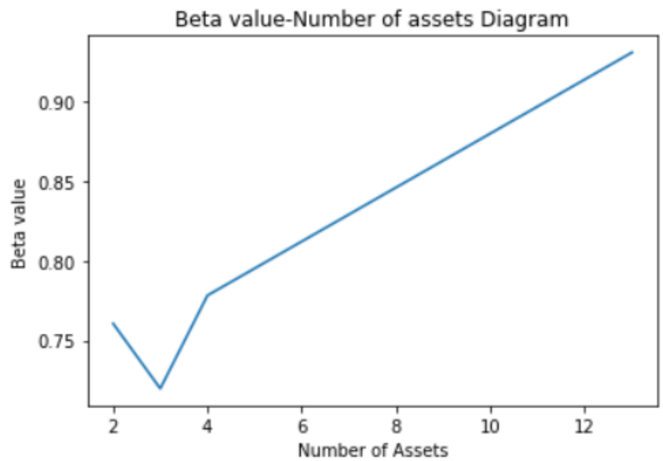
Part a (iii)

How does the variance of the portfolio change as we add assets to the portfolio? How does the beta change as we add assets to the portfolio?





In general, when we add assets to the portfolio, the variance of the portfolio decreases. The variance increased slightly when we added all 13 stocks to the portfolio. Upon deeper analysis, we see that creating a new portfolio of the remaining 9 stocks, this new portfolio has a much higher variance than the first 4 stocks. Hence, adding all 13 stocks results in an increase in portfolio variance as compared to portfolio 3 with only 4 stocks, but it is compensated for with an increase in returns as well.



The beta of the portfolio is a weighted sum of each stock's beta. Hence, the beta of the portfolio changes when adding more stocks will either increase or decrease depending on the beta of the stocks added.

<u>Part b(i)</u>

- Estimate the betas of each of the 13 individual stock excess returns against world market.
- Use the stock and the portfolio betas to verify that the beta of a portfolio is the weighted average of the betas of the underlying stocks.

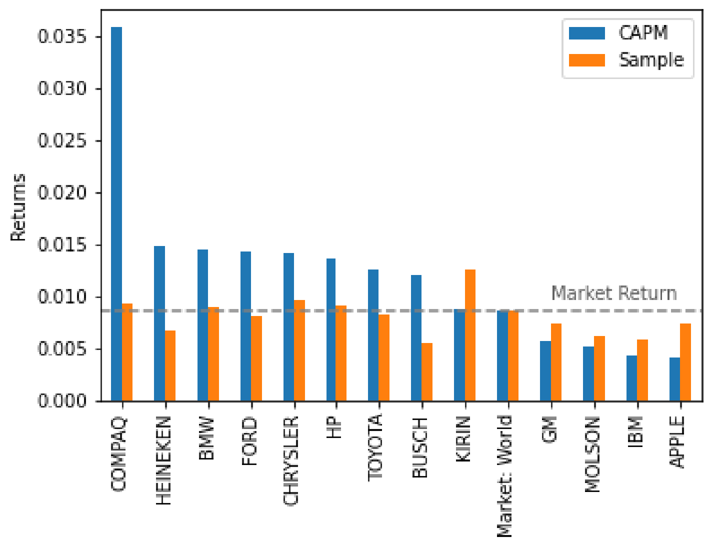
	Beta	**Weighted Beta**
TOYOTA	0.954	-
GM	0.848	-
BMW	1.032	-
FORD	0.929	-
CHRYSLER	1.113	-
APPLE	0.864	-
IBM	0.674	-
COMPAQ	1.070	-
HP	1.049	-
BUSCH	0.638	-
HEINEKEN	0.768	-
KIRIN	1.460	-
MOLSON	0.708	-
Portfolio 1 (GM, IBM)	0.761	0.761
Portfolio 2 (GM, IBM, Anheuser Busch)	0.720	0.720
Portfolio 3 (GM, IBM, Anheuser Busch, Toyota)	0.778	0.778

Portfolio 4 (All stocks)	0.931	0.931

Part b(ii)

For these stocks, compare the sample-average returns and the expected returns implied by CAPM. Do you find any differences in patterns?

	CAPM	Sample	Difference
TOYOTA	0.012476	0.008213	4.263684e-03
GM	0.005559	0.007295	-1.736421e-03
BMW	0.014395	0.008881	5.513932e-03
FORD	0.014260	0.007999	6.261218e-03
CHRYSLER	0.014013	0.009582	4.430866e-03
APPLE	0.004148	0.007435	-3.287294e-03
IBM	0.004303	0.005797	-1.494493e-03
COMPAQ	0.035803	0.009211	2.659109e-02
HP	0.013500	0.009025	4.475326e-03
BUSCH	0.011953	0.005489	6.464055e-03
HEINEKEN	0.014786	0.006607	8.178725e-03
KIRIN	0.008738	0.012567	-3.828218e-03
MOLSON	0.005165	0.006093	-9.281420e-04
Market: World	0.008607	0.008607	-1.734723e-18



With reference to the above bar plot, we notice that the sample average returns are quite different from the returns implied by CAPM. This could be due to estimation errors in the CAPM, or non-stationary issues in the estimations. Upon closer inspection, it appears that stocks with high beta (high CAPM) and above-market returns, tend to experience a higher difference between the two types of returns.