Assignment 3

Due Week 11 beginning of class time. Please submit the following documents in a single zip file through Canvas:

- *Your 'ipynb' source file. (Please make sure that it is runnable to replicate your results.)*

- *A brief report of your answers, take-aways, and thoughts.*

Please name your zip file in the format of "A@_G#.zip", where "@" is the assignment number and "#" is your group number on Canvas. For example, if you are submitting Assignment 2 for Group 1, please name your zip file as "A2_G1.zip". Only one submission per group is needed.

References to resources that are not in the textbook or class handouts should be explicitly mentioned in the write-ups and source codes.

The assignment has 40 points.

1. (5 pts) Today you bought 100 shares of Stock X at $100 per share and 120 shares of Stock Y at $ 120 per share. The stock prices of X and Y a year from now is uncertain and are both relevant to the strictness of certain government regulation. (Think of the travel restriction under the COVID-19 pandemic.) Suppose, a year from now the regulation will either be in "strict", "moderate", or the "mild" with probabilities of 10%, 50%, and 40% respectively. After analyzing the stocks X and Y you determine that the price of X and Y a year from now will be, depending on the strictness of regulation:

Regulation	Probability	Price of X	Price of Y
Strict	10%	$70	$140
Moderate	50%	$105	$125
Mild	40%	$120	$115

   (a) What is the coefficient of correlation between the returns of Stock X and Stock Y?

   (b) What is the expected return over the next year to your investment in stocks X and Y? What is the standard deviation of that return? What is the Sharpe Ratio of your portfolio? Suppose that the T-bill rate is 0.5 %.

2. (7 pts) Your brother comes to you for some investment advice on how to split his money between several portfolios managed by a mutual fund.

- Portfolio A consists entirely of risk-free securities, and has a certain return of 4%.

- Portfolio B has an expected return of 19% and a standard deviation of 25%.

- Portfolio C has an expected return of 10% and a standard deviation of 15%.

Your brother is leaning towards investing his money entirely in portfolio C, since he is unwilling to take the higher risk associated with portfolio B, but wants a higher return than offered by portfolio A.

(a) After taking the class, you suggest to him an alternative portfolio (consisting of a combination of A and B) that has the same standard deviation as portfolio C but higher expected return. Assume he has $20,000 to invest.

    i. How much should he invest in A and how much in B?

    ii. What is his expected return in this case?

(b) Sketch a risk/return diagram to help explain to him why his combination of risk and return is improved by this option and why his risk has not increased. Show all four portfolios on the diagram (the original three plus your suggestion).

3. (10 pts) The file *q3_data.csv* contains the monthly returns on:

- A Salomon Brothers portfolio of non-US bonds (supplied by Ibbotson Associates).

- The Morgan Stanley Capital International (MSCI) EAFE (Europe, Asia, and the Far East) index of world stock market performance.

- The Value-Weighted portfolio of US Common Stocks.

- A portfolio of 100 US Corporate Bonds, from Ibbotson Associates.

- The Money Market rate.

All returns are measured in the unit of percentages. These data series all start in January of 1985 and run through the end of 1992.

(a) For the period 1/1/1985 through 12/31/1992, calculate:

    i. The (in-sample) average **excess return** (i.e. the returns above the money market return) for each of the first four assets.

    ii. The (in-sample) **standard deviation** for each of these excess returns.

    iii. The (in-sample) **covariance matrix** (of the excess returns).

(b) Using the values calculated above, (remember, the excess risk-free rate will be zero from now on, because we already subtracted the market returns)

i. Calculate the weights of each of the four assets in the global minimum variance as well as the tangency portfolio. What does the weights being negative mean?

ii. Calculate the weights of each of the four assets in a Sharpe ratio maximizing portfolio **with no-shorting constraint**. (That is, all weights have to be nonnegative.)

iii. Plot the ex-post (i.e., in-sample) efficient frontier for this set of assets. In the same plot, please also include (1) the global minimum variance portfolio (2) the tangency portfolio (3) Sharpe ratio maximizing portfolio with no-shorting constraint and (4) each of the four assets on the plot.

4. (8 pts) Suppose that the covariance matrix for two stocks and the market portfolio is as follows:

	Stock 1	Stock 2	Market
Stock 1	0.16	0.02	0.064
Stock 2	0.02	0.09	0.032
Market	0.064	0.032	0.04

An investor is considering forming a portfolio with the allocation: 20% in the riskless asset, 40% in Stock 1 and 40% in Stock 2. Call this portfolio P. You have decided to analyze whether this portfolio is a sensible one using the CAPM model.

(a) What is the variance of portfolio P?

(b) What are the betas of assets 1, 2, and P relative to the market (i.e. what are β_1, β_2, and β_P)? Suppose the riskless return r_f is 4% and the expected return on the market $E[r_m]$ is 10%. Assuming that the CAPM is correct,

  i. What is the expected return on portfolio P?

  ii. Find an efficient portfolio consisting of the riskless asset and the market portfolio that has the same standard deviation as portfolio P. What is the expected return on this portfolio?

5. (10 pts) The file *q5_data.csv* contains historical monthly rates of return on several stock indices, corporate and government bonds, and individual stocks (taking dividend and interest payments into consideration) over certain period. All returns are measured in percentages. You may use the T-bill rate as the riskless rate and work with the excess returns (i.e., the additional returns compared to the riskless rate) throughout this question.

(a) Set up four different portfolios, finding monthly rates of return on each. The first portfolio is evenly invested in GM and IBM. The second portfolio is evenly invested in three stocks

(GM, IBM, and Anheuser Busch) while the third portfolio is evenly invested in four stocks (GM, Toyota, IBM, and Anheuser Busch). The last portfolio is equally invested in all 13 common stocks. The rate of return on any portfolio is just the weighted average of the rates of return on the underlying stocks.

   i. Calculate the average monthly excess return and its variance on these four portfolios.

   ii. For each of these four portfolios, use the following two methods to estimate the (sample) beta versus the "World Market" portfolio:

      • Run a linear regression on historical data and use the slope as beta;[1]

      • Use the covariance definition of beta directly.

     Confirm that these two methods lead to the same estimate.

   iii. How does the variance of the portfolio change as we add assets to the portfolio? How does the beta change as we add assets to the portfolio?

(b) Estimate the betas of each of the 13 individual stock excess returns against world market.

   i. Use the stock and the portfolio betas to verify that the beta of a portfolio is the weighted average of the betas of the underlying stocks.

   ii. For these stocks, compare the sample-average returns and the expected returns implied by CAPM. Do you find any differences in pattern?

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[1] Although linear regression is not a topic that we have covered formally in this module, it is relatively easy to pick up and implement. For example, in our case study regarding the volatility surface movement, we have already implemented linear regression as the benchmark method; see the lecture codes *volatility_surface.ipynb*.