Ammar Elbashir (3721991)
Ethan Banovic (3724710)
Arda Sarier (3721030)

1 Predicate Logic — 50 points

1.1 First Order Logic — 14 points

Translate the following sentences into first order logic:

a. There is an ice cream flavor loved by everyone.

> Let *icecream* be a constant.
> $\exists\ icecream\ \forall\ x : Loves(x, icecream)$

b. It is not true that everyone loves some ice cream flavor.

> Let *icecream* be a constant.
> $\exists\ x\ \forall\ icescrem : \neg(Loves(x, icecream))$

c. Anyone who does not shave himself must be shaved by the barber.

> Let *barber* be a constant.
> $\forall x : \neg(Shaves(x, x)) \Rightarrow Shaves(barber, x)$

d. Whomever the barber shaves, must not shave himself.

> Let *barber* be a constant.
> $\forall x : Shaves(barber, x) \Rightarrow \neg(Shaves(x, x))$

e. Rudolph is a reindeer and Rudolph has a red nose.

> Let *Rudolph*, *barber*, *rednose* be constants.
> $Is(Rudolph, barber) \land Is(Rudolph, rednose)$

f. Anyone with a red nose is weird or is a clown.

> Let *weird*, *clown* be constants.
> $\forall x : Is(x, weird) \lor Is(x, clown)$

g. No reindeer is a clown.

Ammar Elbashir (3721991)
Ethan Banovic (3724710)
Arda Sarier (3721030)

Let *reindeer*, *clown* be constants.
$\forall\ reindeer : \neg(Is(reindeer, clown))$

Note: You are not required to specify which object a variable refers to, or what a function returns, if it is obvious from the context. For instance, it's obvious from the context that x is a person and $Likes(x, y)$ is true if x likes some y. However, if you think that the context is insufficient, you might start providing definitions.

1.2 Clausal Forms 6 points

Reduce the following sentences to clausal form:

a. $\forall x \forall y (R(x, y) \to (R(x, y) \land Q(y)))$

$$\begin{aligned}
&\forall x \forall y (R(x, y) \to (R(x, y) \land Q(y))) \\
&\equiv \forall x \forall y (\neg R(x, y) \lor (R(x, y) \land Q(y))) \\
&\equiv \forall x \forall y ((\neg R(x, y) \lor R(x, y)) \land (\neg R(x, y) \lor Q(y))) \\
&\equiv \forall x \forall y (1 \land (\neg R(x, y) \lor Q(y))) \\
&\equiv \forall x \forall y ((\neg R(x, y) \lor Q(y)))
\end{aligned}$$

b. $\forall x \exists y \forall z (P(x, y, z) \to \exists u R(x, u, z))$

$$\begin{aligned}
&\forall x \exists y \forall z (P(x, y, z) \to \exists u R(x, u, z)) \\
&\equiv \forall x \exists y \forall z (\neg P(x, y, z) \lor \exists u R(x, u, z))
\end{aligned}$$

c. $\forall x (\neg \exists y P(x, y) \land \neg(Q(x) \land \neg R(x)))$

$$\begin{aligned}
&\forall x (\neg \exists y P(x, y) \land \neg(Q(x) \land \neg R(x))) \\
&\equiv \forall x (\neg \exists y P(x, y) \land (\neg Q(x) \lor R(x)))
\end{aligned}$$

1.3 Knowledge Extraction via Resolution 30 points

Let $flee$, $stay$, now, $lion$ be constants. You are given the following Knowledge Base:
S1 $\forall t \forall x (Observe(t, x) \land Danger(x) \to Suggestion(t, flee))$
S2 $\forall t (\neg \exists x (Observe(t, x) \land Danger(x)) \to Suggestion(t, stay))$
S3 $Danger(lion)$
S4 $Observe(now, lion)$
Use Resolution to show that the KB entails $\exists x Suggestion(now, x)$ and extract an answer.

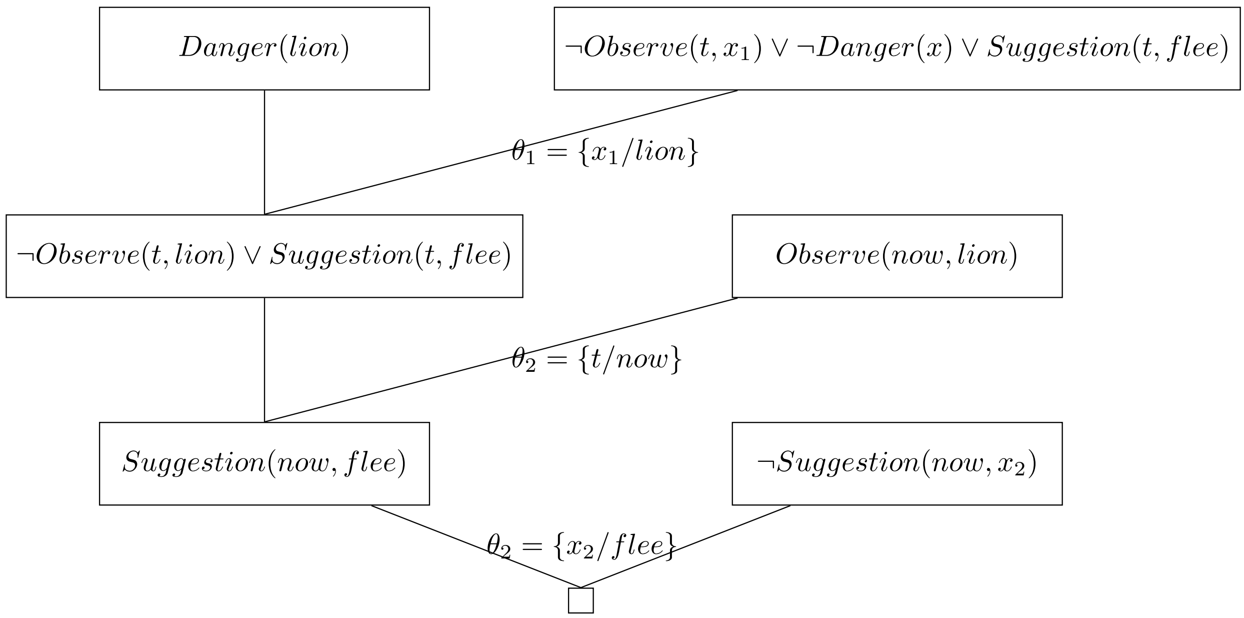

Figure 1: PL-Resolution

2 Predicate Logic **50 points**

2.1 Probability **30 points**

Examiners running the game show *"Ring the golden bell"* have been taking bribes from some participants. A given participant is either allowed to stay or kicked off in each episode.
If the participant has been bribing the examiners she will be allowed to stay with probability 4/5. If the participant has not been bribing the examiners, she will be allowed to stay with probability 1/3.
Suppose that 1/4 of the participants have been bribing the examiners. The same participants bribe the examiners in both rounds, i.e., if a participant bribes them in the first round, she bribes them in the second round too (and vice versa).

a. If you pick a random participant who was allowed to stay during the first episode, what is the probability that she was bribing the examiners?

$B =$ is bribing, $\neg B =$ is not bribing, $S =$ is staying, $\neg S =$ is not staying.

We are looking for and want to compute $P(B|S)$.

Given:

- $P(B) = \frac{1}{4}$
- $P(S \mid B) = \frac{4}{5}$
- $P(S \mid \neg B) = \frac{1}{3}$

With the given information we can create the tree in Figure 2.

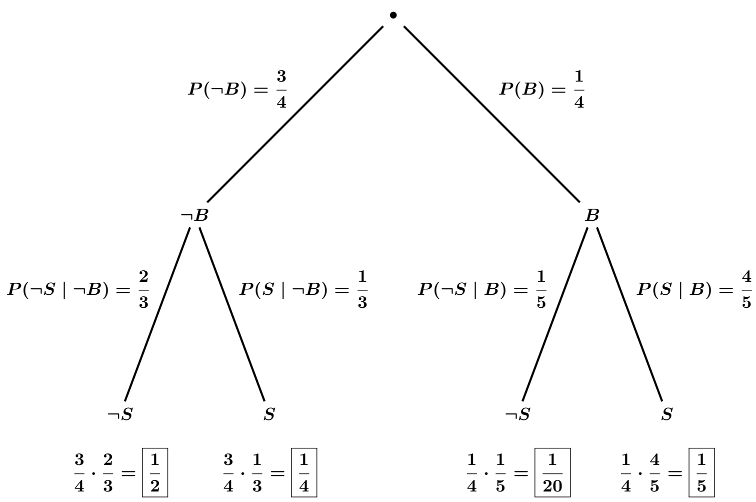

Figure 2: Probability tree

Before computing $P(B|S)$ we need to compute $P(S)$ first:

$$P(S) = P(\neg B) \cdot P(S \mid \neg B) + P(B) \cdot P(S \mid B) = \frac{1}{4} \cdot \frac{4}{5} + \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{5} + \frac{1}{4} = \frac{4+5}{5 \cdot 4} = \frac{9}{20}$$

we computing $P(B \mid S)$ using Bayes' theorem:

$$P(B \mid S) = \frac{P(S|B) \cdot P(B)}{P(S)} = \frac{\frac{4}{5} \cdot \frac{1}{4}}{\frac{9}{20}} = \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{20}{9} = \frac{4}{9}$$

b. If you pick a random participant, what is the probability that she is allowed to stay during both of the first two episodes?

$$\begin{aligned}
P(\text{stay 2 times}) &= P(\neg B) \cdot P(S \mid \neg B)^2 + P(B) \cdot P(S \mid B)^2 \\
&= \frac{3}{4} \cdot \left(\frac{1}{3}\right)^2 + \frac{1}{4} \cdot \left(\frac{4}{5}\right)^2 \\
&= \frac{3}{4} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{4}{5} \cdot \frac{4}{5} \\
&= \frac{1}{4 \cdot 3} + \frac{4}{5^2} \\
&= \frac{1}{12} + \frac{4}{25} \\
&= \frac{25 + 4 \cdot 12}{12 \cdot 25} \\
&= \frac{73}{300}
\end{aligned}$$

c. If you pick random participant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

$$\begin{aligned}
P(\text{kicked off second time}) &= P(\neg B) \cdot P(S \mid \neg B) \cdot P(\neg S \mid \neg B) \\
&\quad + P(B) \cdot P(S \mid B) \cdot P(\neg S \mid B) \\
&= \frac{3}{4} \cdot \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{4} \cdot \frac{4}{5} \cdot \frac{1}{5} \\
&= \frac{1}{2 \cdot 3} + \frac{1}{5^2} \\
&= \frac{1}{6} + \frac{1}{25} \\
&= \frac{25 + 6}{6 \cdot 25} \\
&= \frac{31}{150}
\end{aligned}$$

2.2 Naïve Bayes' Classifier **20 points**

Given this data table about buying a car. Predict the class of the following new example using Naïve Bayes' Classification:
age ≤ 30, income = high, Married = yes, credit-rating = fair

RID	age	income	married	credit_rating	class: buy_car
1	≤ 30	high	no	fair	no
2	≤ 30	high	no	excellent	no
3	31, 32,...,40	high	no	fair	yes
4	> 40	medium	no	fair	yes
5	> 40	low	yes	fair	yes
6	> 40	low	yes	excellent	no
7	31, 32,...,40	low	yes	excellent	yes
8	≤ 30	medium	no	fair	no
9	≤ 30	low	yes	fair	yes
10	> 40	medium	yes	fair	yes
11	≤ 30	medium	yes	excellent	yes
12	31, 32,...,40	medium	yes	excellent	yes
13	31, 32,...,40	high	yes	fair	yes
14	> 40	medium	no	excellent	no

We want to predict the class of the given example:

$$e = (\underbrace{age \leq 30}_{e_0}, \underbrace{income = high}_{e_1}, \underbrace{married = yes}_{e_2}, \underbrace{credit = fair}_{e_3})$$

by classifying it into one of the two categories:

$$buy\_car \in \{yes, no\}.$$

1. Prior over categories ($P(category)$):

$$P(buy\_car = yes) = \frac{9}{14}, \qquad P(buy\_car = no) = \frac{5}{14}.$$

2. Conditional probabilities ($P(e_i \mid category)$):

   - $buy\_car = yes$:

$$P(\underbrace{age \leq 30}_{e_0} \mid buy\_car = yes) = \frac{2}{9}, \qquad P(\underbrace{income = high}_{e_1} \mid buy\_car = yes) = \frac{2}{9},$$

$$P(\underbrace{married = yes}_{e_2} \mid buy\_car = yes) = \frac{7}{9}, \qquad P(\underbrace{credit = fair}_{e_3} \mid buy\_car = yes) = \frac{6}{9}.$$

Ammar Elbashir (3721991)
Ethan Banovic (3724710)
Arda Sarier (3721030)

- $buy\_car = no$:

$$P(\underbrace{age \leq 30}_{e_0} \mid buy\_car = no) = \frac{3}{5}, \qquad P(\underbrace{income = high}_{e_1} \mid buy\_car = no) = \frac{2}{5},$$

$$P(\underbrace{married = yes}_{e_2} \mid buy\_car = no) = \frac{1}{5}, \qquad P(\underbrace{credit = fair}_{e_3} \mid buy\_car = no) = \frac{2}{5}.$$

Using the Naïve Bayes formula

$$P(Category \mid e) = \alpha\, P(Category) \prod_j P(e_j \mid Category)$$

we can compute $P(buy\_car = yes \mid e)$:

$$P(e \mid buy\_car = yes) = \prod_j P(e_j \mid buy\_car = yes) = \frac{2}{9} \cdot \frac{2}{9} \cdot \frac{7}{9} \cdot \frac{6}{9} = \frac{56}{2187} \approx 0.0256,$$

$$P(buy\_car = yes \mid e) = P(buy\_car = yes) \cdot P(e \mid buy\_car = yes) = \frac{9}{14} \cdot \frac{56}{2187} \approx 0.0165.$$

and $P(buy\_car = no \mid e)$:

$$P(e \mid buy\_car = no) = \prod_j P(e_j \mid buy\_car = no) = \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{2}{5} = \frac{12}{625} \approx 0.0192,$$

$$P(buy\_car = no \mid e) = P(buy\_car = no) \cdot P(e \mid buy\_car = no) = \frac{5}{14} \cdot \frac{12}{625} \approx 0.00686.$$

Now we can predict the class of the given example:

$$P(buy\_car = yes \mid e) = 0.0165 \ > \ 0.00686 = P(buy\_car = no \mid e).$$

$$\boxed{\text{Predicted Category: } buy\_car = yes}$$

The Naïve Bayes classifier predicts that the customer will buy a car.