Project Plan — Hybrid Casini + JT Island

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Objective

Reproduce Casini's relative-entropy argument for a Bekenstein-style bound in a simple 1+1D QFT model, compute a concrete worked example (perturbative excitation), and connect the bound conceptually to a JT gravity "island" calculation in an appendix.

Motivation

Casini's approach uses quantum-information tools (relative entropy and modular Hamiltonians) to derive rigorous entropy bounds in QFT. JT gravity and island calculations give a semiclassical, holographic perspective on entropy in gravitational settings. Combining both perspectives demonstrates both analytic rigor and contemporary relevance.

Core Deliverables

- Casini_note.pdf (4–6 pages): step-by-step derivation + a worked perturbative example.
- jt_appendix.pdf (1–2 pages): reproduction of one canonical JT/island result + short commentary linking to the Casini argument.
- notebooks/: reproducible Jupyter notebooks (casini_toy.ipynb, jt_toy.ipynb) with symbolic + numeric checks.
- README.md and 1-page-summary.pdf (concise non-technical summary for supervisors).

Methods & Steps

- Choose a model: Rindler wedge or interval in a 1+1D CFT where the modular Hamiltonian is known.
- Reproduce Casini's derivation: define states ρ and σ, compute ΔS and ΔK, and use positivity of relative entropy to derive $\Delta S \leq \Delta K$.
- Provide a worked example: pick a small-excitation state and compute leading-order relative entropy (symbolic or numeric).
- Reproduce one standard JT island calculation and write a short appendix mapping the relative-entropy viewpoint onto the JT result.
- Polish: write final PDF (4–8 pages), clean notebooks, README, and 1-page summary.

Risks & Mitigations

Risk: A fully analytic integral becomes intractable.

Mitigation: use a perturbative expansion and numeric checks; focus write-up on the conceptual derivation if closed forms are unavailable.

Risk: Gaps in recent notation or techniques (since you've been away from courses).

Mitigation: short targeted refresh of math/QFT/GR essentials and a one-page cheat-sheet; use pedagogical reviews as primary references.

Why this is valuable

Shows ability to read and reproduce recent literature. Produces verifiable outputs (PDF + notebooks) that admissions committees can inspect. Connects rigorous QFT methods to modern holographic topics, appealing to a broad range of supervisors.

Immediate Actions (Do these now)

- Set up Python/Jupyter environment (instructions will follow).
- Create a GitHub repository with folders: /notebooks, /doc, /data, README.md.
- Open the provided starter notebook and run cells to verify environment.
- Prepare a 1-page formula cheat-sheet for quick reference.
- Start the Casini derivation in notebook cells and commit frequently.

Weekly Plan (detailed)

Week(s)	Primary Focus	Key Tasks / Deliverables
Week 0 (setup)	Repo & environment	Create GitHub repo skeleton; save dissertation excerpts
Weeks 1–2	Refresh basics	Math/QI/QFT/GR cheat-sheet; choose QFT model; pick J
Weeks 3–5	Casini study	Work through Casini paper; write step-by-step derivation
Week 6	Worked example	Compute small-excitation relative entropy; numeric chec
Week 7	Polish Casini	Finalize Casini_note draft; tidy notebook; README
Weeks 8–10	JT mini	Read JT/island notes; reproduce canonical island calcula
Weeks 11–12	Synthesis	Write combined PDF (4–8 pp); finalize notebooks; create
Weeks 13–14	Outreach prep	Referees review; prepare outreach email lists; enroll/finis
Weeks 15–16	Applications prep	Draft base SOP; tailor to programs; confirm letters and s

Expanded Week-by-Week Tasks

Week 0 (Today)

• Create repo and notebook skeleton.

• Create 1-page formula cheat-sheet.

Weeks 1–2 (Foundations)

• Targeted refresh: linear algebra, density matrices & von Neumann entropy, modular Hamiltonian intuition, GR basics (black hole thermodynamics).

• Decide on QFT model (Rindler or interval) and pick JT island paper.

Weeks 3–5 (Casini derivation)

• Read Casini 2008 + supporting notes; annotate and map notation.

• Write step-by-step derivation cells in notebook; symbolic manipulation with SymPy.

Week 6 (Worked example)

• Compute leading-order relative entropy for small excitation; perform numeric check; create figure.

Week 7 (Polish Casini)

• Finalize Casini_note.pdf draft; tidy notebooks; prepare README and 1-page summary.

Weeks 8–10 (JT mini)

• Study JT/island pedagogical notes; reproduce canonical calculation; create a toy Page-curve/schematic; write a short JT appendix.

Weeks 11–12 (Synthesis & polish)

• Merge Casini & JT into final 4–8 page PDF; finalize notebooks; prepare video or screencast (optional).

Weeks 13–14 (Referees & outreach)

• Send 1-page summary + repo link to referees; ask for letter updates.

• Contact supervisors with deliverable attached once referees provide feedback.

Weeks 15–16 (Applications prep)

• Finalize SOP base draft; tailor CV and SOP per program; confirm letters and submit applications.

Recommended Reading & Resources

- Casini, 'Relative entropy and the Bekenstein bound' (2008) — core derivation.
- Bousso, 'The holographic principle' — general background.
- Almheiri et al. (2019–2020) papers on replica wormholes / islands — JT context.
- Short QFT entanglement notes (Srednicki), and David Tong QFT lecture notes for intuition.
- Tutorials for SymPy, NumPy, Matplotlib, and basic Jupyter usage.

Tools & Expected Outputs

Python 3 (SymPy, NumPy, SciPy, Matplotlib), Jupyter Notebook/JupyterLab, GitHub for version control. Expected outputs: Casini_note.pdf, jt_appendix.pdf, notebooks in /notebooks, README.md, 1-page summary, optional screencast.

Next immediate steps (what I can do for you now)

- Generate a fully runnable Jupyter notebook file with the starter code (expanded).
- Fetch exact PDFs of the recommended reading (Casini 2008, JT island papers).
- Draft a short email to your supervisor (Hisham) attaching the 1-page plan to request feedback.

Prepared by ChatGPT for Ammar — hybrid Casini + JT project plan (full).