

Homework 2

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CE 661

1. Question 1

a. NWU is a right handed system

$$b. C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

c. The determinant of C is 1. This is expected since this matrix is a rotation about x.

d. The point will be reflected in y and z but not change in x. The point in the NWU frame is (11.521, 215.633, -108.617).

2. Question 2

$$a. C_{ECI}^{ECER} = (C_{ECEF}^{ECI})^T = \begin{bmatrix} \cos(\omega_e t) & -\sin(\omega_e t) & 0 \\ \sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Handwritten derivation of the identity $C_{ECEF}^{ECI} \cdot C_{ECI}^{ECEF} = I$.

Given:

$$C_{ECEF}^{ECI} = \begin{bmatrix} \cos(\omega_e t) & \sin(\omega_e t) & 0 \\ -\sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_{ECI}^{ECEF} = \begin{bmatrix} \cos(\omega_e t) & -\sin(\omega_e t) & 0 \\ \sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then:

$$C_{ECEF}^{ECI} \cdot C_{ECI}^{ECEF} = I$$

$$\begin{bmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C & -S & 0 \\ S & C & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C^2 + S^2 + 0 & -CS + CS + 0 & 0 + 0 + 0 \\ -CS + CS + 0 & S^2 + C^2 + 0 & 0 + 0 + 0 \\ C + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b.

3. Question 3

a. Quaternion = [0.0427 0.0303 0.7070 0.7053]

$$\text{b. } C_{IMU}^{NED} = \begin{bmatrix} -0.0015 & 0.9998 & 0.0177 \\ -0.9947 & -0.0033 & 0.1030 \\ 0.1030 & -0.0174 & 0.9945 \end{bmatrix}$$

4. Question 4 – yes the matrix satisfies the definition of a rotation matrix.

$$\text{a. } A^{-1} = A^T$$

$$\text{b. } \det(R) = 1$$

$$\text{c. } \text{rank}(A) = 3 \text{ (I think this is covered by the previous two statements)}$$

5. Question 5

$$C_{NED}^{IMU} = \begin{bmatrix} -0.1042 & 0.9813 & 0.1619 \\ -0.9938 & -0.0964 & -0.0555 \\ -0.0388 & -0.1667 & 0.9853 \end{bmatrix}$$

6. Point in frame 2 = [3.6863, 5.0151]. Multiple by a rotation matrix that has theta = 30.