

## Kinematic Positioning and Navigation – Winter 2018: Homework #2

### Reference Frames and Transformations in KPN

1. The primary local level frame we use in this class is NED. Say you are working for an organization that has decided to use a local level frame defined as NWU (North-West-Up).
  - a. Is NWU a right-handed or left-handed coordinate system?
  - b. What is the DCM:  $\mathbf{C}_{NED}^{NWU}$  ?
  - c. Is the determinant of the matrix  $\mathbf{C}_{NED}^{NWU}$  what you would expect? Why or why not?
  - d. If a point has coordinates (11.521, -215.633, 108.617) in the NED frame, what are the point's coordinates in the NWU frame?
2. The DCM for transforming from the Earth Centered Inertial (ECI) frame to the Earth Centered Earth Fixed (ECEF) frame is:

$$\mathbf{C}_{ECEF}^{ECI} = \begin{bmatrix} \cos(\omega_e t) & \sin(\omega_e t) & 0 \\ -\sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where  $\omega_e$  is the Earth's rotation rate and  $t$  is time.

- a. What is the DCM:  $\mathbf{C}_{ECI}^{ECEF}$  ?
  - b. Show that  $\mathbf{C}_{ECI}^{ECEF} \mathbf{C}_{ECEF}^{ECI} = \mathbf{I}$ , where  $\mathbf{I}$  is the 3x3 identity matrix. (Hint: try writing out the product and using trig identities to simplify the result.)
3. You measure the following Tait Bryan angles:  $\Phi = 5.912$  deg,  $\Theta = -1.013$  deg,  $\Psi = 90.084$  deg. Using the MATLAB code on the course Canvas site, answer the following:
    - a. What is the corresponding quaternion representation?
    - b. What is  $\mathbf{C}_{IMU}^{NED}$  ?
  4. Does the matrix  $\mathbf{A}$  below satisfy the definition of a rotation matrix?

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

5. One of the records from the POS file you created in last week's HW assignment is given below:

Date (UTC-8.00)	1/8/2018
Time (UTC-8.00)	12:37:35 PM
PosLat(deg)	44.56520792
PosLon(deg)	-123.27161433
PosAlt(m)	95.788
AngleRoll(deg)	-3.222
AnglePitch(deg)	-9.316
Heading(deg)	96.061

OxTS's coordinate frame definitions and conventions for orientation measurement match the Tait-Bryan convention we discussed in class. What is the DCM,  $\mathbf{C}_{NED}^{IMU}$  for this record? (Hint: you can use the MATLAB code on the course site for this one, as well.)

6. The coordinates of point  $\mathbf{p}$  in frame 1 (indicated by solid red axes) below are (5.7, 2.5). What are the coordinates of  $\mathbf{p}$  in frame 2 (indicated by dashed blue axes)?

