ME 533 HW 1

① $G(p) = \dfrac{10}{p(1+0.1p)}$

closed loop: $\dfrac{G(p)}{1+G(p)}$

$$\frac{\frac{10}{p(1+0.1p)}}{1+\frac{10}{p(1+0.1p)}} = \frac{10}{p(1+0.1p)+10} = \frac{y(p)}{u(p)}$$

roots of CE

$p + 0.1p^2 + 10 = 0$

$p^2 + 10p + 100 = 0$

$p = \dfrac{-10 \pm \sqrt{100 - 4(1)(100)}}{2}$

$p = -5 \pm 5\sqrt{-3}$

roots have neg real parts so system is stable

$10u(p) = 0.1p^2 y(p) + p \cdot y(p) + 10y(p)$

$10u = 0.1\ddot{y} + \dot{y} + 10y$ $\quad x_1 = y$, $x_2 = \dot{y}$

$10u = 10\dot{x}_2 + x_2 + 10x_1$

$\dot{x}_2 = -\frac{1}{10}x_2 - x_1 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

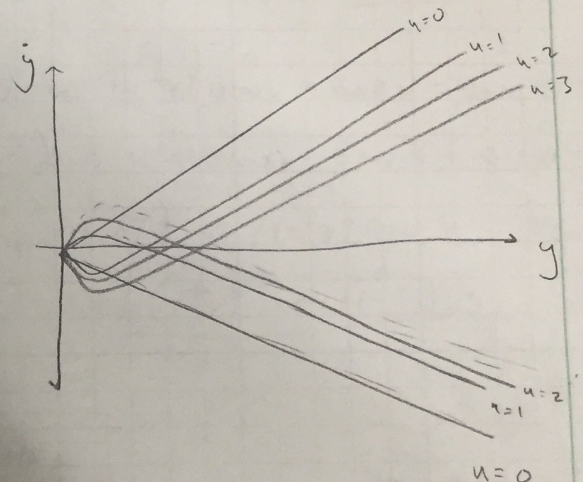
if $u = 0$

$\ddot{y} + 10\dot{y} + 100y = 0$

$\dot{y}\frac{d\dot{y}}{dy} + 10\dot{y} + 100y = 0$

$\frac{1}{2}\dot{y}^2 + 10\dot{y}y + 50y^2 = 0$

$\dot{y}^2 + 20\dot{y}y + 100y^2 = 0$



if $u = 1$

$\ddot{y} + 10\dot{y} + 100y = 1$

$\dot{y}\,d\dot{y}/dy + 10\dot{y} + 100y = 1$

$\frac{1}{2}\dot{y}^2 + 10\dot{y}y + 50y^2 = y$

$\dot{y}^2 + 20\dot{y}y + 100y^2 = 2y$

if $u = -1$

$\dot{y}^2 + 20\dot{y}y + 100y^2 = -2y$

$(\dot{y} + 10y)^2 = -2y$

$\dot{y} + 10y = \sqrt{-2y}$

$\dfrac{dx_2}{dx_1} = \dfrac{-\frac{1}{10}x_2 - x_1 + u}{x_2} = -\frac{1}{10} - \frac{x_1}{x_2} + \frac{u}{x_2} = \alpha$

if u

$\dot{y}^2 + 20\dot{y}y + 100y^2 = 2uy$

$\dot{y} + 10y = \pm\sqrt{2uy}$

$\dot{y} = \pm\sqrt{2uy} - 10y$

(2) $\dot{x} = y + x(x^2+y^2-1)\sin\left(\frac{1}{x^2+y^2-1}\right) \longrightarrow \frac{\dot{x}-y}{x} = (x^2+y^2-1)\sin\left(\frac{1}{x^2+y^2-1}\right)$

$\dot{y} = -x + y(x^2+y^2-1)\sin\left(\frac{1}{x^2+y^2-1}\right) \longrightarrow \dot{y} = -x + y\frac{\dot{x}-y}{x}$

$x = r\cos\theta \qquad y = r\sin\theta \qquad r^2 = x^2+y^2$

$\dot{x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta \qquad \dot{y} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$

$\dot{r}\sin\theta + r\dot{\theta}\cos\theta = -r\cos\theta + \cancel{r}\sin\theta\left[\frac{\dot{r}\cos\theta - r\dot{\theta}\sin\theta - r\sin\theta}{\cancel{r}\cos\theta}\right]$

$\cancel{\dot{r}\sin\theta\cos\theta} + r\dot{\theta}\cos^2\theta = -r\cos^2\theta + \cancel{\dot{r}\sin\theta\cos\theta} - r\dot{\theta}\sin^2\theta - r\sin^2\theta$

$r\dot{\theta}\underbrace{(\sin^2\theta+\cos^2\theta)}_{1} = -r\underbrace{(\sin^2\theta+\cos^2\theta)}_{1}$

$\cancel{r}\dot{\theta} = -\cancel{r}$

$\boxed{\dot{\theta} = -1}$

$\dot{r}\cos\theta - r\dot{\theta}\sin\theta = r\sin\theta + r\cos\theta(r^2-1)\sin\left(\frac{1}{r^2-1}\right)$

$\dot{r}\cos\theta + \cancel{r\sin\theta} = \cancel{r\sin\theta} + r\cos\theta(r^2-1)\sin\left(\frac{1}{r^2-1}\right)$

$\dot{r}\cancel{\cos\theta} = r\cancel{\cos\theta}(r^2-1)\sin\left(\frac{1}{r^2-1}\right)$

$\boxed{\dot{r} = r(r^2-1)\sin\left(\frac{1}{r^2-1}\right)}$

$\frac{1}{r^2-1} = n\pi$

$\frac{1}{n\pi} = r^2-1$

$\frac{1}{n\pi}+1 = r^2$

$r = \pm\sqrt{\frac{1}{n\pi}+1}$

assume $r>0$

when $\sin\left(\frac{1}{r^2-1}\right) = 0$ then $\dot{r} = 0$

So values for r when $\frac{1}{r^2-1} = \pi, 2\pi, \ldots$ are all limit cycles

$0 \le r < 1$

$r=0, \dot{r}=0 \rightarrow$ eq pt

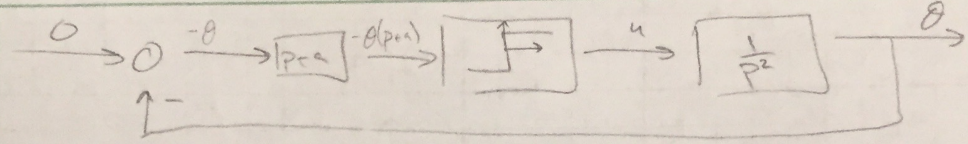
$r=1$ $\dot{r}$ = undefined

Stability

n	δr	$\dot{r}$	
1	$-$	$-$	unstable
1	$+$	$+$	
2	$-$	$+$	stable
2	$+$	$-$	
3	$-$	$-$	unstable
3	$+$	$+$	
4	$-$	$+$	stable
4	$+$	$-$	

when $\frac{1}{r^2-1} = 2\pi, 4\pi, 6\pi, \ldots$ the limit cycle is stable

when $\frac{1}{r^2-1} = \pi, 3\pi, 5\pi, \ldots$ the limit cycle is unstable



$$u(t) = \begin{cases} -U & \text{if } -\dot\theta + a\theta \le 0 \\ U & \text{if } -\dot\theta + a\theta > 0 \end{cases}$$

if $u(t) = U$:

$$\ddot\theta = U$$

$$\dot\theta\, d\dot\theta = U\, d\theta$$

$$\tfrac{1}{2}\dot\theta^2 = U\theta + C_1$$

$$\dot\theta^2 = 2U\theta + C_1$$

if $u(t) = -U$:

$$\ddot\theta = -U$$

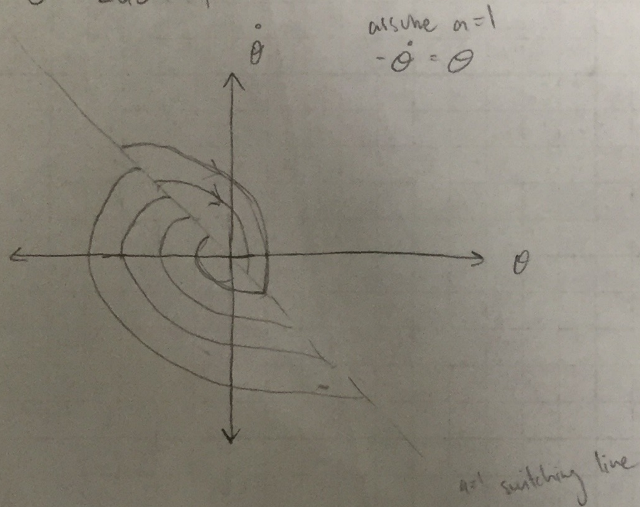
$$\dot\theta\, d\dot\theta = -U\, d\theta$$

$$\tfrac{1}{2}\dot\theta^2 = -U\theta + C_2$$

$$\dot\theta^2 = -2U\theta + C_2$$

assume $a=1$

$-\dot\theta = \theta$



stable that circle in to

$\dot\theta = 0,\ \theta = 0$