Nonlinear Dynamics (and Control)

Lecture #1

What is a Nonlinear system?

- A system that does not satisfy the superposition principle
  - output of the system is not directly proportional to the input.

Why study them?

- they are omnipresent
  - Aircraft control, biomedical engineering, process control, robotics, ...
- Nonlinear control techniques are useful for
  - improvement of existing control techniques
    - linear systems operate only in very small range of the statespace. (such as simplified pendulum dynamics)
    - Non-linear control systems can operate in large ranges typically.
  - Nonlinearities like Columb friction, backlash, hysteresis always exist

- Dealing w/ model uncertainties
    - robust controllers,
    - adaptive "
- Sometimes the system formulation is simpler.

Types of non-linearities → natural → inherent to natural sys.
→ intentional → introduced by designer
→ Discontinuous
→ Continuous.

Linear Systems

$\dot{x} = Ax$ Linear time-invariant Control System (LTI) ⇒ $\dot{x} = 0$

- have unique equilibrium pt if A is non-singular.
- equilibrium pt. is stable if all eigenvalues of A have negative real parts, regardless of initial conditions
- transient response ~~dis~~ composed of natural modes of system

(exponentials corresponding to solution: $e^{\lambda_i t}$, $i = 1, 2, .., N$)

- in the presence of external input

$$\dot{x} = Ax + Bu$$

Asymptotic stability of $\dot{x} = Ax$ $\Rightarrow$ BIBO stability of $\dot{x} = Ax + Bu$

<u>Example</u> (Pg 5 of textbook)

$$\dot{v} + |v|v = u$$

u → input

[Compare w/ linear sys $\dot{x} + x = u$]

v = velocity of underwater vehicle

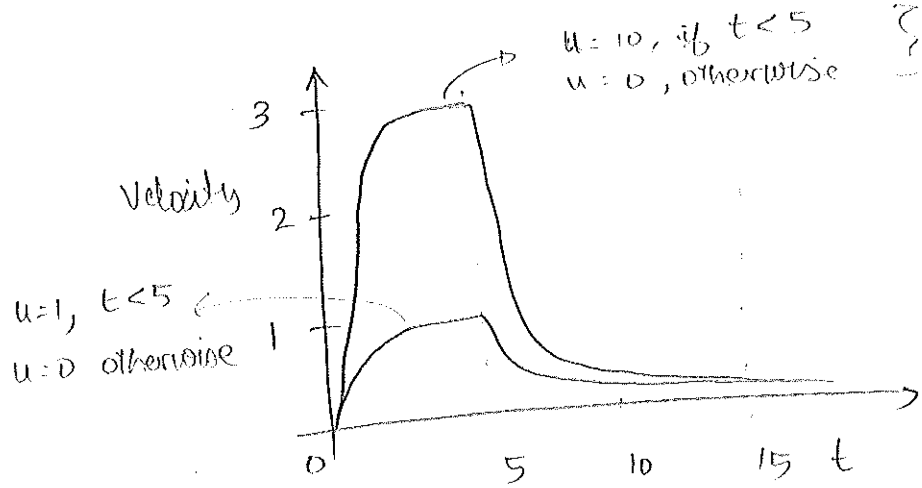
$$\dot{v} = u - \underbrace{|v|v}$$

↓ damping coeff = f (velocity)

Affects Response @ high velocities

- Step response different @ different velocities
- Steady state velocities not proportional to step inputs.
  - You cannot "add" responses. (steady state velocity $= \sqrt{u}$)

[Is $\dot{x} + x^2 = 0$ stable?]



{ Velocity stabilizes when input + drag balance.

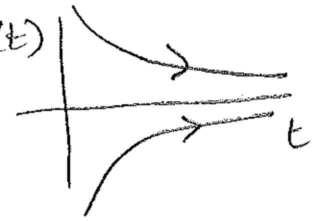
Common Nonlinear System Properties

(1) Multiple <u>Equilibrium points</u> → (point where the system can stay forever)

$$\dot{x} = -x + x^2 \text{ , w/ initial condition } x(0) = x_0$$

<u>Linearization</u>:

$$\left.\begin{aligned}\dot{x} &= -x \\ \text{Solution} \Rightarrow x(t) &= x_0 e^{-t}\end{aligned}\right\} \Rightarrow \text{equilibrium point } x = 0$$



<u>Actual full-order solution</u>

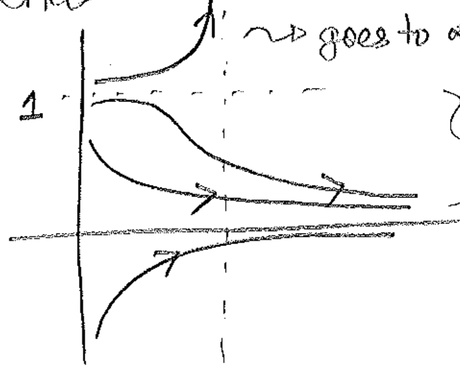
$$x(t) = \frac{x_0 e^{-t}}{1 - x_0 + x_0 e^{-t}}$$

Two equm pts: $\left.\begin{aligned} x &= 0 \\ x &= 1 \end{aligned}\right\}$

$-x + x^2 = 0$

$x(-1+x) = 0$

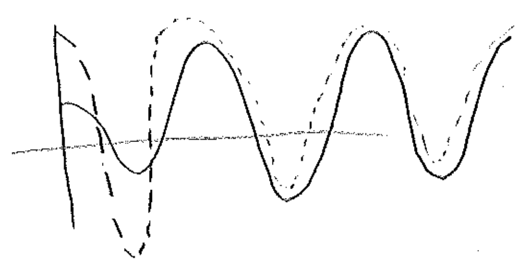
{ and behavior depends on initial conditions



(2) Limit Cycles

Nonlinear systems can display fixed amplitude and fixed period oscillations w/o external excitation.

$$m\ddot{x} + 2c(x^2 - 1)\dot{x} + kx = 0$$

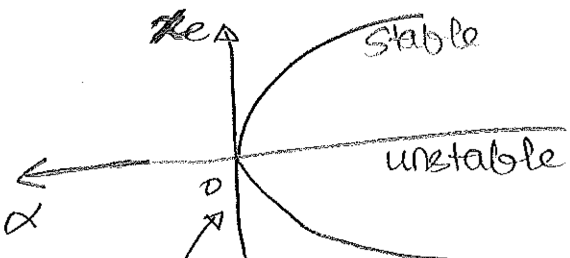


(3) Bifurcations

- Stability depends on system parameters.
- Parameter values @ which system behavior changes are called bifurcation/critical values.
- Bifurcation theory ⇒ quantitative change in parameters leading to qualitative change in system behavior.

<u>Duffing equation</u> : smoke rising from an incense stick

$$\ddot{x} + \alpha x + x^3 = 0$$



$x_e = 0, \sqrt{\alpha}, -\sqrt{\alpha}$

pitchfork bifurcation

[Another type of bifurcation : Hopf bifurcation
P10 in book]

(4) Chaos - System response sensitive to initial conditions

- eg. Turbulence in fluid mechanics.
  Weather phenomena.

[chaos cannot occur in linear systems]
(reason? superposition principle)

Standard methods to analyze nonlinear (control) systems

(1) Phase-plane analysis – graphical method for analyzing 2^{nd} order systems

(2) Lyapunov theory – Indirect method or linearization method (linear around equilibrium)
– Direct method –

(3) Describing functions – Find "linear equivalents" & then use frequency methods
– used to predict limit cycles.

In this course, mainly (1) & (2)