ME 533 Nonlinear Dynamics Analysis

Homework 2

All problems are from the textbook from the textbook "Applied Nonlinear Control" by Slotine and Li, Prentice Hall, 1991.

Problem 1

(Problem 3.1 from textbook)
The norm used in the definition of stability need not be the usual Euclidean norm. If the state-space is of finite dimension n (i.e., the state vector has n components), stability and its type are independent of the choice of norm (all norms are equivalent), although a particular choice of norm simplify analysis. For n=2, draw the unit balls corresponding to the following norms:

(i) ||x||^2 = x_1^2 + x_2^2
(ii) ||x||^2 = x_1^2 + 5 x_2^2
(iii) ||x|| = |x_1| + |x_2|
(iv) ||x|| = Sup(|x_1|,|x_2|)

Recall that a Ball B(x_0, R) with center x_0 and radius R, is the set of x such that ||x-x_0|| <= R and that the unit ball is B(0, 1).

Problem 2

(Problem 3.2 from textbook)
For the following systems, find the equilibrium points and determine their stability. Indicate whether the stability is asymptotic and whether it is global.
(i) x_dot = -x^3 + (sinx)^4
(ii) x_dot = (5 – x)^5
(iii) x_ddot + x_dot^5 + x^7 = x^2 (sinx)^8 cos(3x)^2
(iv) x_ddot + (x-1)^4 x_dot^7 + x^5 = x^3 (sinx)^3
(v) x_ddot + (x-1)^2 x_dot^7 + x = sin(Pi x/2)

Problem 3

(Problem 4.2 from textbook)
Analyze the stability of the dynamics (corresponding to a mass shrinking in a viscous liquid):
v_dot + 2 a |v| v + b v = c

Problem 4

(Problem 4.9 from textbook)
Determine whether the following systems have a stable equilibrium. Indicate whether the stability is asymptotic and whether it is global.
x = (x_1, x_2)^T
x_dot = A x.
(i) A = ((-10, e^(3t)), (0, -2))

(ii) A = ((-1, 2 sin(t)), (0, -(t+1)))
(i) A = ((-1, e^(2t)), (0, -2))