Geometric Mechanics HW 3 Due: 10/30/17

Comprehension

1) first constraint restricts the system to points that are a fixed distance from $(0,0,0) \in \mathbb{R}^3$

Same accessible manifold as

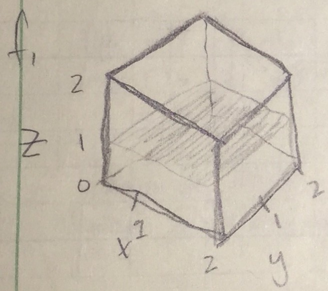
$$f_1(p,t) = z - 1$$

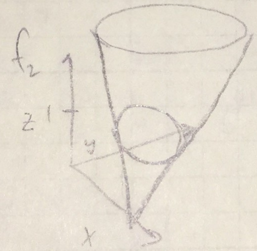
$$f_2(p_1,t) = \sqrt{x^2+y^2} - 1$$

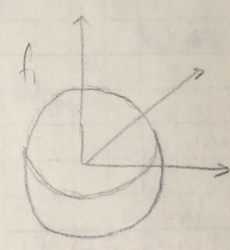
Equation should be zero when constraints satisfied

$$f_1(p,t) = \sqrt{x^2+y^2+z^2} - d$$

$$f_2(p_1,t) = z - 1$$







$0 = z - 1$

$z = 1$

$0 = \sqrt{x^2+y^2+(1)^2} - d$

$d^2 = x^2+y^2+1$

$d^2 - 1 = x^2+y^2$

want unit circle so

$d^2 - 1 = 1$

$d^2 = 2$

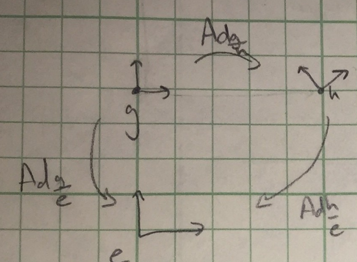
$d = \sqrt{2}$

Sphere with radius d, and center at $c = (0,0,0)$

$$\boxed{\begin{aligned} f_1(p,t) &= \sqrt{x^2+y^2+z^2} - \sqrt{2} \\ f_2(p_1,t) &= z - 1 \end{aligned}}$$

2)

$$\mathrm{Ad}_{\frac{g}{e}}\, \overset{\circ\!\to}{g} = \mathrm{Ad}_{\frac{h}{e}}\, \overset{\circ\!\to}{h} \qquad \overset{\circ\!\to}{h} = \mathrm{Ad}_{\frac{g}{h}}\, \overset{\circ\!\to}{g}$$



$$\overset{\circ\!\to}{h} = \mathrm{Ad}_{\left(\frac{h}{e}\right)}^{-1} \mathrm{Ad}_{\frac{g}{e}}\, \overset{\circ\!\to}{g} \implies \mathrm{Ad}_{\left(\frac{h}{e}\right)}^{-1} \mathrm{Ad}_{\frac{g}{e}} = \mathrm{Ad}_{\frac{g}{h}}$$

$$\mathrm{Ad}_{\left(\frac{e}{h}\right)} \mathrm{Ad}_{\frac{g}{e}} = \mathrm{Ad}_{\frac{g}{h}}$$

$\overset{\leftarrow\!\circ}{g} = \overset{\leftarrow\!\circ}{h}$ because rigidly attached

$$\overset{\leftarrow\!\circ}{g} = (T_g R_{g^{-1}})(T_e L_g)\overset{\circ\!\to}{g}$$

$$\overset{\leftarrow\!\circ}{h} = (T_h R_{h^{-1}})(T_e L_h)\overset{\circ\!\to}{h}$$

equal

$$(T_g R_{g^{-1}})(T_e L_g) = \overset{\mathrm{ad}_g}{\begin{bmatrix} c\theta & -s\theta & y \\ s\theta & c\theta & -x \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & y \\ 0 & 1 & -x \\ 0 & 0 & 1 \end{bmatrix}$$

$\theta = 0$ for g (assume no rotation)

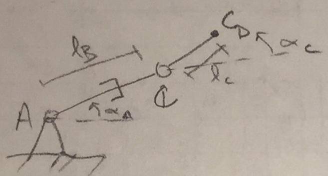
$$(T_h R_h^{-1})(T_e L_h) = \overset{\mathrm{ad}_h^{-1}}{\begin{bmatrix} c\alpha & \sin\alpha & x\sin\alpha - y\cos\alpha \\ -s\alpha & \cos\alpha & x\cos\alpha + y\sin\alpha \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\mathrm{Ad}_h^{-1}\mathrm{Ad}_g = \begin{bmatrix} c\alpha & s\alpha & x\sin\alpha - y\cos\alpha \\ -s\alpha & c\alpha & x\,c\alpha + y\,s\alpha \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & y \\ 0 & 1 & -x \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha & s\alpha & \cancel{y\,c\alpha} - \cancel{x\,s\alpha} + \cancel{x\,s\alpha} - \cancel{y\,c\alpha} \\ -s\alpha & c\alpha & -\cancel{y\,s\alpha} - \cancel{x\,c\alpha} + \cancel{x\,c\alpha} + \cancel{y\,s\alpha} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} c\alpha & s\alpha & 0 \\ -s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}} \checkmark$$

3) Setup matrix calculation using body, spatial, & exponential approaches.



$$s = \begin{bmatrix}\alpha_A \\ \ell_B \\ \alpha_C\end{bmatrix}$$

$$\vec{a}_A = \vec{a}_C = \begin{bmatrix}0 \\ 0 \\ 1\end{bmatrix} \qquad \vec{a}_B = \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix}$$

$$Ad_g^{-1} = \begin{bmatrix} c & s & xs - yc \\ -s & c & xc + ys \\ 0 & 0 & 1 \end{bmatrix} \vec{g}$$

Body approach

$$\vec{g}_{cd} = \overbrace{\begin{bmatrix} Ad^{-1}_{h_{\frac{cd}{0}}} \vec{a}_A & Ad_{h_{\frac{cd}{Ad}}} \vec{a}_B & Ad^{-1}_{h_{\frac{cd}{Bd}}} \vec{a}_C \end{bmatrix}}^{J^b_{cd}} \dot{s}$$

$$Ad^{-1}_{h_{\frac{Ad}{0}}} = Ad^{-1}_{a_A h_A} Ad^{-1}_{a_B h_B} Ad^{-1}_{a_C h_C} \qquad Ad_{h_{\frac{cd}{Ad}}} = Ad^{-1}_{a_C h_C} Ad^{-1}_{a_B h_B} \qquad Ad_{h_{\frac{cd}{Bd}}} = Ad^{-1}_{a_C h_C}$$

$$Ad^{-1}_{a_C h_C} = Ad^{-1}_{h_C} Ad^{-1}_{a_C} \qquad Ad^{-1}_{a_B h_B} = Ad^{-1}_{h_B} Ad^{-1}_{a_B} \qquad Ad^{-1}_{a_A h_A} = Ad^{-1}_{h_A} Ad^{-1}_{a_A}$$

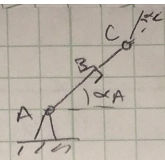
$$Ad^{-1}_{h_C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \ell_C \\ 0 & 0 & 1 \end{bmatrix} \qquad Ad^{-1}_{h_B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \ell_B \\ 0 & 0 & 1 \end{bmatrix} \qquad Ad^{-1}_{h_A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Ad^{-1}_{a_C} = \begin{bmatrix} c\alpha_C & s\alpha_C & 0 \\ -s\alpha_C & c\alpha_C & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Ad^{-1}_{a_B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Ad^{-1}_{a_A} = \begin{bmatrix} c\alpha_A & s\alpha_A & 0 \\ -s\alpha_A & c\alpha_C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{g_{cd}} = T_e L_{g_{cd}} J^b_{g_{cd}} \qquad T_e L_g = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_e L_{g_{cd}} = \begin{bmatrix} c(\alpha_A + \alpha_C) & s(\alpha_A + \alpha_C) & 0 \\ -s(\alpha_A + \alpha_C) & c(\alpha_A + \alpha_C) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Spatial



$$\overset{\leftrightarrow}{g}_n = \sum_{i=1}^{n} Ad_{g_{(i-1)d}} (\dot{\alpha}_i \vec{a}_i)$$

$$ad_g = \begin{bmatrix} c & -s & y \\ s & c & -x \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overset{\leftrightarrow}{g}_3 = Ad_{g_{Bd}} (\dot{\alpha}_C \vec{a}_C) + Ad_{g_{Ad}} (\dot{l}_B \vec{a}_B) + Ad_{g_0} \dot{\alpha}_A \vec{a}_A$$

$$S = \begin{bmatrix} \alpha_A \\ l_B \\ \alpha_C \end{bmatrix} \quad \vec{a}_A = \vec{a}_C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \vec{a}_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$Ad_{g_{Bd}} = \begin{bmatrix} c\alpha_A & -s\alpha_A & l_B \sin\alpha_A \\ s\alpha_A & c\alpha_A & -l_B \cos\alpha_A \\ 0 & 0 & 1 \end{bmatrix} \quad Ad_{g_{Ad}} = \begin{bmatrix} c\alpha_A & s\alpha_A & 0 \\ s\alpha_A & c\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Ad_{g_0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{g_C}^{S} = \begin{bmatrix} Ad_{g_0} (\vec{a}_A) & Ad_{g_{Ad}} (\vec{a}_B) & Ad_{g_{Bd}} (\vec{a}_C) \end{bmatrix}$$

$$T_e R_g = \begin{bmatrix} 1 & 0 & -y \\ 0 & 1 & x \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_e R_{g_{Cd}} = \begin{bmatrix} 1 & 0 & -(l_B \sin\alpha_A + l_C \sin(\alpha_A + \alpha_C)) \\ 0 & 1 & l_B \cos\alpha_A + l_C \sin(\alpha_A + \alpha_C) \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{g_{Cd}} = T_e R_{g_{Cd}} \cdot J_{g_C}^{S}$$