ROB 541 – 4 credits (lecture)

Geometric Mechanics

Monday and Wednesday 10:00-11:50

Bexell Hall 328 Oregon State University

Fall 2017

Instructor Information

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- Office location: Graf 317
- Office hours: Thursday 09:30-10:30

Prerequisites

Graduate standing, along with prior exposure to linear algebra and differential equations.

Course overview

This course serves as an introduction to geometric methods in the analysis of dynamic systems. Using the kinematics of simple robotic systems as a motivating example, we explore topics such as:

- Manifolds and Lie groups
- Representations of velocity
- Holonomic and nonholonomic constraints
- Constraint curvature and response to cyclic inputs
- Distance metrics

The primary goal of this class is to develop an intuitive understanding of these concepts and how they are used, rather than working through a set of formal proofs and derivations. We will, however, incorporate enough mathematical formalism for this class to serve as a starting point for further investigation into this topic area.

Learning outcomes

By the end of this course students will be able to

- Use structured mathematical spaces like manifolds and Lie groups to describe the configuration spaces of physical systems,
- Use constraints on these spaces to model the dynamics of such systems, and
- Use the way in which these constraints change over the configuration spaces to identify and characterize nonlinear aspects of the system dynamics.

Course readings

- *An Introduction to Geometric Mechanics and Differential Geometry* (Draft version), Hatton and Choset. Distributed on course Canvas site.
- Supplemental papers. Distributed on course Blackboard site.

Homework and Grading

There are three components to the course assignments:

1. Comprehension questions: short questions designed to check your understanding of the mathematical ideas introduced in the course.

2. Implementation tasks: Code up examples from the textbook, to become familiar with how these tools work in practice.

3. Term project: In teams of two, select a system (ideally, related to your research). Describe an interesting problem related to this system, and use the topics discussed in this course to bring insight to this problem.

The specific assignment schedule is provided at the end of this syllabus.

- Problem Sets: 40%
- Final Project: 60%

Statement Regarding Students with Disabilities

Accommodations are collaborative efforts between students, faculty and Disability Access Services (DAS). Students with accommodations approved through DAS are responsible for contacting the faculty member in charge of the course prior to or during the first week of the term to discuss accommodations. Students who believe they are eligible for accommodations but who have not yet obtained approval through DAS should contact DAS immediately at 737-4098

Student Conduct Code and Academic Integrity

Link to student conduct code:
`http://studentlife.oregonstate.edu/studentconduct/offenses-0`

Dates	Topic	Subjects	Homework
21-Sep	Introduction and Overview		
26-Sep	Configuration spaces I	Manifolds & Lie groups (Homeomorphism, Diffeomorphism, Direct product, isomorphism)	Configuration space questions and coding Due Oct 7
28-Sep	Configuration spaces II	Rigid bodies (Semi-direct products)	
3-Oct	Velocities I	Tangent spaces (vector fields and differential mappings), Groupwise velocity	Velocities questions and coding Due Oct 14
5-Oct	Velocities II	Rigid body velocities	
10-Oct	Forward Kinematics I	Chains of rigid bodies, exponential map, start Jacobians	Kinematics questions and coding due October 21
12-Oct	Forward Kinematics II	Finish Jacobians, mobile systems	
17-Oct	Kinematic Locomotion I	Fundamentals of kinematic locomotion (nonholonomic constraints)	Locomotion questions and coding due October 28
19-Oct	Kinematic Locomotion II	Full-body locomotion	
24-Oct	Gaits I	Satellite turning (nonconservativity and Stokes' theorem) and Parallel parking (noncommutativity and Lie brackets)	Sysplotter exercises 1 due November 11
26-Oct	Gaits II	Undulatory locomotion (combining nonconservativity and noncommutativity)	
31-Oct	Minimum-perturbation coordinates	Hodge-Helmholtz decomposition	Sysplotter exercises 2 due Nov 18
2-Nov	Continuous-structure kinematics	Modal shapes and the Leibniz rule	
7-Nov	Distance Metrics I	Fundamentals of distance metrics (metric tensors, generating metrics, cartographic projection)	
9-Nov	Distance Metrics II	Efficiency, Soap bubbles and the Leibniz rule	
14-Nov	Inertial Costs		Work on final projects
16-Nov	Continued discussion of previous topics		
21-Nov	Final Project Consultation		
23-Nov	Thanksgiving		
28-Nov	Final Project Presentations		
30-Nov	Final Project Presentations		

Assignment details

Note that the topics are typically spaced a week apart, and are due a week after the lectures on the material that they cover.

Due October 7

Comprehension questions

1. How many charts do we need in the atlas for
   (a) A cylinder?
   (b) A torus?
2. Does the set of integers form a group under multiplication? Why or why not?
3. Generate canonical matrix-multiplication representations for the additive group $(\mathbb{R}, +)$ and the direct-product scale-shift group.

Implementation task Use Matlab or Python to write a set of functions that implement transformations on $SE(2)$. You should specifically be able to:

1. Create 3×3 matrix representations of group elements from their (x, y, θ) parameters.
2. Compose group elements to produce new group elements.
3. Multiply group elements by point locations to get the global positions of points in a local frame.

Demonstrate that you have done this by producing an animation with the following characteristics:

1. Draw an object with an identifiable orientation, starting at the origin and with zero orientation.
2. Move this object so that it follows an S-shaped curve formed by two semi-circles, with orientation tangent to the curve.
3. Also illustrate the path of a second object that is rigidly attached to the first object (e.g., draw this object as moving along with the first object, and trace out its (x, y) position).

Due October 14

Comprehension question A rigid body at $g = (1, 1, -\pi/4)$ is moving with velocity $\dot{g} = [1, 0, 1]^T$. For rigid bodies at locations

- $g_1 = (0, 0, 0)$
- $g_2 = (-1, 1, \pi/2)$
- $g_3 = (-1, -1, -\pi/4)$
- $g_4 = (1, -1, \pi)$

Find:

1. The velocity $\dot{g}_i$ of each body if all of them have the same body velocity $\overset{\circ\!\!\rightarrow}{g}$.
2. The velocity $\dot{g}_i$ of each body if all of them have the same spatial velocity $\overset{\leftarrow\!\!\circ}{g}$
3. The body velocity $\overset{\circ\!\!\rightarrow}{g}_i$ of each body if all of them have the same spatial velocity $\overset{\leftarrow\!\!\circ}{g}$

Implementation task Extend your rigid-body tools to include lifted actions acting on the group velocities. Demonstrate this functionality by:

1. Augmenting your movie from the first implementation task with velocity vectors that show
   (a) the world velocities of the two objects, illustrated as arrows or lines attached to the moving objects
   (b) the body velocities of the two objects, illustrated on separate axes in which each object is at its local origin.
2. Making a set of plots in which spatial generator fields are underlayed behind the trajectories traced out by the object, illustrating that in each half of the curve, the motions of both objects are flows along a common spatial generator.

Due October 21

Comprehension questions

1. Find a set of holonomic constraints that produce the same accessible manifold as that produced by the constraints in (B3.1) and (B3.2), but for which the first constraint restricts the system to points that are a fixed distance from $(0, 0, 0) \in \mathbb{R}^3$.

2. Show (in coordinates) that $Ad_h^{-1} Ad_g$ for $SE(2)$ elements where the x and y values of g and h are equal produces the rotational change of basis operation, as discussed in the greybox about adjoints.

3. Set up the matrix calculations for finding the Jacobian of a rotary-prismatic-rotary arm, using each of the body, spatial, and exponential approaches.

Implementation task Use your rigid-body toolset to construct a multi-body kinematics function. This function should:

1. Take in a list of local joint axes $\overset{\circ}{\vec{a}}$, link transforms h and $h_{\frac{m}{p}}$, and a set of joint values α.

2. Draw the corresponding mechanism in the specified configuration.

3. Place arrows or lines at the center of each link, illustrating the direction in which the center points will move as each joint is moved (color-code or otherwise indicate which joint is associated with each arrow).

Demonstrate this function by producing plots for a rotary-rotary-prismatic arm and a rotary-prismatic-rotary arm, in “interesting” configurations.

Additionally, make an animation of the arm moving through an “interesting” pattern, with a single arrow at the center of each link, illustrating the direction in which it will move.

Due October 28

Comprehension questions

1. Show that the nonholonomic constraints on the differential drive car induce a *holonomic* constraint between its orientation and its wheel angles.

2. Find the Pfaffian constraints on an Ackerman car (modeled as a kinematic bicycle), and use them to calculate the system's local connection

   (a) For the standard rear-wheel drive model

   (b) For a front wheel drive with passive rear wheels

Plot the connection vector fields for these systems.

Implementation task Generate the θ connection vector field for a three-link system, pinned to ground at the center of the middle link and subject to conservation of angular momentum. Plot this vector field, and also generate animations of how the system moves as the joints are moved

1. From straight-out $\alpha = (0, 0)$ to S-shaped $\alpha = (1, -1)$.

2. From straight-out $\alpha = (0, 0)$ to C-shaped $\alpha = (1, 1)$.

Due November 11

1. Download and run the Sysplotter Matlab program.
2. Using the draw-gait tools, find clockwise gaits that move the three-link viscous swimmer positively and negatively in the x direction.
3. Animate the motion of the system during these gaits.
4. Compare the area-rule estimate of the net displacement to the true net displacement for the circle-family of gaits, in both the middle-link and minimum-perturbation coordinates

Due November 18

Use the optimize-gait tool in sysplotter to find the locally-optimal gait corresponding to each local extrema of the x CCF for the three-link viscous swimmer. Animate the swimmers "racing" each other with these gaits, normalized for power consumption.