Comprehension questions

1. How many charts do we need in the atlas for (a) A cylinder? (b) A torus?
    a. (assuming it does not have a top and bottom) 2 charts - on the cylindrical surface with discontinuities as far apart as possible
    b. 4 charts - 2 from the inside, and 2 from the outside to deal with discontinuities around the circumference of the torus and the radius of the torus

2. Does the set of integers form a group under multiplication? Why or why not?

    No it does not form a group.

    1. Closure - ☑ - multiplication of two integers results in another integer
    2. Associativity - ☑ - sequence of multiplication does not affect result
    3. Identity Element - ☑ - 1 is the identity integer
    4. Inverse - ☒ - inverse of an integer is not an integer (except for identity)

3. Generate canonical matrix-multiplication representations for the additive group (R, +) and the direct-product scale-shift group.

For the additive group for up to n dimensions (n=5 in this example):

[1, 0, 0, 0, 0, 0, 0, 0, 0],
[a1, 1, a2, 0, a3, 0, a4, 0, a5],
[0, 0, 1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 1, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 1, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 1, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 1, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 1]

For the scale shift group:

[a1, 0, 0],
[0, 1, a2],
[0, 0, 1]]