Comprehension questions

1. How many charts do we need in the atlas for (a) A cylinder? (b) A torus?
    a. (assuming it does not have a top and bottom) 2 charts - on the cylindrical surface with discontinuities as far apart as possible
    b. 3 charts - 1 from the inside, and 1 from the outside to deal with discontinuities around the circumference of the torus. 1 more is needed to deal with where two of the discontinuities overlaps.

2. Does the set of integers form a group under multiplication? Why or why not?
    No it does not form a group.
    1. Closure - ☑ - multiplication of two integers results in another integer
    2. Associativity - ☑ - sequence of multiplication does not affect result
    3. Identity Element - ☑ - 1 is the identity integer
    4. Inverse - ☒ - inverse of an integer is not an integer (except for identity)

3. Generate canonical matrix-multiplication representations for the additive group (R, +) and the direct-product scale-shift group.

For the additive group for up to n dimensions (n=5 in this example):
The identity matrix is needed to ensure that the resulting matrix has values in all the same locations. The values of the additive group values need to be in one of the rows. In that row, the values should not be adjacent to each other. In the example below, the values are in the second row, but it should work in the 4th, 6th, or 8th row. This configuration ensures that no two values are multiplied together, but values in the same location will be added together, which is what we want from the group action.

[1, 0, 0, 0, 0, 0, 0, 0, 0],
[a1, 1, a2, 0, a3, 0, a4, 0, a5],
[0, 0, 1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 1, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 1, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 1, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 1, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 1]

For the scale shift group:
The value in the (1,1) will be multiplied with the same location in the second matrix. Similar to the example above, the remainder of the diagonals should be one. This will cause the second value in the group to be added together.

[a1, 0, 0],
[0, 1, a2],
[0, 0, 1]]