ELEC214P Interference, Diffraction and Polarization of Electromagnetic Waves

Important: Read this script well before the date of the experiment. You will need to prepare some material **before** coming to the laboratory.

This experiment has two parts. The first part should be completed before coming to the lab. In this part, you are given experimental data already available and you are expected to use Matlab to process and compare it with your theoretical predictions in your own time. At the start of the lab session you will be asked to show your preliminary work, including your Matlab programs and results, and the theoretical predictions for the experimental part (part 2). A mark will be given for this "pre-lab" test and this will be part of your report mark.

In the lab session you will need to work on Part 2 of this script.

Wave Propagation

Wave propagation is a central concept in physics and engineering. It is at the heart of most systems and devices in electronic engineering. Examples include among others, long, medium and short wave radio, microwave links, optical fibre systems, sonar, audio (acoustic) systems, optics and electron waves in semiconductor devices.

In this experiment we shall be concerned with electromagnetic waves, nevertheless many (but not all) of the results and conclusions will apply to other kinds of wave such as acoustic or electron waves. This session has two parts that deal with important aspects of wave propagation: The first part "Interference of waves", is for you to do in your own time and before the lab session. The second part, "Diffraction and Polarisation" will be done in the lab but you will need to prepare some material for this in advance. You will be asked to explain aspects of the theory and to show your theoretical predictions for this part at the start of the lab session. A mark will be given for this and will form part of the report mark. In the report you will have to include answers to the questions or specific tasks indicated in this script.

Part 1 Interference of waves

To be completed before the lab session. The answers to the questions in this part should be available for examination during the lab session and included in the report.

Two waves at the same frequency will give rise to interference effects when they overlap. This effect appears in numerous practical applications. It is the basis of interference fringes in, for example, optical interferometers and in the design of antenna arrays.

In this experiment we investigate a particularly simple case of interference of waves emitted from two narrow aperture sources. The experiment is essentially the microwave counterpart of Young's two-slit experiment in optics, which should be familiar to you. Microwaves are convenient for us because their wavelength is much longer than those of optical waves, and the interference pattern can therefore be measured with reasonable accuracy using a simple apparatus.

Consider the situation shown in Fig.1.

Two sources, S_1 and S_2, with equal amplitude and phase illuminate an observation plane where we consider an arbitrary point at a distance x from the central point denoted by 0. The sources could be 'line sources' (along z) or 'point sources' generating cylindrical or spherical waves, respectively. In either case waves propagating away from a source will suffer some

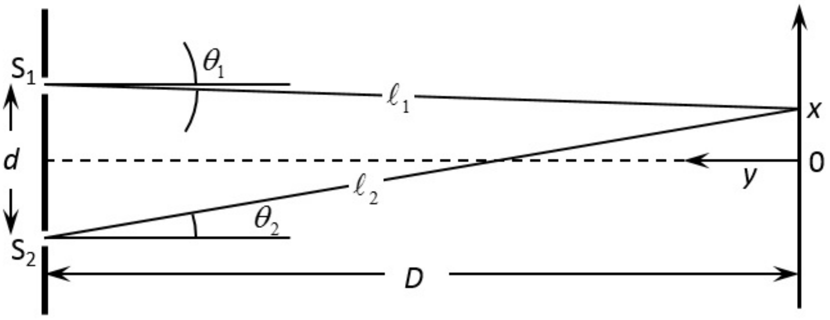

Fig. 1 Two in-phase sources, S_1 and S_2, illuminating an observation plane.

reduction in intensity ($|E|^2$) with $1/r$ or $1/r^2$, respectively. If the distance from the sources to the point x=x, ℓ_1 and ℓ_2, are nearly equal ($(\ell_1 - \ell_2)/D << 1$) we can approximate and neglect the difference in amplitudes at the observation plane. However, we <u>cannot</u> neglect the difference in phase. Hence, we can see that there will be constructive interference at some position x when the path lengths ℓ_1 and ℓ_2 differ by zero or an integer number of wavelengths *i.e.* when $\ell_1 - \ell_2 = n\lambda$. Similarly there will be destructive interference at some position x when the path lengths ℓ_1 and ℓ_2 differ by an odd integer number of half wavelengths *i.e.* when $\ell_1 - \ell_2 = (2n+1)\lambda/2$. Thus, along the x-axis there will be a series of maxima and minima of intensity.

We could obviously calculate a general expression for the intensity along the x-axis. The total field at x, on the observation plane without approximation is given by[1]:

$$E_T = \frac{e^{-jk\ell_1}}{\ell_1} + \frac{e^{-jk\ell_2}}{\ell_2} \tag{1}$$

Here we have considered a $1/r$ fall off of field (*i.e.* a $1/r^2$ fall off of intensity), corresponding to a point source radiating in 3D space.

From Fig. 1 we have:

$$\sin\theta_1 = \left(d/2 - x\right)/\ell_1 \qquad \text{and} \qquad \cos\theta_1 = D/\ell_1$$

also:

$$\sin\theta_2 = \left(d/2 + x\right)/\ell_2 \qquad \text{and} \qquad \cos\theta_2 = D/\ell_2$$

Then, since $\sin^2\theta + \cos^2\theta = 1$ we can write:

$$\begin{aligned} \ell_1 &= D\cos\theta_1 + \left(d/2 - x\right)\sin\theta_1 \\ \ell_2 &= D\cos\theta_2 + \left(d/2 + x\right)\sin\theta_2 \end{aligned} \tag{2}$$

Now if $\ell_1 \approx \ell_2 \approx D$, we can approximate E_T in (1) by:

$$E_T \approx \frac{1}{D}\left(e^{-jk\ell_1} + e^{-jk\ell_2}\right)$$

and substituting (2):

$$E_T \approx \frac{1}{D}\left(e^{-jk\left[(d/2-x)\sin\theta_1 + D\cos\theta_1\right]} + e^{-jk\left[(d/2+x)\sin\theta_2 + D\cos\theta_2\right]}\right) \tag{3}$$

Since θ_1 and θ_2 are small, we can make the following approximations:

[1] Here we assume that the fields originated by both sources are parallel at the point of observation (x) so we can use a scalar sum. This will be the case if their polarisation is along the z-axis.

$$\sin\theta_1 \approx \theta_1 \approx (d/2 - x)/D \quad \text{and similarly,} \quad \sin\theta_2 \approx \theta_2 \approx (d/2 + x)/D \,.$$

Then, for the cosine terms (using the first 2 terms of the Taylor series), we have:

$$\cos\theta_1 \approx 1 - \theta_1^2/2 \quad \text{and} \quad \cos\theta_2 \approx 1 - \theta_2^2/2 \,.$$

<u>Q1.</u> The square of the magnitude of the electric field is known as the intensity. Show in your report that the approximations above lead to:

$$|E_T|^2 \approx \frac{4}{D^2}\cos^2\frac{kdx}{2D} \tag{4}$$

From this expression, what is the distance between consecutive maxima or minima?

<u>Q2.</u> Download and run the Matlab script Interference.m that calculates the intensity versus position *x* using eqn. (1) directly instead of using the approximate expression (4). Modify the script to include a calculation of the intensity as it varies with the displacement *x* of the observation point on the screen using eqn. (4) as well and plot this in the same figure as the results from eqn. (1). Comment on the approximations used in (1)-(4), compare the results and discuss and explain the differences.

Experiment 1.1: Interference of waves

For this part of the experiment, you will be given experimental data obtained using the set-up described schematically in the following diagram.

Two in-phase equi-amplitude 'point' sources are provided by the rectangular waveguide T-junction feeding two open-ended waveguides of equal lengths. The signals from both sources are received by the horn at the right and fed to a microwave detector and amplifier. The horn is moved along a ruler on the *x*-axis and the measured output is observed as the horn is moved. The horn is kept aligned along the *y*-axis throughout this experiment. The input is provided by a microwave generator connected through a coaxial cable to the waveguide T-junction. It generates a microwave signal of 10 GHz modulated in amplitude with a 5 KHz square wave signal. We need to measure the intensity of the total field received by the horn and this is done using the setup indicated in the figure. The dimensions in Fig. 2 are: D = 255 cm and d = 63 cm.

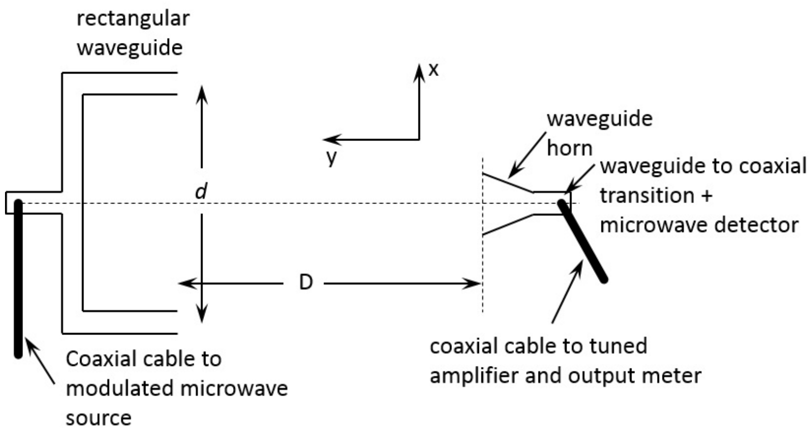

Fig. 2 Experimental arrangement for the interference experiment. Note that the picture is not at scale. In the practical situation D >> d.

At the receiver end there is a horn followed by a "detector", which is a piece of waveguide with a probe or a small antenna inside, connected through a microwave diode to the output coaxial cable. The signal received by the horn is very low and needs to be amplified before is sent to the meter. This is done by an amplifier circuit connected between the detector and the meter.

Q3. Explain the function of the detector diode. What is the frequency of the signal carried by the coaxial cable to the meter in the setup described in the figure above?
What is the relation between the electric field intensity received by the horn and the magnitude of the signal (current) coming from the detector through the coaxial cable? It happens that the output of the detector is proportional to $|E|^2$ *i.e.*, is proportional to the field intensity; can you explain why?

Q4. From the Moodle site of the course, download the file Int1.txt containing measurements done using the setup shown in Fig. 2. The file has two columns; the first is the displacement x of the receiving horn in cm, where the origin is at the centre, that is, when the horn is aligned to the midpoint between the two sources. The second column is the reading in the meter, in mV. Write a Matlab program that loads the file Int1.txt, and plots the received voltage versus x. Explain the relation between this voltage and the received field intensity by the horn. Name the program: Pattern1.m and include it in your report. The program should also plot the theoretical curve from eqn. (1) in the same figure. Compare the two curves and comment in your report. **Hint.** In your program, normalise the intensity values calculated from the given readings to maximum value 1 before plotting and comparing with the theoretical results.

Do the maxima have equal amplitude? Do all the minima have zero amplitude? Which of these, maxima or minima, will give a better, sharper definition of position? Comment on your observations and compare them with the theoretical results obtained for Fig.1 based on two point sources. Are the approximations in the theory justified? The theoretical description and derivations that follow Fig. 1 assumes that the electric field is vertical, that is, along the z-axis and that is also the case of the experiment that produced the results in the file Int1.txt from the set-up in Fig. 2.

Would there be any difference if the sources were transmitting waves with their electric fields in the x-y plane instead? If so, why?

At the observation plane the total field is measured using a horn receiver with a finite aperture. Does this introduce any complication?

Experiment 1.2 Measurement of relative permittivity of a dielectric material

Consider now the modified set-up shown in Fig. 3. A dielectric sheet is now interposed between one of the waveguide sources and the receiving horn. The dielectric slab will introduce an extra phase difference in one of the paths and this will cause a shift in the interference pattern in the observation plane.

Q5. Using simple theory, show that the relative permittivity of the sheet is given by:

$$\varepsilon_r = \left[1 + \frac{\Delta s}{\delta}\frac{d}{D}\right]^2 \qquad (5)$$

where Δs is the shift of the central maximum along the x-axis, and δ is the thickness of the dielectric sheet.

Hint. Consider the phase shift introduced by the thickness δ of the dielectric sheet and the corresponding shift in the position of the minima (or maxima). Since you know the phase difference that corresponds to the distance between minima (or maxima), you can determine the corresponding value of refractive index.

The angle between the propagation direction and the normal to the slab is actually very small (unlike Fig. 3) so assume normal incidence in these calculations.
Clearly state any assumptions you made in deriving this expression.

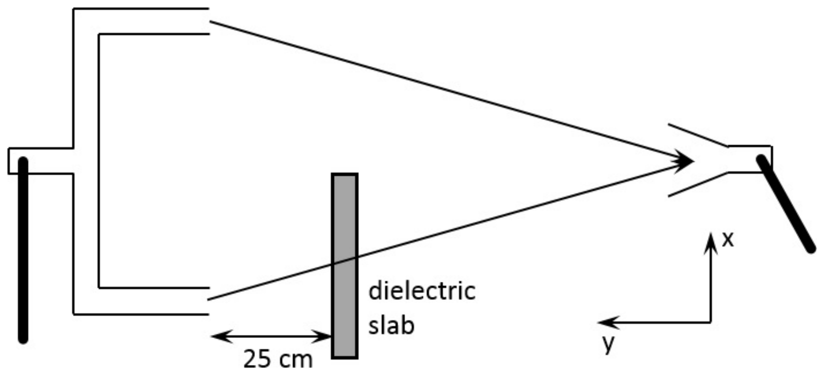

Fig. 3 Experimental arrangement for measurement of relative permittivity of a dielectric sheet.

<u>Q6.</u> Download the file Int2.txt that contains the new measurements of intensity versus *x*. This file has the same format as Int1.txt and the second column lists the readings of the instrument in mV. Create a new Matlab program modifying your Pattern1.m, to read this file and plot both the original and the shifted interference pattern in the same plot. Name this program ShiftedPattern.m and include it in your report. From the plots and the data files, determine the shift as accurately as you can and calculate the relative permittivity of the dielectric slab if its thickness is δ = 1.2 cm.
Would this method be suitable for (a) thick sheets (b) high permittivity sheets? If not, what would be the problem? Is the position of the dielectric sheet between the source and the screen important or relevant? Does it need to be 25 cm as shown in the figure? Discuss.

Exercise 1.3 Antenna Arrays

Antenna arrays are groups of antennas radiating at the same frequency, thus creating an interference pattern around them. This interference pattern can be shaped by varying the position and separation of the individual antennas and the amplitude and phase of their individual excitations.

Consider the arrangement in Fig. 4 that shows two point antennas separated by the distance *d* and excited with currents of value 1 and $e^{j\varphi}$ (equal magnitude, phase difference = φ).

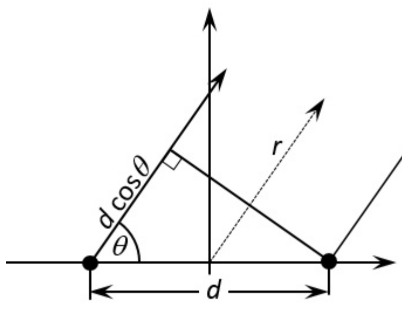

Fig. 4 An array of two point antennas.

In general, we are interested in the field radiated by the antennas at a very large distance from the antennas (known as the “far field”).

Following the same reasoning that leads to eqn. (1) we can write for the field at a very large distance the expression in equation (6). The assumption of a very large distance allows us to consider the rays from the individual antennas to the observation point as parallel to each other and the corresponding distances as equal to r in the amplitude term (denominators in eqn. 1). Note that the path difference cannot be neglected in the calculation of the phase (exponents in (1)). From this and Fig. 4 we can write:

$$E(r,\theta) = \frac{e^{-jk(r+d\cos\theta/2)}}{r} + \frac{e^{j\varphi}e^{-jk(r-d\cos\theta/2)}}{r} \qquad (6)$$

<u>Q7.</u> Starting from equation (6), find an expression for the absolute value of the far field as a function of the angle θ.

The final expression will have the form:

$$\left|E(r,\theta)\right| = \frac{F(\theta)}{r} \qquad (7)$$

where the factor $F(\theta)$ is called the radiation pattern of the array, or more properly, the Array Factor.

Write a Matlab program and name it Array.m, to calculate the array factor of this array as a function of the antenna separation d, the phase difference between the antennas, φ and the inclination angle θ. Use the Matlab command "polar" to plot the <u>normalised</u> array factor versus θ for the cases: (a) $d = \lambda/2$ and $\varphi = 0$, (b) $d = \lambda/2$ and $\varphi = 90$, (c) $d = \lambda/2$ and $\varphi = 180$, (d) $d = \lambda/4$ and $\varphi = 0$, (e) $d = \lambda/4$ and $\varphi = 90$ and (f) $d = \lambda/4$ and $\varphi = 180$. Include these plots and the program Array.m in your report.

Part 2 Diffraction and Polarisation

For this part you will need to work in the Lab with the setup provided and obtain sufficient data to complete the report satisfactorily. Read the script and the references before the lab session to know what to expect, to plan your measurements and to make sure you will have adequate and sufficient data. Plan your measurements carefully.

Many phenomena in electromagnetics can be explained adequately in terms of rectilinear wave propagation. For example, the performance of an optical lens can be studied in great detail by making the assumption that light travels in straight lines. Optical lens design is therefore largely geometry and refraction (Snell's Law). The dimensions of a lens in a camera or microscope are, in terms, in terms of optical wavelengths, very large. When the component or space under discussion has dimensions comparable with or smaller than the wavelength then we must use <u>wave theory</u> to adequately describe the situation. Examples where wave theory is required include optical fibres and optical waveguide components, precise determination of microwave microstrip waveguide components, design of microwave antennas, and modelling of radio propagation over hilly terrain.

For example, consider the problem shown diagrammatically in Fig.5.

A uniform plane wave is partially blocked by an opaque screen. There is an illuminated region and a region that is in shadow. This simple description seems to fit in well with everyday experience of shadows caused by obstacles to sunlight. But such everyday observation is not that precise. Our eyes can distinguish dimensions of the order of 10^{-4} m. The wavelength of light is less than 10^{-6} m. Suppose we ask what is happening at D in Fig. 5, in the transition between the illuminated region and the shadow region. How does the intensity suddenly drop to zero there? Even with only a little knowledge of electromagnetic theory we are forced to ask ourselves how the electric field 'ends' at D. See for example Fig. 6. Clearly, there is not a sharp edge here but there must be a transition region. This is dependent on the wavelength so in the optical range these effects occur is the order of microns so shadows appear to be sharp. In microwaves, though, with wavelengths in the order of cm and mm these effects are clearer to see.

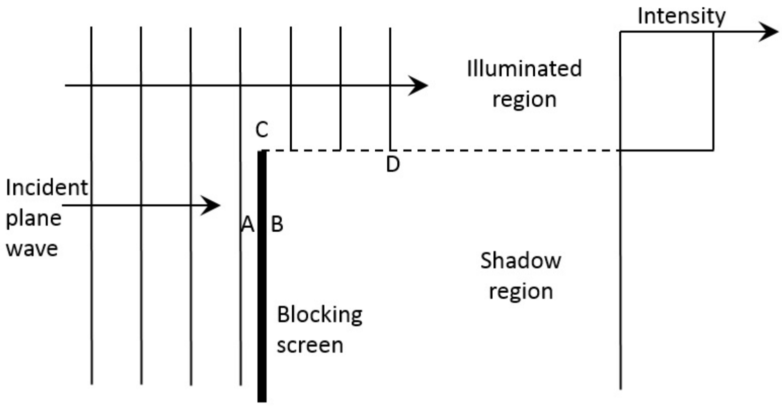

Fig. 5 A simple treatment of a uniform plane wave partially blocked by an opaque screen.

The nature of the screen is also important. If it is absorbing then that part of the incident wave arriving at A will be absorbed. However, if the screen is electrically conducting, that part of the incident wave will be reflected. With a conducting screen there might be a current at C. Will this current not radiate into the shadow region? Will there not be some field in the 'shadow' region whatever the screen type?

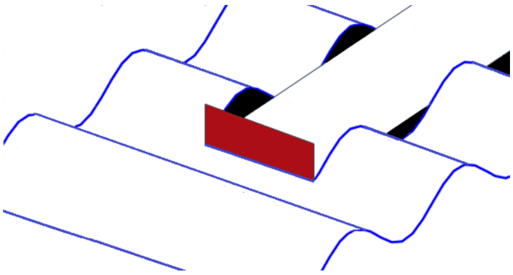

Fig. 6 Detail of Fig. 5 showing unrealistic behaviour for a wave

To understand what happens we can consider each point in a phase front[2] (infinite number of them and infinitesimally close) as a point source (Huygens's principle[3]), so each of them will produce a spherical (or cylindrical) wave, which will superpose or interfere with each other creating the next wave front.

If the points radiate in phase and are on a plane, the next phase front, which is the superposition of the individual wavelets, will also be a plane: the interference pattern of the infinite array of point sources radiating in phase is just a plane, see Fig. 7 below. If we now consider the phase front passing at position C in Fig. 5, there are no source points on the screen side of C and the wavelets at the edge are not "compensated"; the new phase front will then curve at the edge and there will be propagation into the shadow region due to the interference effect of the infinite number of source points. This phenomenon is called diffraction. Additionally, the nature of the screen will affect the result

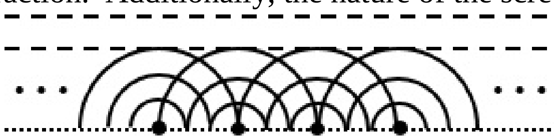

Fig. 7 Each point in a phase front will generate a "wavelet" which will interfere with each other producing the next phase front.

since for example, in a conductive screen, excited currents will radiate towards the screen, contributing to the total field there.

Strictly speaking, a proper vector theory should be used, involving Maxwell's equations in vector form, boundary conditions, and considerable mathematics. Fortunately, in situations

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[2] A <u>*phase front*</u> or wavefront in connection to a propagating wave is a surface of constant phase, like the spherical surfaces surrounding a point source.

[3] Huygens principle states that:" *Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves."*

where the observation plane is many wavelengths away from the diffracting objects, reasonably accurate answers can be obtained using scalar diffraction theory.

Q8. Consult the references indicated for the Fresnel's diffraction theory[1] and/or any other source of information and include in your report a description in your own words of the theory of diffraction, in particular concerning the diffraction by a screen as in Experiment 3. You must include a diagram of the expected results for the comparison with your experimental results (Q.9)

Note: When using material taken from a book, journal, webpage or any other source, textual reproduction (or very similar) is not permissible. Use your own words and explanations and quote the corresponding source using one of the standard formats to quote references.

Experiment 2.1: Diffraction of a wave by a conducting plane

Consider now the set up indicated in Fig. 8 below.

An electromagnetic wave is radiating from a horn antenna and a movable metal screen is placed between this source and the receiver antenna. The receiver is the same described in Part 1 of this script.

Slide the metal screen along the rail, in the *x* direction so that the receiving horn is progressively: fully illuminated by the source; partially blocked from the source by the screen; and fully in the shadow region. Concentrate on your observation on the effects of one edge of the screen; that is, don't slide the screen to pass all the way in front of the source but from far from one side up to the centre the screen.

You should know in advance the shape of the results you are expecting. Before recording any results, observe the output meter as you slowly slide the edge of the screen across the source. The output should range from high (when the screen is fully illuminated) – to high with oscillations – to low – tending to zero (when is fully in the shadow). Record the values trying to represent the expected variation of the amplitude, not just at regular intervals. That is, more points will be needed where the values oscillate. Use sufficient number of points and sufficiently close to follow accurately the expected variations.

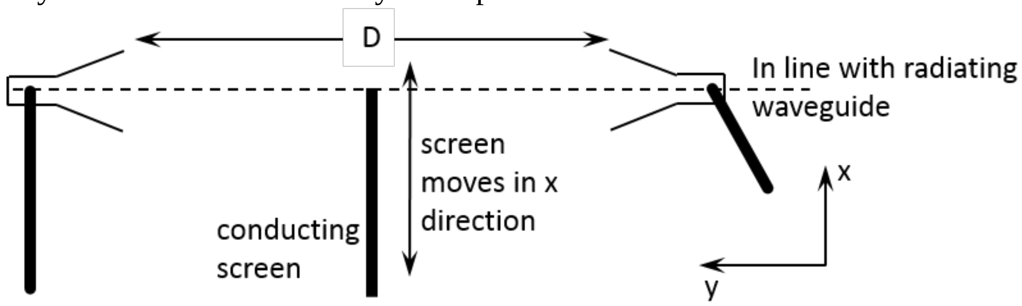

Fig. 8 Experimental arrangement for diffraction experiment.

The situation is somehow different to the case depicted in Fig.5 where the wave incident at the screen was supposed to be a plane wave. In our experimental set-up the incident wave comes from an antenna at a finite distance from the screen, so the equivalent wavelets (see Fig. 7) from points in the aperture (not obstructed part of the plane of the screen) cannot be considered as having equal amplitude and phase. However the resultant effect should be similar.

Q9. Plot the received intensity against the position of the screen edge using the graduated rule provided. In particular, determine the intensity around the point where the source, screen

[1] See for example 'Modern Optics' by Robert D. Guenther, published by John Wiley & Sons or "Introduction to Fourier Optics" by Joseph W. Goodman, (3rd ed. 2005). These books give a very good account of diffraction. Also, for a superb yet simple description of both interference and diffraction, see "The Feynman Lectures of Physics", vol. 1 by R.P. Feynman, R.B. Leighton and M. Sands.

edge, and horn are all in line as shown in Fig. 8 (use that point as the origin) and move the screen sufficiently in both directions to capture the important features of this phenomenon. Compare your results with published results for an opaque semi-infinite diffracting screen obtained using Fresnel's diffraction theory. Comment on the differences due to the use of an excitation different to a plane wave. How could the set-up be changed to approximate better the case of a plane wave?
Does a conducting screen really correspond with an opaque screen? Would the experimental results differ if an orthogonal polarisation were incident on the screen? In the experiment the screen was moved. Suppose the screen was fixed and the receiving horn moved instead. Would the results be different? What do the terms 'near field' and 'far field' mean?

Polarisation of Electromagnetic Waves

Electromagnetic waves can be polarised *i.e.* the electric field may be aligned along one particular direction only (linear polarisation case – any other case is simply a superposition of waves with different polarisation states). For example, with respect to the earth we can talk about the E vector being horizontally or vertically polarised – or in general, at any angle. Polarisation is important in radio propagation and antenna design, and in many other applications in microwaves and optics. Antennas for example, can be designed to transmit (or receive) preferentially in one particular polarisation that depends of the antenna geometry and orientation and is then important that if receiving, the antenna's orientation matches the polarisation of the incoming wave.

Q10. Explain briefly what is polarisation of an electromagnetic wave. What is a polariser? Why polarisation is important in anti-glare sun glasses? What effect is used in this case? Explain what happens when you put a polariser between a source emitting a polarised field and a receiver. How would the intensity of the received wave vary if you rotate the polariser? Assume that the polarisation of the receiver matches that of the source.

Experiment 2.2: Polarisation

In this experiment you will use the same setup used for the diffraction experiment above. The horn used as source is emitting a linearly polarised electromagnetic wave and you will need to determine in which direction the field is polarised. Set up the apparatus as shown in Fig. 9. In this experiment you will have a wire grid and a perforated mesh. Position the grid between the source and receiving horn. Rotate the grid and observe the variation of the received intensity. Detailed measurements are not required in this case. Merely observe which orientations of the grid result in maximum and minimum transmission. Repeat the experiment using the perforated metal screen.

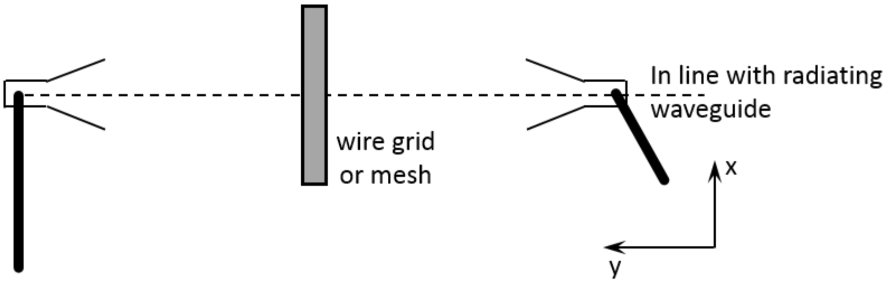

Fig. 9 Showing use of wire grid or mesh to block transmission of a polarised wave.

Q11. Since the excitation comes from a rectangular waveguide, what do you think the polarisation of the emitted wave should be and why? Now, from the observed orientation of the grid and the corresponding received intensity, what is the polarisation of the source?

Explain. Does this coincide with your expectations for this source? Explain your observations. Which orientation of the wire grid blocks the wave? Why? Explain in detail the mechanism involved in the blocking action: how is the polariser working? Which of the two types of screen is more effective as a polariser? Why?

Report

In this course the lab part of the assessment corresponds to 20% of the total course mark, which is double the weight of the lab in other courses. Consequently, it is expected that your reports should be substantially longer and more complete than an ordinary lab report and reflect this weight. This WILL be considered during the marking of the report.

Make sure you have recorded all relevant information in your laboratory book, *e.g.* the setup used, meter scales, orientation of waveguides, dimensions, frequency, any necessary description of apparatus, etc.

Paragraphs marked with a **Q#.** in this script contain questions or topics you must answer in your report. For some of the answers you would need to consult reference material and the course notes (even if the material has not been covered in the lectures yet).

Organise your report following the sections of this script and answer the numbered questions in sequence, indicating in each case which question you are answering.

Include in each section a description in your own words of the relevant theoretical aspects and use your experimental results to illustrate the theory. Use the sources of reference indicated and any other you might find and quote them when and where you are using them. Compare your results with theoretical predictions as asked and add your comments and discussion where relevant.

Your report must also include in full the experimental data you have obtained during the lab session and the conversion of this data to field intensity when necessary. All plots must be of intensity versus position.

Add an introduction, abstract, conclusions and references to complete the report.

DO NOT copy material from books, from this script or from any other source. Use your own words in any description in the report. ANY unattributed textual reproduction of material, from any source, including text, figures, graphs or pictures constitutes plagiarism and will be severely penalised.

Marks in the report will be allocated on:

— Quality and extent of the work done previously to the experiment, including your predictions for the experimental results. This will be evaluated by individual oral examination at the start of the lab session. (20% of report mark)

— Presentation of the report: General presentation: typing, layout, free from grammatical and spelling mistakes; Organisation and clarity of writing; Good figures and plots: axes properly labelled; References: properly cited and listed (appropriate format). (10% of report mark)

— Contents: Your understanding of the subject, description of the theory and your theoretical predictions; Quality of your experimental data and results; Correctness of the Matlab scripts; Proper Analysis and Conclusions; Comparison of the results with theory and the corresponding discussion. (70% of report mark)

A typed, well presented report with clearly and properly labelled graphs and diagrams, is of course expected, but 'good presentation' cannot compensate lack of understanding, poor results, poor interpretation of results, or a poor discussion.