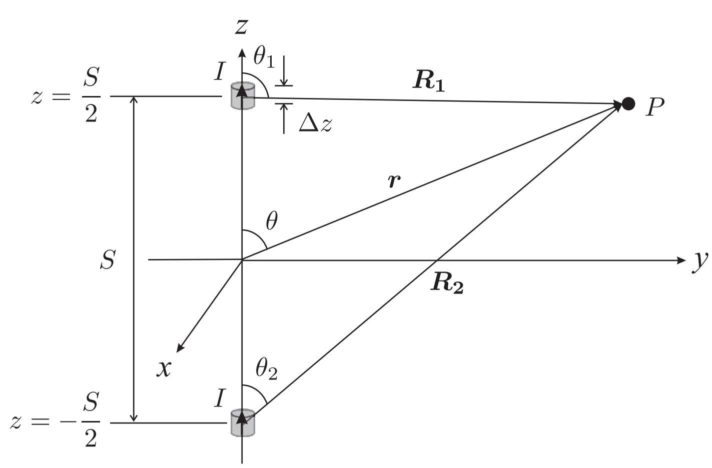
Introduction to Arrays

The antennas we have studied so far have very low directivity / gain. While this is good for broadcast applications (where we want uniform coverage), there are cases where we want a more "focused" antenna pattern to prevent wasting power illuminating areas/direction where we do not need coverage. For example, if we are trying to send a signal to a terminal on the horizon ($\theta = 90°$), a dipole is quite wasteful because even $\pm 45°$ from the horizon we are still broadcasting half the radiation intensity (the HPBW points for an ideal dipole.)

Although we could design more directive antenna elements, one straightforward way to increase the directivity of a single antenna is to assemble it with other antennas to form an *antenna array*. Then, using interference between the fields created by the individual array elements, it is possible to synthesize a variety of directive beam patterns.

1 Two-Element Antenna Arrays

Let's consider a simple case of two ideal dipoles spaced a distance S apart along the z-axis. Since the elements themselves are oriented along the z-axis, we call this a *collinear array*.



This analysis looks very similar to that which we carried out for the $\lambda/2$ dipole. Recall that we divided the dipole into many sections of ideal dipoles and used superposition (the summation of all the elements' responses) to determine the resulting electric and magnetic fields. Here we will use the same approach – except that we only have to worry about the contribution of two segments.

Recall the θ-component of the electric field radiated in the far field by a Hertzian dipole is

$$E_\theta = \underbrace{\frac{jk\eta I \Delta z}{4\pi}}_{E_s} \sin\theta \frac{e^{-jkR}}{R}. \tag{1}$$

Let's call the first fraction E_s since it was previously defined as the *strength factor* of the dipole and did not depend on the geometry of the situation. The total E_θ-field produced by the two dipoles, by superposition, is

$$E_T = E_s \sin\theta_1 \frac{e^{-jkR_1}}{R_1} + E_s \sin\theta_2 \frac{e^{-jkR_2}}{R_2}. \tag{2}$$

In the far field, very far from the array, we can make the following approximations:

$$\theta_1 = \theta_2 = \theta; \tag{3}$$

$$\frac{1}{R_1} = \frac{1}{R_2} = \frac{1}{r}. \tag{4}$$

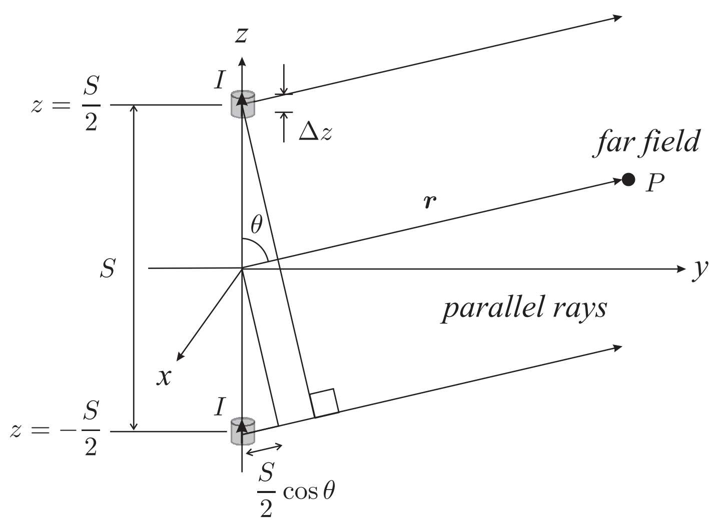
Recall that we cannot simply say that $R_1 = R_2 = r$ from the phase term (the complex exponential term) because even if $R_1 \approx R_2 \approx r$,

$$\exp(-jkR_1) \neq \exp(-jkR_2) \neq \exp(-jkr). \tag{5}$$

But, making the *parallel ray approximation*, we can say

$$R_1 \approx r - \frac{S}{2}\cos\theta \tag{6}$$

$$R_2 \approx r + \frac{S}{2}\cos\theta. \tag{7}$$



Therefore,

$$\begin{aligned} E_T &= \frac{E_s}{r}\sin\theta \left[e^{-jk(r-\frac{s}{2}\cos\theta)} + e^{-jk(r+\frac{s}{2}\cos\theta)}\right] && (8)\\ &= E_s \frac{e^{-jkr}}{r}\sin\theta \left[e^{j\frac{kS}{2}\cos\theta} + e^{-j\frac{kS}{2}\cos\theta}\right] && (9)\\ &= 2E_s \frac{e^{-jkr}}{r}\sin\theta\cos\left(k\frac{S}{2}\cos\theta\right) && (10)\end{aligned}$$

The $E_s \frac{e^{-jkr}}{r} \sin\theta$ term is exactly the pattern of a single dipole, if placed at the origin (the centre of the array.) So what has happened is that the original field of the dipole has been doubled (which we expect, because we have two dipoles driven with the same amplitude as the single dipole previously), and multiplication by a factor

$$2\cos\left(k\frac{S}{2}\cos\theta\right), \tag{11}$$

which we call the *array pattern* or more commonly the *array factor*. The original element pattern is modified by multiplying by this new factor. The array factor results *purely* from summing the phase terms corresponding to the different distances involved in the array. Here, for this specific example,

$$AF = e^{j\frac{kS}{2}\cos\theta} + e^{-j\frac{kS}{2}\cos\theta}. \tag{12}$$

Notice that the array factor is only a function of wavelength (k), element spacing (S) and observation angle (θ.) We also notice that it represents the response of the array if the elements used had been purely isotropic; that is, if

$$E \propto \frac{e^{-jkr}}{r} \tag{13}$$

(a factor which is found using the far-field E and H of the dipole and dropping angular dependence.) Notice that the AF has no dependence on the $\sin\theta$ pattern factor associated with the constituent elements: the AF term is *separable* from the total field expression. The total pattern is the multiplication of the array factor and the field produced by the constituent element. This property is called *pattern multiplication*. Notice also that for the degenerate case of a one-element array, regardless of the element type,

$$AF = 1. \tag{14}$$

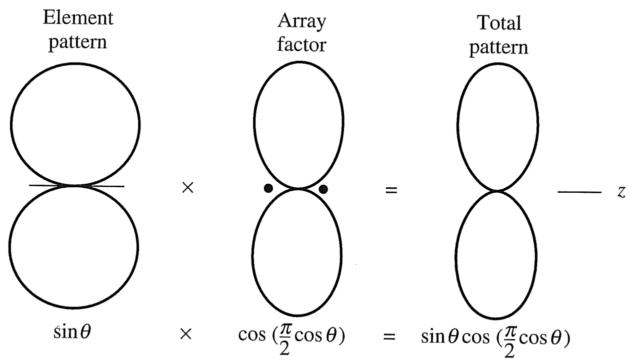
Example: 2-element dipole array with an element spacing of half a wavelength ($S = \lambda/2$).

$$\begin{aligned} AF &= 2\cos(k\frac{S}{2}\cos\theta) = 2\cos(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\cos\theta) && (15)\\ &= 2\cos\left(\frac{\pi}{2}\cos\theta\right) && (16)\end{aligned}$$

The total pattern is the pattern factor multiplied by the array factor. Graphically, this is achieved as follows [1]:

We see that the resulting pattern is slightly more directive than that of the individual elements composing the array.

In general, all sorts of beam possibilities can be achieved by changing the *wavelength*, *element spacing*, and as we will soon see, the *number of elements* as well as the *amplitude and phase* of the *element excitations*. Here we have only considered two elements driven with currents of identical amplitude and phase. However, the analysis technique is identical: the principle of superposition is always used.



2 Interpretation of Array Factor for the Two-Element Case

We have developed a formula for array factor for the two-element case. It is instructive to see physically was is happening for a few examples. Remember, the AF represents the pattern of an array of isotropic elements.

Example: $S = \lambda/2$ (graphics from [1])

$$AF = 2\cos\left(\frac{\pi}{2}\cos\theta\right) \tag{17}$$

Example: $S = \lambda$ (graphics from [1])

The array factor is

$$AF = 2\cos\left(\pi\cos\theta\right) \tag{18}$$

will have nulls wherever

$$\cos(\pi\cos\theta) = 0. \tag{19}$$

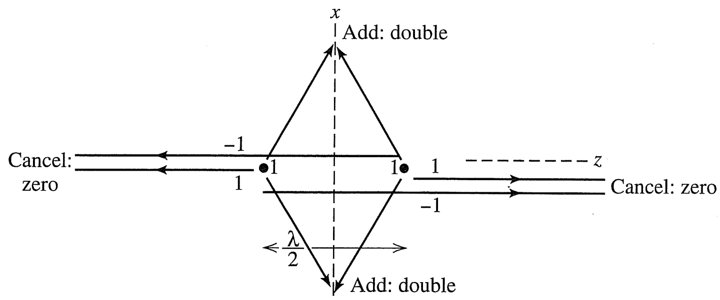
Nulls occur between the additive points in the pattern – that is, wherever the contributions from both sources are $180°$ out of phase. Evaluating,

$$\pi\cos\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \tag{20}$$

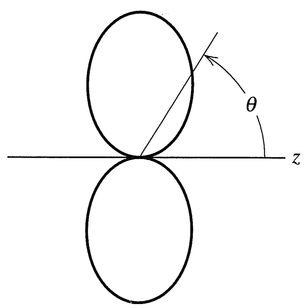
$$\theta = \pm 60°, \pm 120°. \tag{21}$$

References

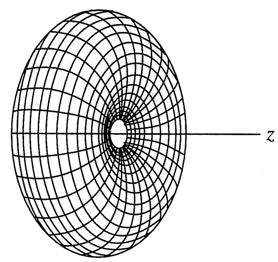
[1] W. L. Stutzman and G. A. Theile, *Antenna theory and design*. John Wiley and Sons, Inc., 1998.



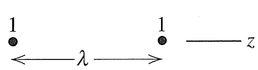
(a) Inspection method.



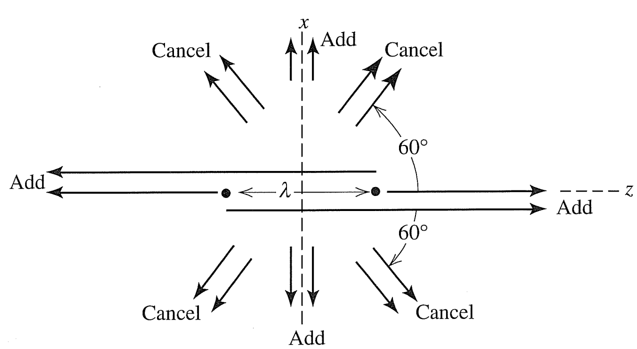
(b) Polar plot of the array factor



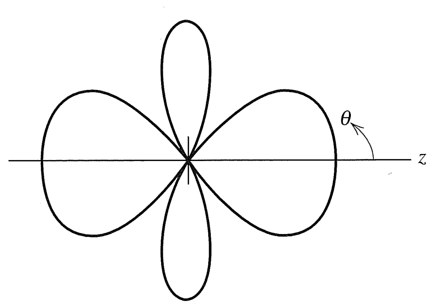
(c) 3D polar pattern.



(a) Array configuration.



(b) Inspection method.



(c) Polar plot of array factor magnitude