1 Grid graph

A comprehensive performance evaluation of our spectral graph topology algorithm was performed considering a grid graph model with 64 nodes $\mathcal{G}_{\mathsf{grid}}^{(64)}$. We compare the performance of the following algorithms: CGL, $\mathsf{CGL}(\mathbf{A})$, and SGL (proposed). We recall that $\mathsf{CGL}(\mathbf{A})$ stands for the CGL algorithm equiped with the knowledge of the connectivity matrix, which gives the information of which nodes are connected and which ones are not.

The experimental setup is as follows. The edges of the grid graph are sampled from $\mathsf{Uniform}(0.1, 3)$. The Laplacian matrix estimation is carried out on the basis of T samples distributed according to $\mathcal{N}(\mathbf{0}, \mathbf{L}_{\mathsf{grid}}^{\dagger})$. We repeat that experiment for 20 times for every value of T and we average out the relative errors and F-scores[1].

Some hyperparameter tunning is required. For the SGL algorithm, we fix $\beta = 10$ for the values of T such that $T/N > 5$. Otherwise, we start with $\beta = 10^{-2}$, and we exponentially increase it up to $\beta = 4$. Additionally, we fix $\alpha = 0$. For the CGL and $\mathsf{CGL}(\mathbf{A})$ algorithms, we choose α from $\{0\} \cup \left\{0.75^r \left(s_{\max}\sqrt{\log(N)/T}\right) | r = 1, 2, ..., 14\right\}$, such that the relative error between the estimated Laplacian and the ground truth is minimized.

Figure 1 compares the performance of the algorithms for different sample size regimes. As it can be noted, the SGL algorithm outperforms the CGL in both relative error and F-score senses. More precisely, for the case when the sample size is equal to the number of nodes, the difference in relative error and in F score are around 15% and 25%, respectively. As expected, with the additional prior knowledge of the connectivity matrix $\mathbf{A}$, the $\mathsf{CGL}(\mathbf{A})$ algorithm basically attains a perfect F-score for $T/N \geq 10$. However, the connectivity matrix is not always available in practical problems, especially in clustering tasks where the goal is precisely to understand the connectivity membership among the nodes. Nonetheless, the proposed SGL algorithm presents a comparable performance against $\mathsf{CGL}(\mathbf{A})$. For instance, at $T/N = 5$, the difference in relative error is around 2.5%, which keeps decreasing until virtually equal performance after $T/N = 100$, where the difference in relative error is around 1.2%. Additionally, we noted that the SGL algorithm requires far less tunning than CGL. At last, we add the relative error of the generalized inverse of the sample covariance matrix (ISCM) for completeness.

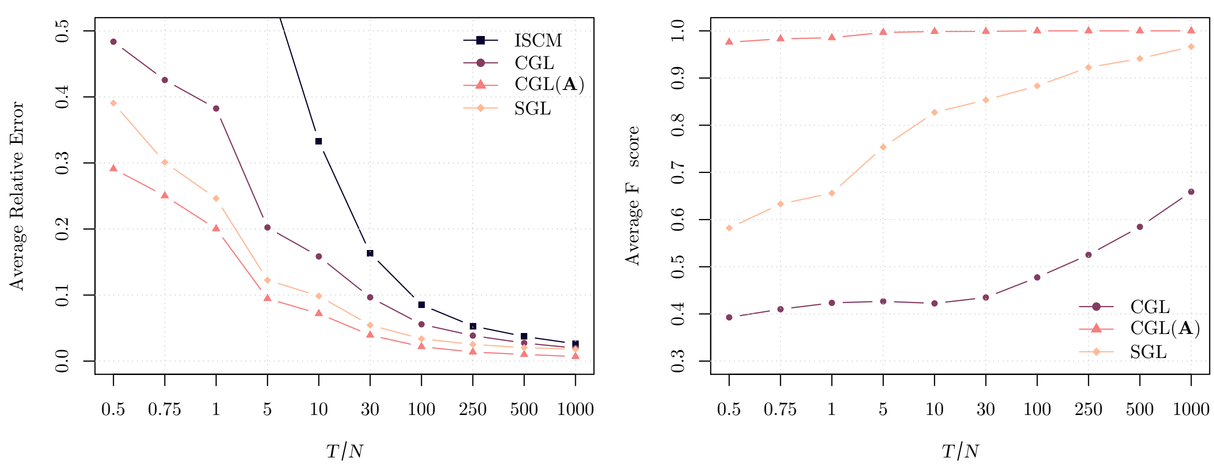

Figure 1: Average performance results for learning Laplacian matrix of a $\mathcal{G}_{\mathsf{grid}}^{(64)}$.

[1]For the computation of the F-score, we ignore edge weight values which are less than 10^{-2}