

1 4-component graph

An experiment with synthetic data generated by a 4-component graph has been conducted. We consider the following model:

$$\mathbf{L}_{\text{noisy}} = (1 - \kappa)\mathbf{L}_{\text{true}} + \kappa\mathbf{L}_{\text{ER}}, \quad (1)$$

where \mathbf{L}_{true} represents the Laplacian matrix of a K -component graph (for this example, $K = 4$) denoted as $\mathcal{G}_K^{(p_1, p_2)}$, in which p_1 and p_2 represent the probabilities of node connections across components and within components, respectively; \mathbf{L}_{ER} represents the Laplacian of an Erdos-Renyi graph $\mathcal{G}_{\text{ER}}^{(p)}$, in which p is the probability of a node connecting to any other node; and $\kappa \in (0, 1)$ controls how much noise is added into \mathbf{L}_{true} by the Erdos-Renyi model. Additionally, the weighted edges of both $\mathcal{G}_K^{(p_1, p_2)}$ and $\mathcal{G}_{\text{ER}}^{(p)}$ were drawn from $\text{Uniform}(0, 1)$. Finally, we set $p_1 = 0$, $p_2 = 1$, $p = 0.35$, and $\kappa = 0.2$.

Then, data has been sampled in the form of $\mathbf{Y} \sim \mathcal{N}(\mathbf{0}, \mathbf{L}_{\text{noisy}}^\dagger)$, where \mathbf{A}^\dagger denotes the generalized inverse of the matrix \mathbf{A} . The total number of nodes N and the number of drawn samples T were set to $N = 64$ (16 nodes per component) and $T/N = 30$.

Figure (1) illustrates the ground truth model, its noisy version, and the model learned by our spectral topology algorithm with $\beta = 10$. We compute the performance of the learning process by means of the relative error (RE) and the F-score (FS). For this example, our algorithm achieves $(\text{RE}, \text{FS}) = (0.131, 0.993)$ which means almost perfect clustering accuracy even in a noisy model that heavily suppress the ground truth weights when κ is much larger than zero.

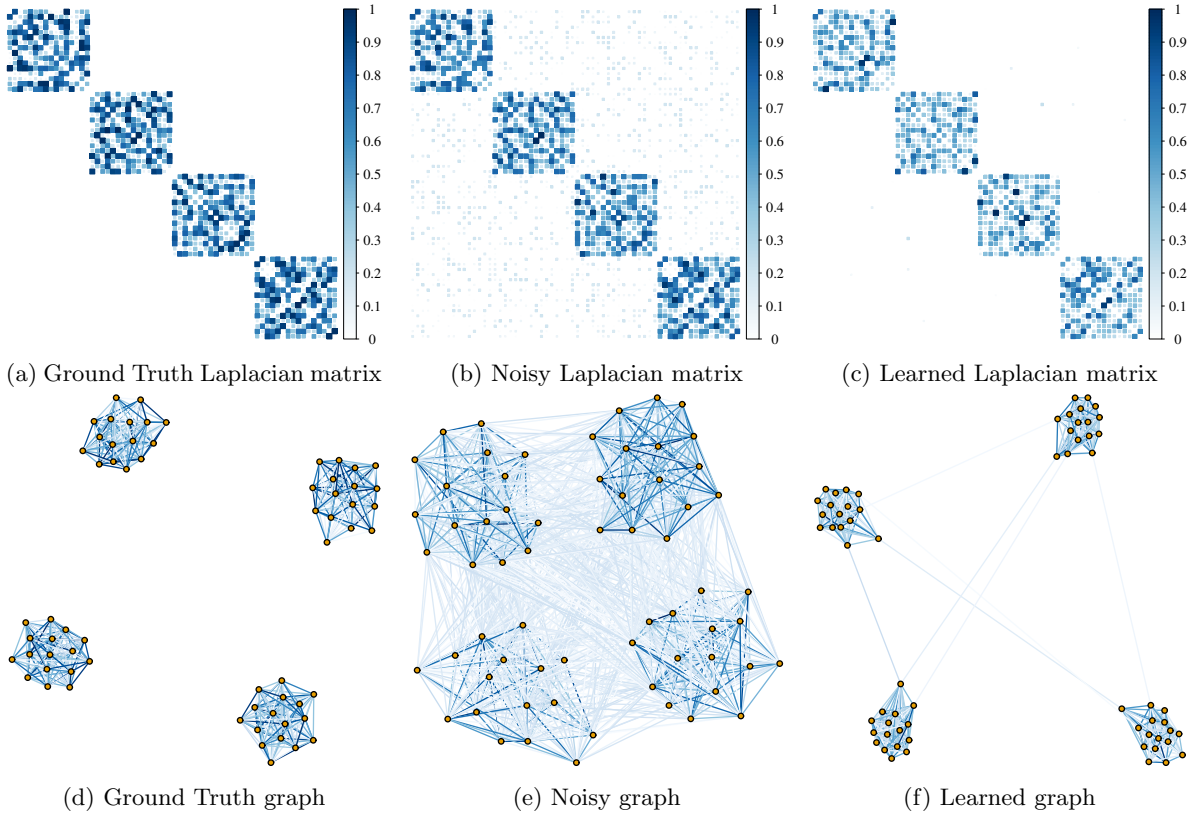


Figure 1: An example of estimating a 4-component graph. (a) the ground truth graph Laplacian matrix (\mathbf{L}_{true}), (b) \mathbf{L}_{true} after being corrupted by noise, (c) the learned graph Laplacian with a performance of $(\text{RE}, \text{FS}) = (0.131, 0.993)$. The panels (d), (e), and (f) correspond to the graphs represented by the Laplacian matrices in (a), (b), and (c), respectively.