Find T.F. HW #4: Simulink & Matlab

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MAE 376 - Sec 01
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1) $L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(t)\,dt = v(t)$

$i(t) \rightarrow$ output

$v(t) \rightarrow$ input

↓ take deriv.

↓

$L\ddot{i} + R\dot{i} + \frac{1}{C}i(t) = \dot{v}(t) \xrightarrow{L.T} \left(Ls^2 + Rs + \frac{1}{C}\right) I(s) = sV(s)$

$$G(s) = \frac{\mathcal{L}\{i(t)\}}{\mathcal{L}\{v(t)\}} = \frac{I(s)}{V(s)} = \frac{s}{Ls^2+Rs+\frac{1}{C}} = \boxed{\frac{s\cdot C}{LCs^2+RCs+1}}$$

ii

2) Find T.F. $A = \begin{bmatrix} 0 & 1 \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix}$, $C = \begin{bmatrix} -\frac{1}{9} & -\frac{2}{9} \end{bmatrix}$, $D = \begin{bmatrix} \frac{1}{3} \end{bmatrix}$

↓

(a) Using $G(s) = C(sI-A)^{-1}B + D =$

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} s & -1 \\ \frac{1}{3} & s+\frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{9}, & -\frac{2}{9} \end{bmatrix} (sI-A)^{-1} \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix} + \frac{1}{3} \qquad (sI-A)^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \cdot \frac{1}{ad-bc} = \frac{1}{+\frac{1}{3}+s^2+\frac{2}{3}s} \begin{bmatrix} s+\frac{2}{3} & 1 \\ -\frac{1}{3} & s \end{bmatrix}$$

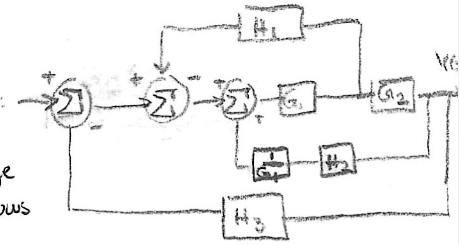
$$= \left(\frac{-\frac{1}{9}(s+\frac{2}{3}) - \frac{2}{9}(-\frac{1}{3})}{+(s^2+\frac{2}{3}s+\frac{1}{3})}, \frac{-\frac{1}{9}(1) - \frac{2}{9}(s)}{+(s^2+\frac{2}{3}s+\frac{1}{3})} \right) \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix} + \frac{1}{3}$$

$$G(s) = \frac{-\frac{2}{27} - \frac{4}{27}s}{s^2+\frac{2}{3}s+\frac{1}{3}} + \frac{1}{3} = \frac{-(2+4s)}{27s^2+18s+9} + \frac{1}{3} = \frac{-(2+4s)+9s^2+6s+3}{27s^2+18s+9} = \boxed{\frac{9s^2+2s+1}{27s^2+18s+9}}$$

(b) NUM = [0.3333, 0.0741, 0.0370] , DEN = [1.000, 0.667, 0.333]

i

3) Block Reduction & Find TF

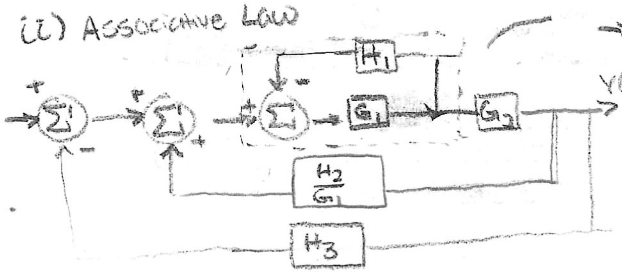


i) Move S.J. of positive feedback with H_2 outside negative loop containing H_1:

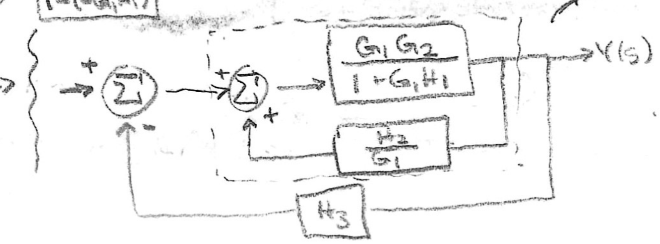
ii) Replace Loop w/ $H_1(s)$ with a single block: $\frac{G_1(s)}{1-(-G_1H_1)}$ * must ~~change~~ / interchange sum points * (this allows H_1 to be reduced)

iii) Replace two remaining loops w/ single blocks

ii) Associative Law



$\rightarrow \Sigma \rightarrow \frac{G_1}{1-(-G_1H_1)} \rightarrow \cdots$ Positive Feedback Reduction



$$\frac{G(s)}{1-G(s)H(s)}$$

Do we multiply?

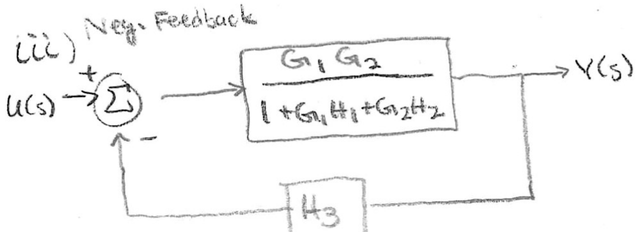
↓

$$\frac{\frac{G_1G_2}{1+G_1H_1}}{1 - \frac{G_1G_2}{1+G_1H_1}\cdot\frac{H_2}{G_1}}$$

↓

$$\frac{G_1G_2}{1+G_1H_1-G_2H_2}$$

iii) Neg. Feedback



$$\rightarrow \frac{\frac{G_1G_2}{1+G_1H_1-G_2H_2}}{1-\left(-\frac{G_1G_2}{1+G_1H_1-G_2H_2}\right)H_3}$$

$$\boxed{\frac{G_1G_2}{1+G_1H_1-G_2H_2+G_1G_2H_3}}$$

$$\therefore U(s) \longrightarrow \frac{G_1G_2}{1+G_1H_1-G_2H_2+G_1G_2H_3} \longrightarrow Y(s)$$

iii

(4) $\begin{cases} \dot{x}_1 = x_2 - x_1 \\ \dot{x}_2 = 2x_2^{-1} + 1 + t \end{cases}$, $x_1(0) = 0$, $x_2(0) = -1$ $\rightarrow$ Linearize

1. Replace $x_1 = \bar{x}_1$ & $x_2 = \bar{x}_2$; set time varying $= 0$

$$\begin{cases} \dot{\bar{x}}_1^{\,\to 0} = \bar{x}_2 - \bar{x}_1 \rightarrow \bar{x}_2 = \bar{x}_1 \nearrow \bar{x}_1 = -2 \\ \dot{\bar{x}}_2^{\,\to 0} = 2\bar{x}_2^{-1} + 1 + 0 \rightarrow \bar{x}_2 = -2 \end{cases}$$

operating point $(\bar{x}_1, \bar{x}_2) = (-2, -2)$

2. Non-linear portion $2x^{-1}$ linearized by $(-2,-2)$ $\rightarrow f(x_1, x_2) = 2x_2^{-1}$

$$f(x_1,x_2) = f(-2,-2) + \frac{\partial f}{\partial x_1}\Big|_{(-2,-2)} \Delta x_1 + \frac{\partial f}{\partial x_2}\Big|_{(-2,-2)} \Delta x_2 = \frac{2}{-2} + 0 + \left[-2x_2^{-2}\right]_{(-2,-2)} \Delta x_2 = -1 - \frac{1}{2}\Delta x_2$$

3. In original model, replace x_1 with $\bar{x}_1 + \Delta x_1 \Rightarrow -2 + \Delta x_1$ & x_2 with $\bar{x}_2 + \Delta x_2$

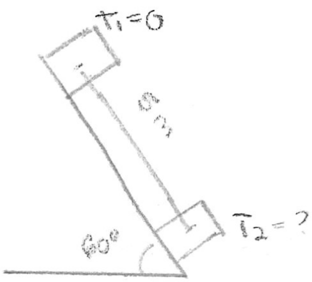
$$\dot{\bar{x}}_1^{\,\to 0} + \Delta\dot{x}_1 = \bar{x}_2 + \Delta x_2 - \bar{x}_1 - \Delta x_1 \Rightarrow \Delta\dot{x}_1 = -2 + \Delta x_2 - (-2) - \Delta x_1 \Rightarrow \Delta\dot{x}_1 = \Delta x_2 - \Delta x_1$$

$$\dot{\bar{x}}_2^{\,\to 0} + \Delta\dot{x}_2 = -1 - \tfrac{1}{2}\Delta x_2 + 1 + t \rightarrow \Delta\dot{x}_2 = -\tfrac{1}{2}\Delta x_2 + t$$

4.
$\Delta x_1(0) = x_1(0) - \bar{x}_1 = 0 - (-2) = 2$
$\Delta x_2(0) = x_2(0) - \bar{x}_2 = -1 - (-2) = 1$

$$\therefore \begin{cases} \Delta\dot{x}_1 = \Delta x_2 - \Delta x_1 , & \Delta x_1(0) = 2 \\ \Delta\dot{x}_2 = -\tfrac{1}{2}\Delta x_2 + t , & \Delta x_2(0) = 1 \end{cases}$$ (i)

(5) $m = 50$ kg , Find K.E. & V after $d = 5$ m @ 60°



$T_1 + V_1 = T_2 + V_2$

$0 + mgh_1 = T_2 + mgh_2$ $\rightarrow$ $h_1 - h_2 = 5 \sin 60$

$h_1 - h_2 = 4.33$ m

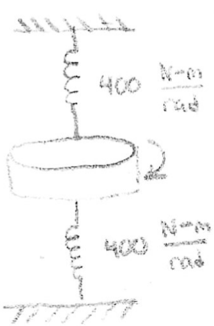
$mg(h_1 - h_2) = T_2$

$(50\,kg \cdot 9.81\,\frac{m}{s^2} \cdot 4.33\,m) = T_2 = 2123.93$ J $\rightarrow T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(50\,kg)\,v_2^2$

(ii)

$$\boxed{V_2 = \sqrt{\frac{2123.93\,J}{25\,kg}} = \sqrt{84.957\,\frac{m^2}{s^2}} = 9.22\,\frac{m}{s}}$$

(6) Disk rotates CW by 5°, determine elastic potential energy



$V_e = \frac{1}{2}k_{eq}(x)^2 \longrightarrow \frac{1}{2}\left(800\,\frac{N\text{-}m}{rad}\right)(0.0873\,rad)^2 = \boxed{3.046\ N\text{-}m\ rad}$

(iii)

$x = \theta = 5\,deg \cdot \frac{\pi\,rad}{180} = 0.0873\,rad$

$k_{eq} = k_1 + k_2 = 400\,\frac{N\text{-}m}{rad} + 400\,\frac{N\text{-}m}{rad} = 800\,\frac{N\text{-}m}{rad}$

¿Why are Springs in parallel?

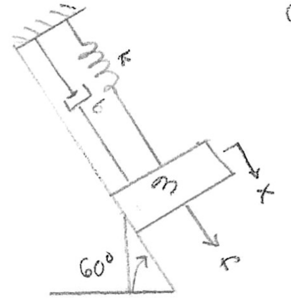
Note that springs are not in parallel or series since there is an object between them

$\cancel{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$

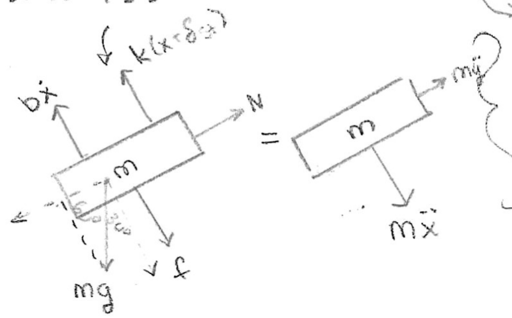
$\cancel{\frac{1}{k_{eq}} = \frac{1}{400} + \frac{1}{400} = \frac{2}{400}}$

$\cancel{k_{eq} = 200\,\frac{N\text{-}m}{rad}}$

(7) Input force f, output displacement x



a) Draw FBD & derive EOM



$\sum F_y = ma_y \rightarrow N - mg\cos 60 = 0;\ N = mg\cos 60$

$\sum F_x = ma_x$ $\quad \delta_{st} = mg\sin 60$

$\rightarrow f + mg\sin 60 - k(x+\delta_{st}) - b\dot{x} = m\ddot{x}$

$\boxed{f - kx - b\dot{x} = m\ddot{x}}$ E.O.M.

$f(t) = m\ddot{x} + kx + b\dot{x}$

b) Diff. eq. $\rightarrow$ Find T.F.

IC $\begin{cases} x(0)=0 \\ \dot{x}(0)=0 \end{cases}$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{m\ddot{x} + kx + b\dot{x}\}$

$F(s) = (ms^2 + bs + k)\,X(s) \rightarrow \dfrac{X(s)}{F(s)} = \dfrac{1}{ms^2+bs+k}$

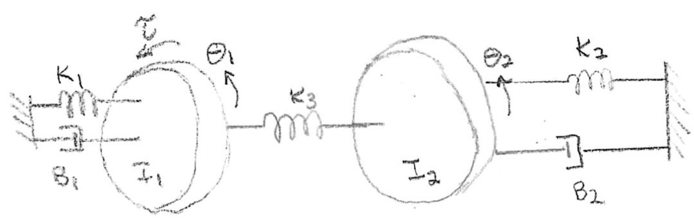
c) Diff. eq. $\rightarrow$ Find SS-representation

$x_1 = x;\ \dot{x}_1 = \dot{x} = x_2$

$x_2 = \dot{x}_1;\ \dot{x}_2 = \ddot{x} = \frac{1}{m}(-kx_1 - bx_2 + f)$

$\begin{bmatrix} 0x_1 + 1x_2 \\ -\frac{k}{m}x_1 - \frac{b}{m}x_2 \end{bmatrix} = A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$

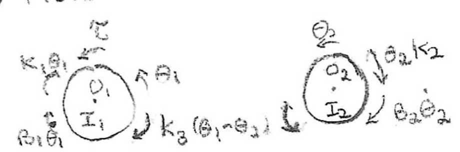
(8)



$\circlearrowleft + \ \sum \vec{M}_0 = I_0 \cdot \vec{\alpha} = I_0 \cdot \ddot{\theta}$

a)

1) F.B.D & E.O.M



EOM 1

$-K_1\theta_1 - B_1\dot{\theta}_1 + K_3(\theta_1 - \theta_2) + \tau = I\ddot{\theta}_1$

$\Rightarrow \boxed{I_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + (K_1+K_3)\theta_1 + K_3\theta_2 = \tau}$

EOM 2

$-K_2\theta_2 - B_2\dot{\theta}_2 + K_3(\theta_1 - \theta_2) = I_2\ddot{\theta}_2$

$\boxed{I_2\ddot{\theta}_2 + B_2\dot{\theta}_2 - K_3\theta_1 + (K_2+K_3)\theta_2 = 0}$

$\downarrow \mathcal{L}(t)$

$(I_1s^2 + B_1s + K_1 + K_3)\Theta_1(s) - K_3\Theta_2(s) = T(s)$

$(I_2s^2 + B_2s + K_2 + K_3)\Theta_2(s) + K_3\Theta_1(s) = 0$

b) Find T.F (use cramer's rule)

2nd-order matrix form

$\begin{bmatrix} I_1s^2 + B_1s + K_1 + K_3 & -K_3 \\ K_3 & I_2s^2 + B_2s + K_2 + K_3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$

$$\Theta(s) = \frac{\begin{vmatrix} F_1(s) & -K_3 \\ F_2(s) & I_2s^2+B_2s+K_2+K_3 \end{vmatrix}}{\begin{vmatrix} I_1s^2+B_1s+K_1+K_3 & -K_3 \\ K_3 & I_2s^2+B_2s+K_2+K_3 \end{vmatrix}}$$

set $F_2(s) = 0$

$$\Theta_1(s) = \frac{T(s)\cdot(I_2s^2+B_2s+K_2+K_3)}{(I_1s^2+B_1s+K_1+K_3)(I_2s^2+B_2s+K_2+K_3)+K_3^2}$$

$$\frac{\Theta_1(s)}{T(s)} = \frac{I_2s^2+B_2s+K_2+K_3}{(I_1s^2+B_1s+K_1+K_3)(I_2s^2+B_2s+K_2+K_3)+K_3^2}$$

Set $F_1(s) = 0$

$$\Theta_2(s) = \frac{-(F_2(s)\cdot(-K_3))}{(I_1s^2+B_1s+K_1+K_3)(I_2s^2+B_2s+K_2+K_3)+K_3^2}$$

$$\frac{\Theta_2(s)}{F_2(s)} = \frac{K_3}{(I_1s^2+B_1s+K_1+K_3)(I_2s^2+B_2s+K_2+K_3)+K_3^2}$$

c) SS-rep. with θ_1 & θ_2 output

$\dot{x}_1 = \dot{\theta}_1 \rightarrow 0x_1 + 0x_2 + 1x_3 + 0x_4$

$\dot{x}_2 = \dot{\theta}_2 \rightarrow 0x_1 + 0x_2 + 0x_3 + 1x_4$

$\dot{x}_3 = \ddot{\theta}_1 \rightarrow -\frac{(K_1+K_3)}{I_1}x_1 - \frac{K_3}{I_1}x_2 - \frac{B_1}{I_1}x_3 + 0$

$\dot{x}_4 = \ddot{\theta}_2 \rightarrow \frac{1}{I_2}(K_3x_1 - (K_2+K_3)x_2 + 0 - B_2x_4)$

$\rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_1-K_3}{I_1} & -\frac{K_3}{I_1} & -\frac{B_1}{I_1} & 0 \\ \frac{K_3}{I_2} & \frac{-K_2-K_3}{I_2} & 0 & -\frac{B_2}{I_2} \end{bmatrix} = A$

☺

(9) Simulink – Transfer Fcn & SS (why does Spring & damper have to be flipped. $m=0.8\ kg$, $L=0.6\ m$, $k=100\ N/m$, $B=0.5\ \frac{N\cdot s}{m}$, $f=10\ N$, $t=0.1\ s$, $g=9.81\ m/s^2$

Construct Block diagram for a) Linearized EOM b) T.F. c) SS from prob 7

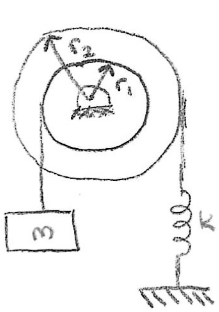
$$0.024\ddot{\theta}+0.5\dot{\theta}+9\theta=0.3f$$

$$\frac{\Theta(s)}{F(s)}=\frac{3.6}{0.288s^2+6s+108}$$

$$\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$$

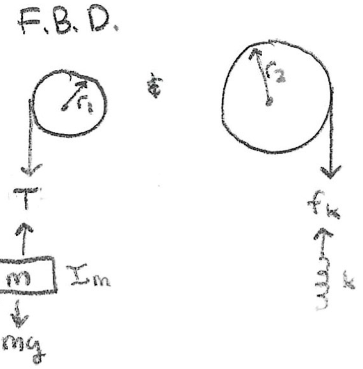
Done on Simscape

(10)

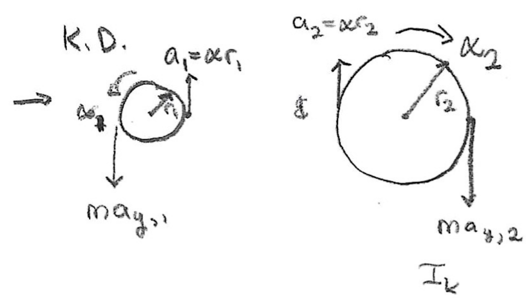


with I_o, k, m

F.B.D.



K.D.



$a_{y,1}=a_{y,2}=0$

$a_1=a_2=0$

$$\Sigma\vec{M}_O=\Sigma I_o\vec{\alpha}=(I_o+I_m)\ddot{\theta}+I_k\theta=0$$

$$\boxed{0=(I_o+mr_1^2)\ddot{\theta}+kr_2^2\theta}\quad (ii)$$