MAE 376

Homework 2

Write MATLAB commands in the editor for the problems 1,2,3,5,6, 8, and 10 to verify your hand calculations and create a m file for each problem.

Please submit all the m files as well as one pdf file of your entire hand calculations into the dropbox.

1- (a) Perform $z1$ / $z2$ and express the result in rectangular form,

(b) Verify that $z1$ / $z2$ = $z1$ / $z2$,

(c) Repeat Part (a) in MATLAB.

$$\frac{-3-j}{2j}$$

$$(a)\ \frac{-3-j}{2j}\cdot\frac{-j}{-j}=\frac{-1+3j}{2}=\frac{-1}{2}+\frac{3}{2}j$$

$$(b)\ \left|\frac{-1}{2}+\frac{3}{2}j\right|=\sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{3}{2}\right)^2}=\sqrt{\frac{10}{4}}=\frac{\sqrt{10}}{2}\ ,\quad \frac{|-3-j|}{|2j|}=\frac{\sqrt{10}}{2}$$

(c)

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>> z1 = -3-j; z2 = 2*j; z1/z2
ans =
  -0.5000 + 1.5000i
```

2- Express the following complex number in its polar form.

$$-\sqrt{3}-3j$$

To calculate phase, we first find $\tan^{-1}\sqrt{3}=\frac{1}{3}\pi$. Since $-\sqrt{3}-3j$ is located in the 3rd quadrant, the phase is taken as either $\pi+\frac{1}{3}\pi$ in the positive sense (counterclockwise) or $\frac{1}{2}\pi+\frac{1}{6}\pi=\frac{2}{3}\pi$ in the negative (clockwise). In summary, $-\sqrt{3}-3j\overset{\text{3rd quadrant}}{=}2\sqrt{3}\ e^{-(2\pi/3)j}$.

3- Perform using polar form and express the result in rectangular form.

$$\frac{3+2j}{-1+3j}$$

$$\frac{3+2j}{-1+3j} = \frac{\sqrt{13}e^{0.5880j}}{\sqrt{10}e^{1.8925j}} = \frac{\sqrt{13}}{\sqrt{10}}e^{-1.3045j} = 0.3-1.1j$$

4- Find all possible values for each expression.

$$(-1)^{1/6}$$

The goal is to find $w=\sqrt[6]{z}$ where $z=-1$. Noting that $z=-1$ is located on the negative real axis, one unit from the origin, we have $r=1$ and $\theta=\pi$, hence $z=-1=e^{j\pi}$. Then,

$$\sqrt[6]{-1}=\sqrt[6]{1}\left(\cos\frac{\pi+2k\pi}{6}+j\sin\frac{\pi+2k\pi}{6}\right), \qquad k=0,1,2,3,4,5$$

Therefore, the six roots are $\pm j$, $\frac{\sqrt{3}}{2}\pm\frac{1}{2}j$, $-\frac{\sqrt{3}}{2}\pm\frac{1}{2}j$, located on the unit circle, the vertices of a six-sided polygon.

5- Solve the following initial-value problem.

$$\dot{x}+x=\sin t\ ,\ \ x(0)=-1$$

Since $g(t)=1$, we have $h(t)=t$ and

$$x(t)=e^{-t}\left[\int e^{t}\sin t\,dt+c\right]=e^{-t}\left[\tfrac{1}{2}e^{t}(\sin t-\cos t)+c\right]=\tfrac{1}{2}(\sin t-\cos t)+ce^{-t}$$

Using the initial condition $c=-\frac{1}{2}$ so that $x(t)=\frac{1}{2}\left[\sin t-\cos t-e^{-t}\right]$.

6- Solve the following initial-value problem.

$$\ddot{x}+4\dot{x}=17\cos t\ ,\ \ x(0)=-1\ ,\ \ \dot{x}(0)=0$$

Characteristic values are $\lambda=0,-4$ hence $x_h \overset{\text{Case (1)}}{=} c_1+c_2e^{-4t}$. Pick $x_p=A\cos t+B\sin t$ and insert into the original ODE to find $(4B-A)\cos t-(B+4A)\sin t\equiv 17\cos t$. This implies $A=-1$, $B=4$ so that a general solution is $x=c_1+c_2e^{-4t}-\cos t+4\sin t$. By the initial conditions, $c_1=-1$, $c_2=1$ and thus $x=-1+e^{-4t}-\cos t+4\sin t$.

7- Write the following expression in the form $D\sin(\omega t + \varphi)$.

$$\cos t + 3\sin t$$

Write $\cos t + 3\sin t = D\sin(t+\phi) = D\sin t\cos\phi + D\cos t\sin\phi$ and compare the two sides to find

$$\begin{array}{l} D\sin\phi = 1 \\ D\cos\phi = 3 \end{array} \overset{D=\sqrt{10}}{\Rightarrow} \begin{array}{l} \sin\phi > 0 \\ \cos\phi > 0 \end{array} \Rightarrow \tan\phi = \tfrac{1}{3} \overset{\text{1st quadrant}}{\Rightarrow} \phi = 0.3218 \text{ rad}$$

Therefore, $\cos t + 3\sin t = \sqrt{10}\sin(t + 0.3218)$.

8-

(a)Find the Laplace transform of the following function. Use Table 2.2 when applicable.

(b) Confirm the result in MATLAB.

$$e^{at-b}, \; a,b = const$$

$$(a) \; \mathcal{L}\left\{e^{at-b}\right\} = \mathcal{L}\left\{e^{at}e^{-b}\right\} \overset{\text{linearity}}{=} \mathcal{L}\left\{e^{at}\right\}e^{-b} = \frac{e^{-b}}{s-a}$$

(b)

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>> syms a b t
>> laplace(exp(a*t-b))
```

9-

(a) Express the following signal in terms of unit-step functions.

(b) Find the Laplace transform of the expression in (a) using the shift on t−axis.

$$g(t) = \begin{cases} -1 & \text{if} \quad 0 < t < 1 \\ 1 & \text{if} \quad 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$(a) \;\; g(t) = -u(t) + 2u(t-1) - u(t-2)$$

$$(b) \;\; G(s) = \frac{-1 + 2e^{-s} - e^{-2s}}{s} = \frac{-(1-e^{-s})^2}{s}$$

10-

(a) Find the inverse Laplace transform using the partial-fraction expansion method.

(b) Repeat in MATLAB.

$$\frac{3s+4}{s(s+1)}$$

(a) Expand as

$$\frac{3s+4}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{(A+B)s + A}{s(s+1)} \quad \Rightarrow \quad \begin{array}{l} A + B = 3 \\ A = 4 \end{array} \quad \Rightarrow \quad \begin{array}{l} A = 4 \\ B = -1 \end{array}$$

Therefore,

$$\frac{3s+4}{s(s+1)} = \frac{4}{s} - \frac{1}{s+1} \quad \overset{\mathcal{L}^{-1}}{\Rightarrow} \quad 4 - e^{-t}$$

(b)

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>> syms s
>> ilaplace((3*s+4)/s/(s+1))
```