MAE 376: HW 2 Aramis Kelkelyan

1. $\frac{-3-j}{2j}$

a) Perform $\frac{z_1}{z_2}$ & express in rect. form

$$\frac{z_1 \Rightarrow -3-j}{z_2 \Rightarrow 2j} \cdot \left(\frac{-j}{-j}\right) = \frac{3j+j^2}{-2j^2} = \frac{3j-1}{2} = \frac{3}{2}j - \frac{1}{2} \Rightarrow -\frac{1}{2}+\frac{3}{2}j$$

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b) Verify $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$$\left|-\frac{1}{2}+\frac{3}{2}j\right| = \sqrt{(-\tfrac{1}{2})^2+(\tfrac{3}{2})^2} = \sqrt{(\tfrac{1}{4}+\tfrac{9}{4})} = \sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{2}$$

c) MATLAB ✓

2) $-\sqrt{3}-3j$

$x = -\sqrt{3}$

$y = -3$

$$r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{3})^2+(-3)^2} = \sqrt{12} = 2\sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-3}{-\sqrt{3}}\right) = \tan^{-1}(\sqrt{3}) = -120^\circ \text{ or } -2\pi/3$$

signs in 3rd Quadrant

$$2\sqrt{3}\, e^{-(2\pi/3)j}$$

i

3) $\frac{3+2j}{-1+3j}$ perform in polar, express in rect.

$z_1 = 3+2j \rightarrow |z_1| = r_1 = \sqrt{3^2+2^2} = \sqrt{13}$

$\tan^{-1}\left(\frac{2}{3}\right) = \theta_1 = 0.187\pi = 0.588$

$\Rightarrow z_1 = \sqrt{13}\, e^{0.187\pi}$

$z_2 = -1+3j \rightarrow |z_2| = r_2 = \sqrt{(-1)^2+3^2} = \sqrt{10}$

$\tan^{-1}\left(\frac{3}{-1}\right) = \theta_2 = -0.397\pi = -1.249$

$\Rightarrow z_2 = \sqrt{10}\, e^{-0.397\pi}$

$$\frac{z_1}{z_2} = \frac{\sqrt{13}}{\sqrt{10}} e^{(0.187+0.397)\pi}$$

$$= \sqrt{\frac{13}{10}}\, e^{0.5848\pi}$$

$$\sqrt{\frac{13}{10}}\, e^{0.5848\pi} \longrightarrow 0.5848\pi \cdot \frac{180}{\pi} = 105.26^\circ \rightarrow z = \sqrt{\frac{13}{10}} \cdot \cos(105.26) + j\sqrt{\frac{13}{10}} \sin(105.26)$$

$$z = -0.3 + 1.10j$$

iii

4) Find all possible roots of $(-1)^{1/6}$

$$\sqrt[6]{-1} = \sqrt[6]{-1+0j} \rightarrow r = |z_1| = \sqrt{(-1)^2+0^2} = 1 \;;\; \theta = \tan^{-1}\left(\frac{0}{1}\right) = 180^\circ \text{ or } \pi$$

$$\therefore \sqrt[6]{1}\left(\cos\frac{\theta+2\pi k}{n} + j\sin\frac{\theta+2k\pi}{n}\right) \text{ where } k = 0, 1, \dots, n-1$$

$1\left(\cos\frac{\pi+0}{6} + j\sin\frac{\pi+0}{6}\right)$

$1\left(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}\right)$

Roots: $\frac{\sqrt{3}}{2} + \frac{1}{2}j$

$1\cdot\left(\cos\frac{\pi+2\pi(1)}{6} + j\sin\frac{\pi+2\pi(1)}{6}\right)$

$1\cdot\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)$

$0 + j$

$1\cdot\left(\cos\left(\frac{\pi+4\pi}{6}\right) + j\sin\left(\frac{\pi+4\pi}{6}\right)\right)$

$\cos\left(\frac{5\pi}{6}\right) + j\sin\left(\frac{5\pi}{6}\right)$

$-\frac{\sqrt{3}}{2} + \frac{1}{2}j$

ii

5) Solve the IVP

$\dot{x} + x = \sin(t)$, $x(0) = -1$

$\mu(t) = e^{\int dt} = e^{t}$

$\int \frac{d}{dt}(e^{t} x) = \int e^{t} \sin(t)\, dt$

let $u = \sin t \rightarrow du = \cos t\, dt$

let $dv = e^{t} dt \rightarrow v = e^{t}$

$\Rightarrow \int u\, dv = uv - \int v\, du$

$= \sin(t) \cdot e^{t} - \int \cos t \cdot e^{t}\, dt$

let $u = \cos t \rightarrow du = -\sin t\, dt$

let $dv = e^{t} \cdot dt \rightarrow v = e^{t}$

$\sin(t) \cdot e^{t} - (\cos(t) \cdot e^{t} - \int e^{t} \cdot (-\sin(t))\, dt$

$\therefore \int e^{t} \cdot \sin(t)\, dt = \sin t \cdot e^{t} - \cos t\, e^{t} - \int e^{t} \sin t\, dt$

$2\int e^{t} \cdot \sin t\, dt = \sin t \cdot e^{t} - \cos t \cdot e^{t}$

$\int e^{t} \cdot \sin t\, dt = \frac{e^{t}(\sin t - \cos t)}{2} + C$

$xe^{t} = \frac{e^{t}(\sin t - \cos t)}{2} + C$

$x = \frac{\sin t - \cos t}{2} + Ce^{-t}$

$x(0) = \frac{0-1}{2} + C \cdot 1 = -1$

$C = -\frac{1}{2}$

$$\boxed{\therefore x = \frac{\sin t - \cos t}{2} - \frac{1}{2}e^{-t}}$$

$\Rightarrow$ iii

6) $\ddot{x} + 4\dot{x} = 17\cos t$, $x(0) = -1$, $\dot{x}(0) = 0$

$r^2 + 4r = 0$

$r(r+4) = 0$

$r = 0, r = -4$

$x_c = c_1 + c_2 e^{-4t}$

$g(t) = 17\cos t \Rightarrow x_p = A\cos t + B\sin t$; $x_p' = -A\sin t + B\cos t$ $x_p'' = -A\cos t - B\sin t$

Plug in: $-A\cos t - B\sin t + 4(-A\sin t + B\cos t) = 17\cos t$

$\hookrightarrow -A + 4B = 17$

$\hookrightarrow -B - 4A = 0 \rightarrow B = -4A$

$-A - 16A = 17$; $A = -1$ $\rightarrow B = -4(-1) = 4$

$\therefore x_p = -\cos t + 4\sin t$

$x = c_1 + c_2 e^{-4t} - \cos t + 4\sin t \rightarrow x(0) = c_1 + c_2 - 1 = -1$ $c_1 = -1$

$\dot{x} = -4c_2 e^{-4t} + \sin t + 4\cos t \rightarrow \dot{x}(0) = -4c_2 + 4 = 0$; $c_2 = 1$

$$\boxed{\therefore x(t) = e^{-4t} - \cos t + 4\sin t - 1}$$

7) $\cos t + 3\sin t \Rightarrow D\sin(t + \phi)$

in $D = \sin(\omega t + \phi)$ form

$D\cos(\phi) = B = 3 \rightarrow \cos(\phi) = \frac{3}{\sqrt{10}} \Rightarrow \phi = 18.435°$

$D\sin(\phi) = A = 1 \rightarrow \sin(\phi) = \frac{1}{\sqrt{10}} \Rightarrow \phi = 18.435°$

$\left.\begin{array}{l}\sin(\phi) > 0 \\ \cos(\phi) > 0\end{array}\right\}$ Q1

$D = \sqrt{A^2 + B^2} = \sqrt{9+1} = \sqrt{10}$

$\phi = 0.3218$ rad

$$\boxed{\therefore \cos t + 3\sin t \Rightarrow \sqrt{10}\sin(t + 0.3218)}$$

i

8) $\mathcal{L}\{e^{-at-b}\}$ (Table $e^{ct} \cdot f(t) \rightarrow F(s-c)$)

$= \mathcal{L}\{e^{-at} \cdot e^{-b}\} = \frac{1}{s-(-a)} \cdot e^{-b} = \boxed{\frac{e^{-b}}{s+a}}$ i

9) $g(t) = \begin{cases} -1 & \text{if } 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$

a) Express the signal in unit-step funct.

$g(t) = -u(t) + 2u(t-1) - u(t-2)$

Amplitude   Time Delay

a) iii

b) iii

$\mathcal{L}\{g(t)\} = -\frac{e^{-0s}}{s} + \frac{2e^{-1s}}{s} - \frac{1e^{-2s}}{s}$

$\mathcal{L}\{g(t)\} = \frac{-1 + 2e^{-s} - e^{-2s}}{s}$

$= \frac{-(e^{-2s} - 2e^{-s} + 1)}{s} = \frac{-(e^{-s} - 1)^2}{s}$

(10) $\mathcal{L}^{-1}\left\{\frac{3s+4}{s(s+1)}\right\} = ?$

$$\frac{3s+4}{s(s+1)} \rightarrow \frac{A}{s} + \frac{B}{s+1} \rightarrow \frac{A(s+1)+Bs}{s(s+1)}$$

s: $A+B=3 \rightarrow B=-1$

C: $A=4$

$$\frac{3s+4}{s(s+1)} \rightarrow \frac{4}{s} - \frac{1}{s+1}$$

$$\mathcal{L}^{-1}\left\{\frac{4}{s} - \frac{1}{s+1}\right\} = \boxed{4-e^{-t}}$$

i