MAE 376 – Modeling & Analysis of Dynamic Systems

HW 4 (Simulink & Matlab)

In the following problems show your work and select the correct answer(s). All the parts in your selected answers must be correct. Your work must conclude the correct answer. Otherwise, receives no credit.

1-

The governing equation for an electric circuit is $L\frac{di}{dt}+Ri+\frac{1}{C}\int_0^t i(t)dt = v(t)$ where L, R, and C are the inductance, resistance, and capacitance, all constants, $i(t)$ is the current and $v(t)$ is the applied voltage. Assuming $i(t)$ and $v(t)$ are the system output and input, respectively, find the transfer function.

i- $\frac{Cs}{Ls^2+RCs+1}$

ii- $\frac{Cs}{LCs^2+RCs+1}$

iii- $\frac{Cs}{LCs^2+Rs+1}$

2- Given matrices **A**, **B**, **C**, and **D** in the state-space description of a system model, find the transfer function or transfer matrix using

(*a*) Equation (4.21),

(*b*) The "ss2tf" command n MATLAB. (Write your **Matlab** code here and also submit the file into the drop box)

$$\mathbf{A}=\begin{bmatrix}0 & 1\\ -\frac{1}{3} & -\frac{2}{3}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}0\\ \frac{2}{3}\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}-\frac{1}{9} & -\frac{2}{9}\end{bmatrix},\ D=\frac{1}{3}$$

Show all the steps of part a and then indicate what are the numerator and denominator of your transfer function?

i- n= [0.3333, 0.0741, 0.0370] and d=[1.0000, 0.6667, 0.3333]

ii- n= [0.2533, 0.0741, 0.0370] and d=[0.85000, 0.6637, 0.3533]

iii- n= [0.4533, 0.0341, 0.0350] and d=[0.97000, 0.6267, 0.3233]

3-

Consider the block diagram in Figure 4.27. Use block diagram reduction techniques listed below to find the overall transfer function.

(*i*) Move the summing junction of the positive feedback with H_2 outside of the negative loop containing H_1.

(*ii*) In the ensuing diagram, replace the loop containing $H_1(s)$ with a single block.

(*iii*) Similarly, replace the two remaining loops with single equivalent blocks to obtain one single block whose input is $U(s)$ and whose output is $Y(s)$.

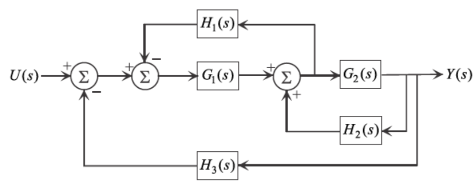

Figure 4.27 Problems 4 and 5.

i) $$U(s) \longrightarrow \boxed{\dfrac{G_2(s)}{1+G_1(s)H_1(s)-G_2(s)H_2(s)+G_1(s)G_2(s)H_3(s)}} \longrightarrow Y(s)$$

ii) $$U(s) \longrightarrow \boxed{\dfrac{G_1(s)}{1+G_1(s)H_1(s)-G_2(s)H_2(s)+G_1(s)G_2(s)H_3(s)}} \longrightarrow Y(s)$$

iii) $$U(s) \longrightarrow \boxed{\dfrac{G_1(s)G_2(s)}{1+G_1(s)H_1(s)-G_2(s)H_2(s)+G_1(s)G_2(s)H_3(s)}} \longrightarrow Y(s)$$

4- the mathematical model of a nonlinear dynamic system is given. Follow the procedure outlined in this section to derive the linearized model.

$$\begin{cases} \dot{x}_1 = x_2 - x_1 \\ \dot{x}_2 = 2x_2^{-1} + 1 + t \end{cases}, \quad \begin{matrix} x_1(0) = 0 \\ x_2(0) = -1 \end{matrix}$$

i) $\begin{cases} \Delta\dot{x}_1 = \Delta x_2 - \Delta x_1 \\ \Delta\dot{x}_2 = -\frac{1}{2}\Delta x_2 + t \end{cases}, \quad \begin{matrix} \Delta x_1(0) = 2 \\ \Delta x_2(0) = 1 \end{matrix}$

ii) $\begin{cases} \Delta\dot{x}_1 = \Delta x_2 - 2\Delta x_1 \\ \Delta\dot{x}_2 = -\frac{1}{2}\Delta x_2 + t \end{cases}, \quad \begin{matrix} \Delta x_1(0) = 0 \\ \Delta x_2(0) = 1 \end{matrix}$

iii) $\begin{cases} \Delta\dot{x}_1 = \Delta x_2 - 2\Delta x_1 \\ \Delta\dot{x}_2 = -2\Delta x_2 + t \end{cases}, \quad \begin{matrix} \Delta x_1(0) = 0 \\ \Delta x_2(0) = 1 \end{matrix}$

5-

If the 50-kg block in Figure 5.12 is released from rest at A, determine its kinetic energy and velocity after it slides 5 m down the plane. Assume that the plane is smooth.

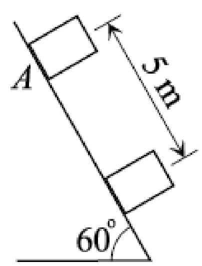

Figure 5.12 Problem 1.

i) V_B=8.22 m/s
ii) V_B=9.22 m/s
iii) V_B=10.14 m/s

6-

If the disk in Figure 5.16 rotates in the clockwise direction by 5°, determine the elastic potential energy of the system. Assume that the springs are originally undeformed.

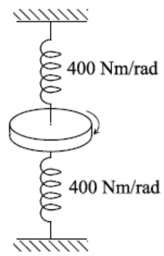

Figure 5.16 Problem 6.

i) V_e=4.2 J
ii) V_e=2.5 J
iii) V_e=3.05 J

7-

For the system shown in Figure 5.38, the input is the force f and the output is the displacement x of the mass.

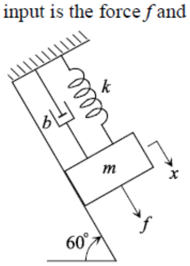

Figure 5.39 Problem 2.

a. Draw the necessary free-body diagram and derive the differential equation of motion.

b. Using the differential equation obtained in Part (a), determine the transfer function. Assume initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$.

c. Using the differential equation obtained in Part (a), determine the state-space representation.

i) $$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3 & 1 & 1 \\ 1 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

ii) $$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 1 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

iii) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3 & 1 & 1 \\ 1 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

8-

Consider the torsional mass–spring–damper system in Figure 5.63. The mass moments of inertia of the two disks about their longitudinal axes are I_1 and I_2, respectively. The massless torsional springs represent the elasticity of the shafts and the torsional viscous dampers represent the fluid coupling.

a. Draw the necessary free-body diagrams and derive the differential equations of motion. Provide the equations in the second-order matrix form.

b. Determine the transfer functions $\Theta_1(s)/T(s)$ and $\Theta_2(s)/T(s)$. All the initial conditions are assumed to be zero.

c. Determine the state-space representation with the angular displacements θ_1 and θ_2 as the outputs.

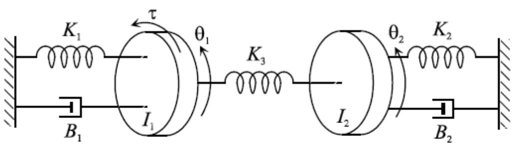

Figure 5.63 Problem 3.

Show all your work in above question then indicate your matrix A.

i) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1+K_3}{I_1} & \frac{K_3}{I_1} & -\frac{B_1}{I_1} & 0 \\ \frac{K_3}{I_2} & -\frac{K_2+K_3}{I_2} & 0 & -\frac{B_2}{I_2} \end{bmatrix}$

ii) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1}{I_1} & \frac{K_3}{I_1} & -\frac{B_1}{I_1} & 0 \\ \frac{K_3}{I_2} & -\frac{K_2+K_3}{I_2} & 0 & -\frac{B_2}{I_2} \end{bmatrix}$

iii) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_1}{I_1} & \frac{K_3}{I_1} & \frac{B_1}{I_1} & 0 \\ \frac{K_3}{I_2} & -\frac{K_2+K_3}{I_2} & 0 & -\frac{B_2}{I_2} \end{bmatrix}$

Simulink:

9-

Example 5.4 Part (d) shows how one can represent a linear system in Simulink based on the differential equation of the system. A linear system can also be represented in transfer function or state-space form. The corresponding blocks in Simulink are `Transfer Fcn` and `State-Space`, respectively. Refer to Problem 7. Construct a Simulink block diagram to find the output $\theta(t)$ of the system, which is represented using (a) the linearized differential equation of motion, (b) the transfer function, and (c) the state-space form obtained in Problem 7. The parameter values are $m = 0.8$ kg, $L = 0.6$ m, $k = 100$ N/m, $B = 0.5$ N·s/m, and $g = 9.81$ m/s^2. The input force f is the unit-impulse function, which has a magnitude of 10 N and a time duration of 0.1 s.

Show your work here and submit the Simulink in drop box.

10-

The double pulley system shown in Figure 5.86 has an inner radius of r_1 and an outer radius of r_2. The mass moment of inertia of the pulley about point O is I_O. A translational spring of stiffness k and a block of mass m are suspended by cables wrapped around the pulley as shown. Draw the free-body diagram and kinematic diagram, and derive the equation of motion using the force/moment approach.

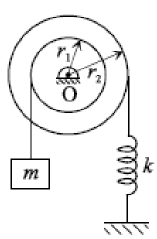

Figure 5.86 Problem 2.

i) $\left(I_O + m r_1\right)\ddot{\theta} + k r_2^2 \theta = 0$

ii) $\left(I_O + m r_1^2\right)\ddot{\theta} + k r_2^2 \theta = 0$

iii) $\left(I_O + m r_1\right)\ddot{\theta} + k r_2 \theta = 0$