

HW 5B: Watermelon Project

Given

$T_i = 5^\circ\text{C}$, Sphere, parameters

$T_\infty = 27^\circ\text{C}$

$\rho = 120 \frac{\text{kg}}{\text{m}^3}$

$D = 40\text{cm} = 0.4\text{m}$

$c = 4200 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}}$

$h = 15 \frac{\text{W}}{\text{m}^2\cdot^\circ\text{C}}$

a. $\frac{du}{dt} = q_{hi} - q_{ho}$ ← P. 355

where $u = mCT = \rho V c T$ $q_{ho} = \frac{T_\infty - T(t)}{R}$
 $\therefore \frac{du}{dt} = \frac{d}{dt}(\rho V c T) = \rho V c \frac{dT}{dt}$ $q_{ho} = 0$

Substituting,

$$\rho V c \frac{dT}{dt} = \frac{T_\infty - T(t)}{R_{conv}}$$

$$\boxed{\frac{dT}{dt} = \frac{T_\infty - T(t)}{R_{conv} \cdot \rho V c}}$$

where $R_{conv} = \frac{1}{hA_s} = \frac{1}{15 \frac{\text{W}}{\text{m}^2\cdot^\circ\text{C}} \cdot \pi \cdot (0.4\text{m})^2} = 0.133 \frac{^\circ\text{C}}{\text{W}}$

$$\rho V c = 120 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\pi (0.4\text{m})^3}{6} \cdot 4200 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}} = 16,889.20 \frac{\text{J}}{^\circ\text{C}}$$

b. $\frac{dT}{dt} = \frac{27^\circ\text{C} - T(t)}{0.133 \frac{^\circ\text{C}}{\text{W}} \cdot 16,889.20 \frac{\text{J}}{^\circ\text{C}}}$

$$\boxed{\frac{dT}{dt} = \frac{1}{2240} (27 - T(t)) \frac{^\circ\text{C}}{\text{s}}}$$

→ Input in Simulink

c. What is t at $T(t) = 20$?

$$\int_5^{20} 2240 \cdot \frac{1}{27 - T(t)} dT = \int_0^t dt$$

let $u = 27 - T(t)$
 $du = -1 dT$

$$\int_{22}^7 2240 \cdot \frac{1}{u} \cdot (-1 du) = t - 0$$

$$-(-2240) \int_7^{22} \frac{1}{u} du = t$$

$$2240 \cdot [\ln|u|]_7^{22} = t$$

$$\boxed{2565.09 \text{ s} = t}$$