MAE 376 – Modeling & Analysis of Dynamic Systems- HW 3

In the following problems show your work and select the correct answer(s). All the parts in your selected answers must be correct. Your work must conclude the correct answer. Otherwise, receives no credit.

1- Evaluate the determinant.

$$\begin{vmatrix} s+1 & 1 & -1 \\ 0 & s+2 & 2 \\ -1 & 2 & s \end{vmatrix} \ , \ s = parameter$$

i- $s^3 + 2s^2 - 3s - 5$

ii- $s^3 + 3s^2 - 3s - 8$

iii- $s^3 + 4s^2 - 3s - 2$

2- Find the inverse of the matrix.

$$\mathbf{A} = \begin{bmatrix} a & 0 & -1 \\ 0 & a+1 & 2 \\ 1 & 0 & a+2 \end{bmatrix} \ , \ a = parameter$$

i- $$\mathbf{A}^{-1} = \frac{1}{(a+1)^3} \begin{bmatrix} (a+1)(a+2) & 0 & a+1 \\ 2 & (a+1) & -2a \\ -(a+1) & 0 & a(a+1) \end{bmatrix}$$

ii- $$\mathbf{A}^{-1} = \frac{1}{(a+1)^3} \begin{bmatrix} (a+1) & 0 & a+1 \\ 2 & (a+1)^2 & -2a \\ -(a+1) & 0 & a(a+1) \end{bmatrix}$$

iii- $$\mathbf{A}^{-1} = \frac{1}{(a+1)^3} \begin{bmatrix} (a+1)(a+2) & 0 & a+1 \\ 2 & (a+1)^2 & -2a \\ -(a+1) & 0 & a(a+1) \end{bmatrix}$$

3- Solve the linear system $\mathbf{Ax} = \mathbf{b}$ using A^{-1} and write the MATLAB code below it.

$$\mathbf{A} = \begin{bmatrix} a & 1 & -2 \\ -1 & 2a & 1 \\ 0 & 1 & 3a \end{bmatrix}, \quad \mathbf{b} = \begin{Bmatrix} a \\ 4a \\ 3a+2 \end{Bmatrix}, \quad a = parameter$$

i) $$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} = \frac{1}{6a^3+2a+2}\begin{bmatrix} 6a^2-1 & 3a-2 & 4a+1 \\ 3a & 3a^2 & -a+2 \\ -1 & -a & 2a^2+1 \end{bmatrix}\begin{Bmatrix} a \\ 4a \\ 3a+2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix}$$

ii) $$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} = \frac{1}{6a^3+2a+2}\begin{bmatrix} 6a^2-1 & -3a-2 & 4a+1 \\ 3a & 3a^2 & -a+2 \\ -1 & -a & 2a^2+1 \end{bmatrix}\begin{Bmatrix} a \\ 4a \\ 3a+2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix}$$

iii) $$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} = \frac{1}{6a^3+2a+2}\begin{bmatrix} 6a^2-1 & 3a-2 & 4a+1 \\ 3a & 3a^2 & -a+2 \\ -1 & a & 2a^2+1 \end{bmatrix}\begin{Bmatrix} a \\ 4a \\ 3a+2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix}$$

4- Solve the linear system $\mathbf{Ax} = \mathbf{b}$ using Cramer's rule and write the MATLAB code below it.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 0 \\ 1 & 4 & -2 \end{bmatrix}, \quad \mathbf{b} = \begin{Bmatrix} 3 \\ 5 \\ 4 \end{Bmatrix}$$

i) X=[-2,-1,1]

ii) X=[-2,1,-1]

iii) X=[-2,-2,-1]

5- Find the eigenvalues and eigenvectors of the matrix,

$$\mathbf{A} = \begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix}, \quad a = parameter$$

i) λ1= a, λ2=1, v=[1,1;1,-1]

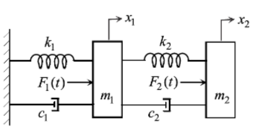
ii) λ1= a, λ2=-a, v=[1,1;-1,1]

iii) λ1= a, λ2=-1, v=[1,1;1,-1]

6- Express the system model, assuming general initial conditions, in

(*a*) Configuration form,

(*b*) Standard, second-order matrix form.



$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1) = F_1(t) \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) = F_2(t) \end{cases}$$; Mechanical system in Figure 4.3.

The generalized coordinates are $q_1 = x_1$ and $q_2 = x_2$

i) a) $\begin{cases} \ddot{q}_1 = \dfrac{1}{m_1}[-c_1\dot{q}_1 - k_1 q_1 + k_2(q_2 - q_1) + c_2(\dot{q}_2 - \dot{q}_1) + F_1(t)] = f_1(q_1, q_2, \dot{q}_1, \dot{q}_2, t) \\ \ddot{q}_2 = \dfrac{1}{m_2}[-k_2(q_2 - q_1) - c_2(\dot{q}_2 - \dot{q}_1) + F_2(t)] = f_2(q_1, q_2, \dot{q}_1, \dot{q}_2, t) \end{cases}$, b) $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$

ii) a) $\begin{cases} \ddot{q}_1 = \dfrac{1}{m_1}[-c_1\dot{q}_1 - k_1 q_1 + k_2(q_2 + q_1) + c_2(\dot{q}_2 - \dot{q}_1) + F_1(t)] = f_1(q_1, q_2, \dot{q}_1, \dot{q}_2, t) \\ \ddot{q}_2 = \dfrac{1}{m_2}[-k_2(q_2 - q_1) - c_2(\dot{q}_2 - \dot{q}_1) + F_2(t)] = f_2(q_1, q_2, \dot{q}_1, \dot{q}_2, t) \end{cases}$, b) $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$

iii) a) $\begin{cases} \ddot{q}_1 = \dfrac{1}{m_1}[-c_1\dot{q}_1 - k_1 q_1 + k_2(q_2 + q_1) + c_2(\dot{q}_2 - \dot{q}_1) + F_1(t)] = f_1(q_1, q_2, \dot{q}_1, \dot{q}_2, t) \\ \ddot{q}_2 = \dfrac{1}{m_2}[\;k_2(q_2 - q_1) - c_2(\dot{q}_2 - \dot{q}_1) + F_2(t)] = f_2(q_1, q_2, \dot{q}_1, \dot{q}_2, t) \end{cases}$, b) $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$

7-Find a suitable set of state variables, derive the state-variable equations, and form the state equation.

Show all the step. What would be your matrix A?

$$\begin{cases} \ddot{x}_1 + \dot{x}_1 + 2x_1 - \dot{x}_2 - 3x_2 = f_1(t) \\ 2\ddot{x}_2 - \dot{x}_1 - 2x_1 + \dot{x}_2 + 3x_2 = f_2(t) \end{cases}$$

i) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3 & 1 & 1 \\ 1 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

ii) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3 & -1 & 1 \\ 1 & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

iii) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 3 & 1 & 1 \\ 1 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

8- Find the state-space form of the mathematical model.

$$\begin{cases} \ddot{x}_1 + 2(x_1 - x_3) - 2(\dot{x}_2 - \dot{x}_1) - \frac{1}{2}(x_2 - x_1) = f(t) \\ \ddot{x}_2 + 2(\dot{x}_2 - \dot{x}_1) + \frac{1}{2}(x_2 - x_1) = 0 \\ x_3 - 2(x_1 - x_3) = 0 \end{cases}, \quad \text{outputs are } x_2 \text{ and } \dot{x}_2.$$

Show all the steps. What would be your A matrix?

i) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{7}{6} & \frac{1}{2} & -2 & 2 \\ \frac{1}{2} & -\frac{1}{2} & 2 & -2 \end{bmatrix}$

ii) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{7}{6} & \frac{1}{2} & 2 & 2 \\ \frac{1}{2} & -\frac{1}{2} & 2 & -2 \end{bmatrix}$

iii) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{7}{6} & \frac{1}{2} & 2 & 2 \\ \frac{1}{2} & -\frac{1}{2} & -2 & -2 \end{bmatrix}$

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A dynamic system model is described by $\ddot{x} + 4\dot{x} + 3x = f(t)$, where x is the output.

(a) Find the state-space form.

(b) Decouple the state equation and obtain the transformed state-space form.

Show all the steps. What would be your V matrix?

i) $\mathbf{V} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$

ii) $\mathbf{V} = \begin{bmatrix} 1 & -1 \\ 1 & -3 \end{bmatrix}$

iii) $\mathbf{V} = \begin{bmatrix} 1 & 1 \\ -1 & -3 \end{bmatrix}$

10- Find all possible input-output equations. What are the I/O equations when transforming the data back to the time domain?

$$\begin{cases} \ddot{x}_1 + \dot{x}_1 + 2(x_1 - x_2) = 0 \\ \ddot{x}_2 + \dot{x}_2 - 2(x_1 - x_2) = f(t) \end{cases}, \quad f(t) = \text{input}, \; x_1 = \text{output}$$

i) $2x_1^{(4)} + 2\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = 2f$

ii) $x_1^{(4)} + 2\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = 2f$

iii) $2x_1^{(4)} + 2\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = f$