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MATH 25800
Quiz 2
January 20, 2010

**Problem 1.** *Let $R$ be a ring, $I$ and ideal in $R$. Prove that factor ring $R/I$ is commutative iff $rs - sr$ is an element of $I$ for all $r$, $s$ in $R$.*

*Proof.* Suppose $R/I$ is commutative. Then $rs + I = (r + I)(s + I) = (s + I)(r + I) = sr + I$ for all $r, s \in R$. But these two additive cosets are equal precisely when $rs - sr \in I$. Conversely, suppose that $rs - sr \in I$ for all $r, s \in R$. Then $0 + I = rs - sr + I$ and adding the coset $sr + I$ to both sides gives $sr+I = (rs-sr+I)+(sr+I) = rs-sr+sr+I = rs+I$. Thus $(r+I)(s+I) = (rs+I) = (sr+I) = (s+I)(r+I)$ for all $r, s \in R$ and $R/I$ is commutative. □

**Problem 2.** *Let $R$ be the ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ (the reals to the reals). Let $A$ be the set $A = \{f \in R \mid f(0)$ is an even integer$\}$. Show $A$ is a subring of $R$ but not an ideal of $R$.*

*Proof.* Let $f, g \in A$. Then $f(0) = 2n$ and $g(0) = 2m$ for some integers $n$ and $m$. Thus $(f - g)(0) = f(0) - g(0) = 2(n - m)$ is also an even integer and $A$ is closed under subtraction. Likewise $(fg)(0) = f(0)g(0) = 2(2nm)$ is an even integer and $A$ is closed under multiplication. The zero function shows that $A$ is nonempty and thus $A$ is a subring of $R$.

Now let $h$ be the constant function $1/2$ and $f$ be the constant 2 function so that $h(x) = 1/2$ and $f(x) = 2$ for all $x \in \mathbb{R}$. It's clear that $h$ is a member of $R$, $f$ is a member of $A$, $h(0) = 1/2$ and $f(0) = 2$. But then $(hf)(0) = h(0)f(0) = 1$. Therefore $A$ is not closed under left multiplication by elements from $R$ and is not an ideal. □