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MATH 25800

Quiz 4

February 22, 2010

**Problem 1.** *Reduce the following matrix to reduced row echelon form:*

$$\begin{pmatrix} 2 & 4 & -2 \\ 4 & 8 & 3 \\ -1 & -3 & 0 \end{pmatrix}.$$

*Proof.* First multiply the first row by 1/2 and then add this row to the last row to get

$$\begin{pmatrix} 1 & 2 & -1 \\ 4 & 8 & 3 \\ 0 & -1 & -1 \end{pmatrix}.$$

Now add $-4$ times the first row to the second row to get

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 7 \\ 0 & -1 & -1 \end{pmatrix}.$$

Add twice the last row and then 3/7 times the second row to the first row to get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 7 \\ 0 & -1 & -1 \end{pmatrix}.$$

Interchange the second and third rows, multiply the new second row by $-1$, multiply the new third row by 1/7 to get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Finally, add $-1$ times the third row to the second row to end up with the identity matrix. $\square$

**Problem 2.** *Let $T$ have the matrix*

$$\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$

*in the standard basis $e_1$, $e_2$, $e_3$. Find the matrix of $T$ in the basis*

$$u_1 = (1,1,1) \quad u_2 = (0,1,1) \quad u_3 = (0,0,1).$$

*Proof.* Let $\mathcal{B}$ be the standard basis and $\mathcal{E}$ be the new basis. We form the matrix $P = M_{\mathcal{E}}^{\mathcal{B}}(I)$ where $I$ is the identity map. The columns of $P$ are then the coefficients of $u_i$ when written in terms of $e_i$. Thus

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

We also need to find $P^{-1}$ which can be done using row reduction:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

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$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right)$$

So

$$P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}.$$

Now we compute the product

$$\begin{aligned} T' &= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 2 & 3 & 1 \\ 4 & 4 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 3 & 2 \\ -2 & 0 & -1 \\ 2 & 1 & 2 \end{pmatrix}. \end{aligned}$$

$\square$