| Distribution | $f_X(x)$ | $F_X(x)$ | $E(X)$ | $\mathrm{Var}(X)$ |
|---|---|---|---|---|
| Bernoulli $(p)$ | $(1-p)^{1-k}p^k$ | $(1-p)^{1-k}$ | $p$ | $p(1-p)$ |
| Binomial $(n,p)$ | $\binom{n}{k}(1-p)^{n-k}p^k$ | $I_{1-p}(n-k,k+1)$ | $np$ | $np(1-p)$ |
| Hypergeometric(N,m,n) | $\frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$ | $\approx \Phi\left(\frac{k-np}{\sqrt{np(1-p)}}\right)$ | $\frac{nm}{N}$ | $\frac{nm(N-n)(N-m)}{N^2(N-1)}$ |
| Negative Binomial $(r,p)$ | $\binom{k-1}{r-1}(1-p)^r p^k$ | $1-I_p(k+1,r)$ | $r\frac{p}{1-p}$ | $r\frac{p}{(1-p)^2}$ |
| Geometric $(n,p)$ | $(1-p)^{k-1}p$ | $1-(1-p)^k$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ |
| Poisson $(\lambda)$ | $\frac{\lambda^k}{k!}e^{-\lambda}$ | $e^{-\lambda}\sum_{i=0}^{k}\frac{\lambda^i}{i!}$ | $\lambda$ | $\lambda$ |
| Uniform | $\frac{I(a\le x\le b)}{b-a}$ | $\frac{x-a}{b-a}$ | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ |
| Normal $(\mu,\sigma^2)$ | $\phi(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ | $\Phi(x)=\int_{-\infty}^{x}\phi(t)dt$ | $\mu$ | $\sigma^2$ |
| Exponential $(\lambda)$ | $\lambda e^{-\lambda x}$ | $1-e^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ |
| Gamma $(\alpha,\lambda)$ | $\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$ | $\frac{\gamma(\alpha,x\lambda)}{\Gamma(\alpha)}$ | $\frac{\alpha}{\lambda}$ | $\frac{\alpha}{\lambda^2}$ |
| Beta $(\alpha,\beta)$ | $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$ | $I_x(\alpha,\beta)$ | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ |

$\Gamma(\alpha)=\int_0^\infty t^{\alpha-1}e^{-t}dt$
$\gamma(\alpha,x)=\int_0^x t^{\alpha-1}e^{-t}dt$
$B(\alpha,\beta)=\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$
$B(x;\alpha,\beta)=\int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt$
$I_x(\alpha,\beta)=\frac{B(x;\alpha,\beta)}{B(\alpha,\beta)}$

$P(X\ge t)\le E(X)/t$
$P(|X-\mu|>t)\le\sigma^2/t^2$

$P(A)=\sum_{i=1}^n P(A\mid B_i)P(B_i)$ $\quad P(B_j\mid A)=\frac{P(A|B_j)P(B_j)}{\sum_{i=1}^n P(A|B_i)P(B_i)}$ $\quad f_Y(y)=f_X(g^{-1}(y))|(g'(g^{-1}(y)))^{-1}|$

$f_{Y|X}(y\mid x)=\frac{f_{XY}(x,y)}{f_X(x)}$ $\quad f_Y(y)=\int_{-\infty}^{\infty}f_{Y|X}(y\mid x)f_X(x)dx$ $\quad f_Z(z)=\int_{-\infty}^{\infty}f_X(x)f_Y(z-x)dx$

$E(g(X))=\int_{-\infty}^{\infty}g(x)f(x)dx$ $\quad E(a+bX+cY)=a+bE(X)+cE(Y)$ $\quad E(g(Y)\mid X=x)=\int_{-\infty}^{\infty}g(y)f_{Y|X}(y\mid x)dy$

$E(Y+Z\mid X)=E(Y\mid X)+E(Z\mid Z)$ $\quad E(g(X)Y\mid X)=g(X)E(Y\mid X)$ $\quad E(Y)=E(E(Y\mid X))$

$\mathrm{Var}(X)=E(X^2)-E(X)^2$ $\quad \mathrm{Cov}(X,Y)=E((X-\mu_X)(Y-\mu_Y))$ $\quad$ Independence $\implies \mathrm{Var}(X+Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)$

$\mathrm{Var}(X)=E((X-E(X))^2)$ $\quad \mathrm{Var}(a+bX)=b^2\mathrm{Var}(X)$ $\quad \mathrm{Var}\left(a+\sum_{i=1}^n b_iX_i\right)=\sum_{i=1}^n\sum_{j=1}^n b_ib_j\mathrm{Cov}(X_i,X_j)$

$\mathrm{Var}(Y)=\mathrm{Var}(E(Y\mid X))+E(\mathrm{Var}(Y\mid X))$ $\quad V(Y\mid X)=E(Y^2\mid X)-E(Y\mid X)^2$

$\rho=\frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}$ $\quad \rho=\pm1\iff\exists(ab)(P(Y=a+bX))$ $\quad \mathrm{Cov}\left(a+\sum_{i=1}^n b_iX_i,c+\sum_{j=1}^m d_jY_j\right)=\sum_{i=1}^n\sum_{j=1}^m b_id_j\mathrm{Cov}(X_i,Y_j)$

$\mathrm{Cov}(X,X)=\mathrm{Var}(X)$ $\quad \mathrm{Cov}(aX,bY)=ab\mathrm{Cov}(X,Y)$

Let $X_1,\ldots,X_i,\ldots$ be a sequence of independent random variables with $E(X_i)=\mu$ and $\mathrm{Var}(X_i)=\sigma^2$. Let $\overline{X}_n=n^{-1}\sum_{i=1}^n X_i$. Then, for any $\varepsilon>0$,

$$P(|\overline{X}_n-\mu|>\varepsilon)\to 0\quad\text{as } n\to\infty.$$

Let $X_1,X_2,\ldots$ be a sequence of random variables with mean $\mu$ and variance $\sigma^2$ and a common distribution. Let $S_n=\sum_{i=1}^n X_i$. Then

$$\lim_{n\to\infty}P\left(\frac{S_n-n\mu}{\sigma\sqrt{n}}\le x\right)=\Phi(x).$$

$\mathrm{lik}(\theta)=\prod_{i=1}^n f(X_i\mid\theta)$ $\quad l(\theta)=\sum_{i=1}^n\log(f(X_i\mid\theta))$ $\quad$ asymptotic variance is $\frac{1}{nI(\theta_0)}=-\frac{1}{E(l''(\theta_0))}$

$I(\theta)=E\left(\frac{\partial}{\partial\theta}\log(f(X\mid\theta))\right)^2$ $\quad I(\theta)=-E\left(\frac{\partial^2}{\partial\theta^2}\log(f(X\mid\theta))\right)$

$f_{\Theta|X}(\theta\mid x)=\frac{f_{X,\Theta}(x,\theta)}{f_X(x)}$ $\quad f_{\Theta|X}(\theta\mid x)=\frac{f_{X|\Theta}(x|\theta)f_\Theta(\theta)}{\int f_{X|\Theta}(x|\theta)f_\Theta(\theta)d\theta}$

Under smoothness conditions on $f$, the probability distribution of $\sqrt{nI(\theta_0)}(\hat\theta-\theta_0)$ tends to a standard normal distribution.

$\frac{P(H_0|x)}{P(H_1|x)}=\frac{P(H_0)P(x|H_0)}{P(H_1)P(x|X_1)}>1$ $\quad \frac{P(x|H_0)}{P(x|H_1)}>c$ $\quad \alpha=P(\text{reject } H_0\mid H_0)$ $\quad \beta=P(\text{accept } H_0\mid H_1)$

$\Lambda^*=\frac{\max_{\theta\in\omega_0}(\mathrm{lik}(\theta))}{\max_{\theta\in\omega_1}(\mathrm{lik}(\theta))}$ $\quad \Lambda=\frac{\max_{\theta\in\omega_0}(\mathrm{lik}(\theta))}{\max_{\theta\in\Omega}(\mathrm{lik}(\theta))}$

Suppose that $H_0$ and $H1$ are simple hypotheses and consider the test that rejects $H_0$ whenever the likelihood ratio is less than $c$ and significance level $\alpha$. Then any other test for which the significance level is less than or equal to $\alpha$ has power less than or equal to that of the likelihood ratio test.

Suppose that for every value $\theta_0$ in $\Theta$ there is a test at level $\alpha$ of the hypothesis $H_0:\theta=\theta_0$. Denote the acceptance region of the test by $A(\theta_0)$. Then the set

$$C(\mathbf{X})=\{\theta\mid\mathbf{X}\in A(\theta)\}$$

is a $100(1-\alpha)\%$ confidence region for $\theta$.

Suppose that $C(\mathbf{X})$ is a $100(1-\alpha)\%$ confidence region for $\theta$; that is, for every $\theta_0$,

$$P(\theta_0\in C(\mathbf{X})\mid\theta=\theta_0)=1-\alpha.$$

Then an acceptance region for a test at level $\alpha$ of the hypothesis $H_0:\theta=\theta_0$ is

$$A(\theta_0)=\{\mathbf{X}\mid\theta_0\in C(\mathbf{X})\}.$$