EE2801

Assignment - 1 [Sampling Theorem]

Q Describe the sampling theorem and Nyquist sampling rate. Find out an appropriate sampling rate for the below mentioned signals and explain your answer with proper reasoning

1. Step signal: $s_1(t) = u(t)$ for a duration of $T = 1$ sec
2. Sinc pulse: $s_2(t) = \dfrac{\sin(2\pi f t)}{\pi t}$.

Soln :→ · Sampling Theorem: The sampling theorem also known as Nyquist theorem. It is the principle to accurately reproduce a pure sine wave measurement, or sample, rate, which must be at least twice its frequency. Mathematically, if f_{max} is highest frequency component in the signal, then the Nyquist sampling rate (f_s) should be greater than or equal to $2 \times f_{max}$.

i.e. $\boxed{f_s \geq 2 f_{max}}$

Nyquist sampling rate (f_s) : The theoretical minimal sampling rate at which a finite bandwidth signal can be sampled to retain all information and reconstructed from its sample without any distortion is called Nyquist sampling rate (f_s).

① Nyquist rate for $s_1(t) = u(t)$ for $T = 1$ sec.

(i) step signal : $s_1(t) = u(t)$ for a duration of $T = 1$ sec.

i.e. $s_1(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{elsewhere} \end{cases}$

Now, if we check $s_1(t)$ in frequency domain i.e taking Fourier transform

$$S_1(j\omega) = \int_0^1 s_1(t)\, e^{(-j\omega t)}\, dt$$

$$= \int_0^1 e^{-j\omega t}\, dt.$$

$$= \frac{e^{-j\omega t}}{-j\omega}\Big|_0^1$$

$$= \frac{(e^{-j\omega} - 1)j}{\omega}$$

So; Magnitude of $S_1(j\omega) = \sqrt{\frac{(1-\cos\omega)^2 + (\sin\omega)^2}{\omega^2}}$

$$= \frac{\sqrt{2(1-\cos\omega)}}{\omega}$$

$$= \frac{\sin(\omega/2)}{(\omega/2)}$$

: which is a sinc function.

So; $f_m = \infty$.

and hence $f_s = \infty$

Now; we know that in higher bandwitch, signals changes a lot in short interval of time and to capture these details we sample them quickly to reconstruct it again.

So; $\boxed{f_s = \infty}$

(2) sinc Pulse ($s_2(t)$):

→ The sinc pulse given by $s_2(t) = \frac{\sin(2\pi f t)}{\pi t}$.

The highest frequency component in the sinc pulse is $2f$.

As, the period (T) is $\frac{1}{2f}$.

So; $f_s \geq 2 f_{max}$

Here, $f_{max} = 2f$

$$\boxed{f_s \geq 4f}$$