

Assignment

EE23010: Probability and Random Processes

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Question: Let X be a random variable with probability density function

$$f(x; \lambda) = \begin{cases} \frac{1}{\lambda} e^{-\frac{x}{\lambda}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $\lambda > 0$ is an unknown parameter. Let Y_1, Y_2, \dots, Y_n be a random sample of size n from a population having the same distribution as X^2 . If

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i \quad (2)$$

then which of the following statements is true?

- 1) $\sqrt{\frac{\bar{Y}}{2}}$ is a method of moments estimator of λ
- 2) $\sqrt{\bar{Y}}$ is a method of moments estimator of λ
- 3) $\frac{1}{2} \sqrt{\bar{Y}}$ is a method of moments estimator of λ
- 4) $2 \sqrt{\bar{Y}}$ is a method of moments estimator of λ

(GATE ST 2023)

Solution:

- 1) Using PDF in (??) we need to find an estimator for the unknown parameter λ in terms of sample mean \bar{Y}
we know $Y_i = X_i^2$ then,

$$E(Y_i) = E(X_i^2) \quad (3)$$

$$= \int_0^{\infty} x^2 \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \quad (4)$$

$$= 2\lambda^2 \quad (5)$$

Method of moment is defined by (??) which gives,

$$\bar{Y} = E(Y_i) \quad (6)$$

$$= 2\lambda^2 \quad (7)$$

where

$$\lambda = \sqrt{\frac{\bar{Y}}{2}} \quad (8)$$

\therefore Option (??) is correct.

- 2) The simulation steps to estimate λ using method of moment estimator in python.
 - a) Generate a random value of λ within the specified range using **np.random.uniform(1,10)**
 - b) Use the generated λ to create a random sample of X values following the given PDF using **np.random.exponential()**
 - c) Then, generate Y as $Y = X^2$
 - d) calculate the mean (\bar{Y}) as **np.mean(Y)**
 - e) Hence, the estimated value of λ is **np.sqrt($\frac{\bar{Y}}{2}$)**

Graph of simulated CDF vs Theoretical CDF

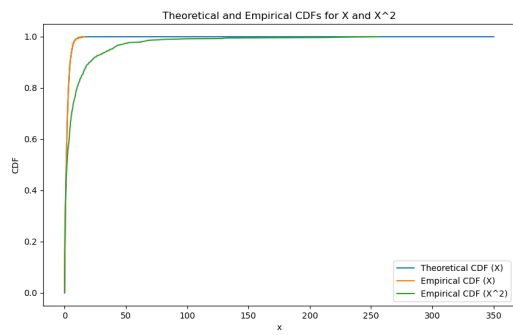


Fig. 2. Figure1