Assignment

EE23010: Probability and Random Processes
Indian Institute of Technology,Hyderabad

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Question: For a real signal, which of the following is/are valid power spectral density/-densities?

1) $s_X(w) = \frac{2}{9+w^2}$

2) $s_X(w) = e^{-w^2} cos^2 w$

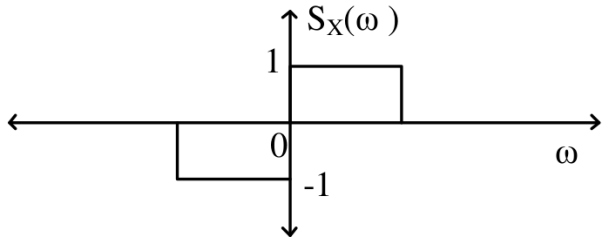

Fig. 3. Figure1

3)

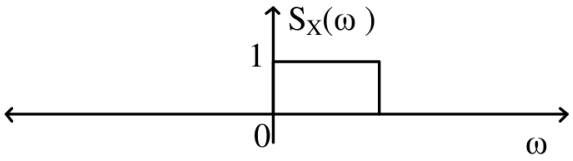

Fig. 4. Figure2

4)

Solution:

The S_X and $R(\tau)$ of a power signal form a Fourier transform pair, i.e.,

$$R(\tau) \overset{\mathcal{F}}{\rightleftharpoons} S_X \tag{1}$$

which gives :

$$S_X = \int R_X(\tau) e^{(-jw\tau)} d\tau \tag{2}$$

Now,For a real signal,the $R_X(\tau)$ is real and even the fourier transform of $R_X(\tau)$ will also exhibits the same properties:

$$Im(s_X(w)) = 0 \tag{3}$$

$$s_X(-w) = s_X(w) \tag{4}$$

Now,

1) Plot for $S_X(w) = \frac{2}{9+w^2}$

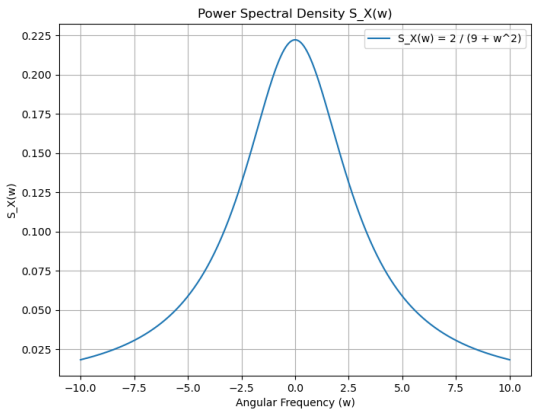

Fig. 1. plot1

$$Im(\frac{2}{9+w^2}) = 0 \tag{5}$$

Also,

$$\frac{2}{9+w^2} = \frac{2}{9+(-w)^2} \tag{6}$$

So, S_X is valid.

2)

$$Im(e^{-w^2}cos^2 w) = 0 \tag{7}$$

And,

$$e^{-w^2}cos^2 w = e^{-(-w)^2}cos^2(-w) \tag{8}$$

It is also a valid S_X.

3)

$$S_X = \begin{cases} 1 & 0 < w < w_o \\ 0 & -w_o < w < 0 \end{cases} \tag{9}$$

$$Im(S_X) = 0 \tag{10}$$

$$S_X(-w) = -S_X(w) \tag{11}$$

As, It is real but odd function.So, it is not a valid S_X.

4)

$$S_X = \begin{cases} 1, 0 < w < w_o \\ 0, \text{otherwise} \end{cases} \tag{12}$$

$$R_X(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X e^{jw\tau} dw \tag{13}$$

$$= \frac{1}{2\pi j\tau}(e^{jw_0\tau} - 1) \tag{14}$$

So, $R_X(\tau)$ is complex and cannot give a valid S_X

$\therefore$ Option (1) and (2) are correct.