

# Assignment - 4

## EE23010: Probability and Random Processes

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Question 12.13.10.6 - How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

**Solution:** Let,  $\Pr(X_i)$  be the sequence of independent Bernoulli random variables.

$$X_i = \begin{cases} 1, & \text{coin toss result in a Heads} \\ 0, & \text{result in Tails} \end{cases} \quad (1)$$

which means

$$\Pr(X_i = 1) = 0.5 \quad (2)$$

$$\Pr(X_i = 0) = 0.5 \quad (3)$$

Let, the total number of trials be  $n$  and  $Z$  be the random variable that represents the number of heads in  $n$  trials which is given by:

$$\Pr(Z = k) = {}^nC_k \times q^{n-k} \times p^k \quad (4)$$

where

$$Z = X_1 + X_2 + \dots + X_n \quad (5)$$

For atleast 1 Heads

$$\Pr(Z \geq 1) > 0.9 \quad (6)$$

$$1 - \Pr(Z = 0) > 0.9 \quad (7)$$

$$1 - \Pr(X_1 = 0, X_2 = 0, \dots, X_n = 0) > 0.9 \quad (8)$$

$$1 - {}^nC_0 (0.5)^n (0.5)^0 > 0.9 \quad (9)$$

$$1 - (0.5)^n > 0.9 \quad (10)$$

$$0.1 > (0.5)^n \quad (11)$$

$$(2)^n > 10 \quad (12)$$

On solving we get

$$n > 0.33 \quad (13)$$

As we know,  $n$  can be a positive integer value.

So,  $n = 4$ .