

# Assignment - 4

## EE23010: Probability and Random Processes

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Question 12.13.10.6 - How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

**Solution:** Let,  $X_i$  be the sequence of independent Bernoulli random variables.

$$X_i = \begin{cases} 1, & \text{coin toss result in a Heads} \\ 0, & \text{result in Tails} \end{cases} \quad (1)$$

which means

$$p = p_{X_i}(1) = 0.5 \quad (2)$$

$$q = p_{X_i}(0) = 0.5 \quad (3)$$

Let, the total number of trials be  $n$  and the pmf of getting  $k$  heads is given by:

$$p_X(k) = \Pr(X = k) \quad (4)$$

$$= {}^nC_k \times (p)^k \times (q)^{n-k} \quad (5)$$

$$= {}^nC_k \times (0.5)^k \times (0.5)^{n-k} \quad (6)$$

The cdf for the following pmf:

$$F_X(k) = p_X(0) + p_X(1) + \dots + p_X(k) \quad (7)$$

$$= {}^nC_0 (0.5)^n (0.5)^0 + {}^nC_1 (0.5)^{n-1} (0.5)^1 + \dots + {}^nC_k (0.5)^{n-k} (0.5)^k \quad (8)$$

Then the probability of getting atleast 1 heads is:

$$\Pr(X \geq 1) > 0.9 \quad (9)$$

$$1 - p_X(0) > 0.9 \quad (10)$$

$$1 - F_X(0) > 0.9 \quad (11)$$

$$1 - {}^nC_0 (0.5)^n (0.5)^0 > 0.9 \quad (12)$$

$$1 - (0.5)^n > 0.9 \quad (13)$$

$$0.1 > (0.5)^n \quad (14)$$

$$(2)^n > 10 \quad (15)$$

On solving we get

$$n > 0.33 \quad (16)$$

As we know,  $n$  can be a positive integer value.

So,  $n = 4$ .