Assignment - 4
EE23010: Probability and Random Processes
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Question 12.13.10.6 - How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution: Let, X_i be the sequence of independent Bernoulli random varibles.

$$X_i = \begin{cases} 1, & \text{coin toss result in a Heads} \\ 0, & \text{result in Tails} \end{cases} \tag{1}$$

which means

$$p = p_{X_i}(1) = 0.5 \tag{2}$$
$$q = p_{X_i}(0) = 0.5 \tag{3}$$

Let, the total number of trials be n and the pmf of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \tag{4}$$
$$= {}^nC_k \times (p)^k \times (q)^{n-k} \tag{5}$$
$$= {}^nC_k \times (0.5)^k \times (0.5)^{n-k} \tag{6}$$

The cdf for the following pmf:

$$F_X(k) = p_X(0) + p_X(1) + ... + p_X(k) \tag{7}$$
$$= {}^nC_0\,(0.5)^n\,(0.5)^0 + {}^nC_1\,(0.5)^{n-1}\,(0.5)^1 + ... + {}^5C_k\,(0.5)^{n-k}\,(0.5)^k \tag{8}$$

Then the probability of getting atleast 1 heads is:

$$\Pr(X \geq 1) > 0.9 \tag{9}$$
$$1 - \Pr(X = 0) > 0.9 \tag{10}$$
$$1 - F_X(0) > 0.9 \tag{11}$$
$$1 - {}^nC_0\,(0.5)^n\,(0.5)^0 > 0.9 \tag{12}$$
$$1 - (0.5)^n > 0.9 \tag{13}$$
$$0.1 > (0.5)^n \tag{14}$$
$$(2)^n > 10 \tag{15}$$

On solving we get

$$n > 0.33 \tag{16}$$

As we know, n can be a positive integer value.
So, n = 4.