

Assignment - 4

EE23010: Probability and Random Processes

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Question 12.13.6.10 - How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Taking ln both side we get:

$$n > \log_2(10) \quad (12)$$

$$\Rightarrow n > 3.32 \quad (13)$$

Solution: Let, X_i be the sequence of independent Bernoulli random variables.

As we know, n can be a positive integer value. So,

$$\Rightarrow X = \sum_{i=0}^n X_i \quad (1)$$

$$\Rightarrow n = 4 \quad (14)$$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \quad (2)$$

which means

$$p_X(k) = \begin{cases} 0.5 = p, & k = 0 \\ 0.5 = q, & k = 1 \end{cases} \quad (3)$$

Let, the total number of trials be n and the pmf of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \quad (4)$$

$$= {}^nC_k (p)^k (q)^{n-k} \quad (5)$$

$$= {}^nC_k (0.5)^k (0.5)^{n-k} \quad (6)$$

The cdf for the following pmf:

$$F_X(k) = \sum_{i=0}^k p_X(i) \quad (7)$$

$$= \sum_{i=0}^k {}^nC_i (0.5)^{n-i} (0.5)^i \quad (8)$$

Then the probability of getting atleast 1 heads is:

$$\Pr(X \geq 1) > 0.9 \quad (9)$$

$$\Rightarrow 1 - p_X(0) > 0.9 \quad (10)$$

$$(2)^n > 10 \quad (11)$$