Assignment - 4

EE23010: Probability and Random Processes
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Question 12.13.10.6 - How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution: Let, X_i be the sequence of independent Bernoulli random varibles.

$$\implies X = \sum_{0}^{k} X_i \tag{1}$$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \tag{2}$$

which means

$$(p, q) = \begin{cases} p_{X_i}(1) = 0.5 \\ p_{X_i}(0) = 0.5 \end{cases} \tag{3}$$

Let, the total number of trials be n and the pmf of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \tag{4}$$
$$= {}^{n}C_k (p)^k (q)^{n-k} \tag{5}$$
$$= {}^{n}C_k (0.5)^k (0.5)^{n-k} \tag{6}$$

The cdf for the following pmf:

$$F_X(k) = \sum_{i=0}^{k} p_X(i) \tag{7}$$
$$= \sum_{i=0}^{k} {}^{n}C_i (0.5)^{n-i} (0.5)^i \tag{8}$$

Then the probability of getting atleast 1 heads is:

$$\Pr(X \geq 1) > 0.9 \tag{9}$$
$$\implies 1 - p_X(0) > 0.9 \tag{10}$$
$$1 - {}^{n}C_0 (0.5)^n (0.5)^0 > 0.9 \tag{11}$$
$$1 - (0.5)^n > 0.9 \tag{12}$$
$$(2)^n > 10 \tag{13}$$

Taking ln both side we get:

$$n > \log_2(10) \tag{14}$$
$$\implies n > 3.32 \tag{15}$$

As we know, n can be a positive integer value. So, n = 4.