Assignment - 4

EE23010: Probability and Random Processes

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Question 12.13.10.6 - How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

Solution: Let, X_i be the sequence of independent Bernoulli random varibles.
$\implies$ X = $\sum_0^k X_i$

$$X_i = \begin{cases} 1, & \text{Heads} \\ 0, & \text{Tails} \end{cases} \tag{1}$$

which means

$$p = p_{X_i}(1) = 0.5 \tag{2}$$
$$q = p_{X_i}(0) = 0.5 \tag{3}$$

Let, the total number of trials be n and the pmf of getting k heads is given by:

$$p_X(k) = \Pr(X = k) \tag{4}$$
$$= {}^nC_k (p)^k (q)^{n-k} \tag{5}$$
$$= {}^nC_k (0.5)^k (0.5)^{n-k} \tag{6}$$

The cdf for the following pmf:

$$F_X(k) = p_X(0) + p_X(1) + ... + p_X(k) \tag{7}$$
$$= {}^nC_0 (0.5)^n (0.5)^0 + {}^nC_1 (0.5)^{n-1} (0.5)^1 + ... + {}^nC_k (0.5)^{n-k} (0.5)^k \tag{8}$$
$$= \sum_{i=0}^{k} {}^nC_i (0.5)^{n-i} (0.5)^i \tag{9}$$

Then the probability of getting atleast 1 heads is:

$$\Pr(X \geq 1) > 0.9 \tag{10}$$
$$1 - p_X(0) > 0.9 \tag{11}$$
$$1 - F_X(0) > 0.9 \tag{12}$$
$$1 - {}^nC_0 (0.5)^n (0.5)^0 > 0.9 \tag{13}$$
$$1 - (0.5)^n > 0.9 \tag{14}$$
$$0.1 > (0.5)^n \tag{15}$$
$$(2)^n > 10 \tag{16}$$

Taking ln both side we get:

$$n \ln(2) > \ln(10) \tag{17}$$
$$n > \log_2(10) \tag{18}$$
$$\implies n > 3.32 \tag{19}$$

As we know, n can be a positive integer value.
So, n = 4.