Assignment

EE23010: Probability and Random Processes
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Question: Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that

1) C will be selected?
2) A will not be selected?

Solution: Let X_A, X_B, X_C, X_D be the random variables representing A,B,C,D respectively.

$$X_A = \begin{cases} 1 & \text{A is selected} \\ 0 & \text{A is not selected} \end{cases} \tag{1}$$

$$X_B = \begin{cases} 1 & \text{B is selected} \\ 0 & \text{B is not selected} \end{cases} \tag{2}$$

$$X_C = \begin{cases} 1 & \text{C is selected} \\ 0 & \text{C is not selected} \end{cases} \tag{3}$$

$$X_D = \begin{cases} 1 & \text{D is selected} \\ 0 & \text{D is not selected} \end{cases} \tag{4}$$

Given,

$$\Pr(X_A = 1) = 2\Pr(X_B = 1) \tag{5}$$
$$\Pr(X_B = 1) = \Pr(X_C = 1) \tag{6}$$
$$\Pr(X_C = 1) = 2\Pr(X_D = 1) \tag{7}$$

representing all in terms of 1 varible x,

$$\Pr(X_B = 1) = x \tag{8}$$
$$\Pr(X_A = 1) = 2x \tag{9}$$
$$\Pr(X_B = 1) = \Pr(X_C = 1) = x \tag{10}$$
$$\Pr(X_D = 1) = \frac{x}{2} \tag{11}$$

and we know sum of probabilities of all candidates must be equal to 1

$$\Pr(X_A = 1) + \Pr(X_B = 1) + \Pr(X_C = 1) + \Pr(X_D = 1) = 1 \tag{12}$$

So,

$$2x + x + x + \frac{x}{2} = 1 \tag{13}$$
$$\implies \Pr(X_B = 1) = \frac{2}{9} \tag{14}$$

1) For C getting selected:

$$\Pr(X_C = 1) = \Pr(X_B = 1) \tag{15}$$
$$\implies \Pr(X_C = 1) = \frac{2}{9} \tag{16}$$

2) For A not getting selected:

$$\Pr(X_A = 0) = 1 - \Pr(X_A = 1) \tag{17}$$
$$= 1 - 2\Pr(X_B = 1) \tag{18}$$
$$= 1 - \frac{4}{9} \tag{19}$$
$$\implies \Pr(X_A = 0) = \frac{5}{9} \tag{20}$$