Assignment

EE23010: Probability and Random Processes

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Question: For a real signal, which of the following is/are valid power spectral density/-densities?

1) $S_X(\omega) = \frac{2}{9+\omega^2}$
2) $S_X(\omega) = e^{-\omega^2} cos^2\omega$
3) See Fig. 3

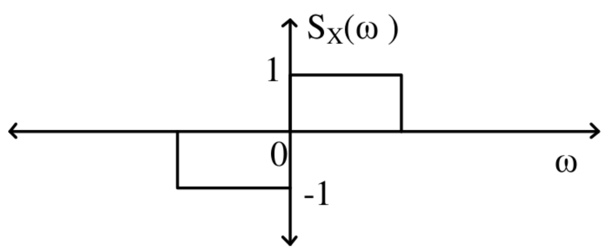

Fig. 3. Figure1

4) See Fig. 4

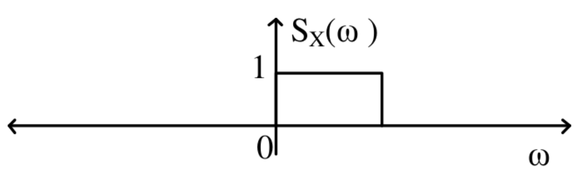

Fig. 4. Figure2

(GATE EC 2023)

Solution:

The S_X and $R(\tau)$ of a power signal form a Fourier transform pair, i.e.,

$$R(\tau) \overset{\mathcal{F}}{\rightleftharpoons} S_X \tag{1}$$

which gives :

$$S_X = \int R_X(\tau)e^{(-j\omega\tau)}d\tau \tag{2}$$

$$= \int [R_X(\tau)cos(\omega\tau)] - j\,[R_X(\tau)sin(\omega\tau)]d\tau \tag{3}$$

Now,For a real signal,the $R_X(\tau)$ is real and even the fourier transform of $R_X(\tau)$ will also exhibits the same properties:

$$\text{Im}(S_X(\omega)) = -\int j\,[R_X(\tau)sin(\omega\tau)d\tau] \tag{4}$$

$$\implies 0 \tag{5}$$

$$\text{and, } S_X(-\omega) = S_X(\omega) \tag{6}$$

$$\int R_X(\tau)e^{j\omega\tau}d\tau = \int R_X(\tau)e^{-j\omega\tau}d\tau \tag{7}$$

So, the properties of S_X are:

(a) $\text{Im}S_X(\omega) = 0$
(b) $S_X(-\omega) = S_X(\omega)$

Now,

1) Plot for $S_X(\omega) = \frac{2}{9+\omega^2}$

$$\text{Im}\left(\frac{2}{9+\omega^2}\right) = 0 \tag{8}$$

Also,

$$\frac{2}{9+\omega^2} = \frac{2}{9+(-\omega)^2} \tag{9}$$

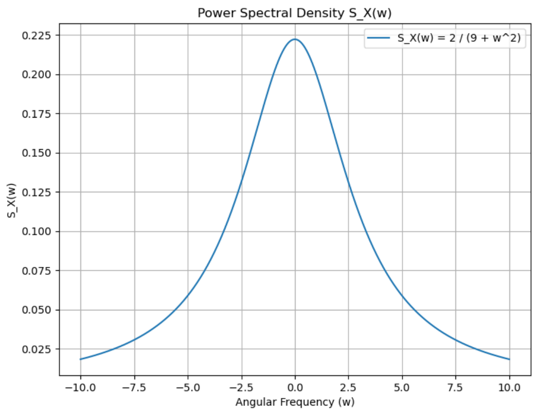

Fig. 1. plot1

So, S_X is valid.

2) Using ((a)) & ((b))

$$\text{Im}(e^{-\omega^2} cos^2\omega) = 0 \tag{10}$$

$$e^{-\omega^2} cos^2\omega = e^{-(-\omega)^2} cos^2(-\omega) \tag{11}$$

It is also a valid S_X.

3) Using ((a)) & ((b))

$$S_X(-\omega) = -S_X(\omega) \tag{12}$$

As, It is real but odd function.So, it is not a valid S_X.

4)

$$S_X = \begin{cases} 1, 0 < \omega < \omega_o \\ 0, \text{otherwise} \end{cases} \tag{13}$$

Here, S_X is neither odd nor even. So, it is not valid.

$\therefore$ Option (1) and (2) are correct.