Toward a 2D Formulation

ErSE-299: Directed Research
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This repository currently targets the **1D acoustic wave equation** using a NISQ-oriented, gate-based workflow that combines variational state preparation with Fourier (spectral) time evolution. Extending this approach to **2D** is not a drop-in modification: in 1D, the Fourier index is a single integer k, whereas in 2D each mode is indexed by a pair (k_x, k_y). This change alters the dispersion relation and, critically, breaks the simple phase-accumulation structure that enables efficient Eq. (14)-style circuits in the 1D case.

In 1D, time evolution in the Fourier basis is diagonal, with mode-dependent phases

$$e^{\pm i\omega_k t},$$

allowing for shallow circuits that avoid explicit arithmetic. In 2D, however, the physically correct dispersion depends jointly on both components, for example

$$\omega(k_x, k_y) \propto \sqrt{\omega_x(k_x)^2 + \omega_y(k_y)^2},$$

which generally necessitates either (i) precomputed phase tables, (ii) controlled approximations, or (iii) explicit reversible arithmetic.

Future Perspectives on 2D Formulation Approaches

Option A — Precomputed 2D diagonal propagator (exact, small-scale). Classically compute

$$\theta_{k_x,k_y} = \omega(k_x, k_y)\, t$$

for all modes, and compile a diagonal unitary

$$U = \mathrm{diag}\left(e^{-i\theta_{k_x,k_y}}\right).$$

This approach preserves the correct 2D physics but scales exponentially with the number of system qubits, making it practical only for small grids. It is best suited for **validation**, **unit testing**, and **proof-of-concept** demonstrations of 2D correctness.

Option B — Separable (Eq. (14)-like) approximation (scalable, approximate). Approximate the 2D dispersion such that the phase factorizes, enabling two independent 1D-style evolution blocks acting separately on the x- and y-registers. This maintains shallow, scalable circuits compatible with NISQ constraints, but it does *not* represent a fully coupled 2D wave equation in general. This option is most appropriate for **algorithmic scaling studies** and qualitative analyses, particularly when spectra are narrow or propagation is aligned with the grid axes.

Option C — Arithmetic-based 2D dispersion within the circuit (exact, challenging). Compute $\omega(k_x, k_y)$ directly on the quantum computer using reversible arithmetic, for example by evaluating Laplacian eigenvalues, followed by squaring, addition, and square-root operations, and applying phases via phase kickback. While exact and scalable in principle, this approach requires **ancilla qubits**, deep circuits, and extensive uncomputation, placing it well beyond near-term NISQ feasibility.

Recommendation

Given the repository's emphasis on NISQ feasibility and benchmarking, a practical development path is:

1. Implement **Option A** for a small (n_x, n_y) grid to establish a correct 2D reference and testing framework.
2. Add **Option B** as the NISQ-oriented, scalable "2D-like" mode, clearly labeled as an approximation.
3. Treat **Option C** as a longer-term research direction (fault-tolerant or resource-estimation track), rather than a near-term target.