Progress Report I

ErSE-299: Directed Research
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28 September 2025

Over the past two weeks, I have consolidated my understanding of quantum approaches to seismic forward modeling. My efforts have included:

- A detailed study of Wen & Wang (2025) on annealing-based stencil optimization.
- A careful reading of Wright et al. (2024) on gate-based NISQ simulations, with lingering questions about discretization-to-state encoding where *PhysRevA* (2019) is referenced in the paper as a theoretical reference for time-domain mappings.
- The beginning of a close reading of Zhang & Chen (2025) on Schrödingerization in the frequency domain.

Looking ahead, my next steps build naturally from these readings. Two complementary research directions remain open:

1. **Mathematical development:** Deepen the study of derivations in *PhysRevA* (2019) and Zhang & Chen (2025). This would clarify how PDEs are systematically mapped into Schrödinger form in both time and frequency domains, highlighting assumptions, error sources, and boundary-condition treatments.
2. **Prototype simulations:** Implement small-scale tests, such as reproducing Wen & Wang (2025)'s annealing-based dispersion optimization or Wright et al. (2024)'s 1D gate-based benchmark. These prototypes would provide hands-on evidence of feasibility, particularly regarding numerical dispersion, circuit depth, and noise effects.

At present, I am undecided between prioritizing theoretical depth (time vs. frequency mappings) or practical implementation. Both directions are strategically important: one would produce a rigorous theoretical notebook, while the other would yield evidence-driven benchmarks.

Research Insights

1. Seismic Wave Propagation with Quantum Annealing (Wen & Wang, 2025)

Wen & Wang (2025) reformulate seismic forward modeling as an optimization problem solvable by *quantum annealing*. Traditionally, finite-difference (FD) coefficients a_m are obtained via Taylor expansions to ensure local accuracy. In contrast, they pose coefficient determination as a *least-squares inverse problem.*

Starting from the 3D acoustic wave equation with constant density, they approximate derivatives using weighted differences. Extending across all wavenumbers and propagation angles, the forward model is written as

$$F(a) \approx d_k,$$

with the least-squares objective

$$\phi(a) = \|F(a) - d\|_2^2.$$

Linearization leads to the update system

$$A\Delta a = b, \qquad A^T A\Delta a = A^T b.$$

Embedding this into a quantum framework requires discretizing coefficients into binary vectors, yielding a QUBO problem:

$$\min_{q\in\{0,1\}^N} q^T Q q,$$

which is solved on a quantum annealer. The optimization minimizes *dispersion error*, i.e., the mismatch between the true and numerical wavenumbers.

Key learning outcomes:

- Dispersion error arises from stencil approximations; annealing provides a direct way to minimize this error in spectral space.
- Binary expansion allows continuous coefficient updates to be encoded as qubits, linking classical least-squares optimization to quantum annealing.
- Classical DRP/high-order FD baselines remain essential for fair performance comparisons.

2. Gate-Based NISQ Simulation of the 1D Wave Equation (Wright et al., 2024)

Wright et al. (2024) present a gate-based simulation of the one-dimensional acoustic wave equation on NISQ hardware. The governing PDE

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

is recast into a quantum circuit that evolves a discretized wavefield in time, with each timestep corresponding to applications of unitary operations analogous to the propagator

$$\psi(t + \Delta t) = e^{-iH\Delta t}\psi(t).$$

In practice, the construction of such circuits draws on standard gates (Hadamard, Pauli-X/Z, and controlled operations) whose compositions approximate differential operators. A key challenge is fidelity: real hardware operations deviate from their ideal unitaries, with infidelity defined as

$$\text{Infidelity} = 1 - \text{Fidelity}.$$

Typical values are 10^{-5}–10^{-4} for single-qubit gates and 10^{-3}–10^{-2} for two-qubit gates. Since PDE simulation requires many gates, errors accumulate, making shallow circuits essential for feasibility on current devices.

A remaining ambiguity is how the discretized PDE degrees of freedom are embedded into a quantum register. Wright et al. (2024) assert that the wavefield is represented in the amplitudes of the quantum state but do not fully derive this mapping. Here, *PhysRevA* (2019) provides the necessary foundation, rigorously showing how the time-domain wave equation can be expressed in Schrödinger form. This makes clear that Wright et al. (2024) build directly on the earlier formalism, while adapting it to an applied seismic context and real hardware constraints.

Key insights:

- Learned how PDE discretization can be approximated within quantum state evolution, though the formal mapping requires reference to *PhysRevA* (2019).
- Understood the central role of gate fidelity and noise, and why circuit depth is the limiting factor for NISQ simulations.
- Recognized Wright et al. (2024) as an applied benchmark: bridging rigorous derivations with practical hardware demonstrations, illustrating both the potential and limitations of quantum seismic solvers today.

3. Schrödingerization of Helmholtz and Wave Equations (Zhang & Chen, 2025; *PhysRevA*, 2019) - currently studying

Zhang & Chen (2025) recast the Helmholtz equation

$$\nabla^2 u + k^2 u = f$$

as a time-independent Schrödinger Hamiltonian, allowing frequency-domain seismic problems to be addressed as eigenvalue problems. By contrast, *PhysRevA* (2019) begins with the wave equation and derives a time-dependent Schrödinger form:

$$i\frac{\partial}{\partial t}\psi(t) = H\psi(t).$$

Together, these perspectives are complementary:

- *PhysRevA (2019)*: hyperbolic PDEs in the time domain, simulated via evolution operators e^{-iHt}.

- *Zhang & Chen (2025)*: elliptic PDEs in the frequency domain, reformulated as Hamiltonian eigenproblems.

My current focus is Zhang & Chen (2025), as frequency-domain seismic modeling introduces challenges of conditioning and indefiniteness.

Key learning outcomes:

- Hamiltonian formalism provides a unifying framework for both time-dependent and frequency-domain PDEs.
- Quantum Fourier Transform (QFT) and time-evolution operators are central in the time-domain approach.
- Together, *PhysRevA* (2019) and Zhang & Chen (2025) form complementary theoretical foundations for seismic quantum simulation.

References

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- Childs, A. M., Liu, J.-P., Ostrander, A., & Su, Y. (2019). *Quantum algorithm for the wave equation. Physical Review A, 101*(2), 022318.