MECH 539 Computational Aerodynamics

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1 Assignment 3

1.1 Question 1

Here, I solved the Transonic Small Disturbance (TSD) theory over a circular arc airfoil using the Murman-Cole method. The simplified TSD equation is written as,

$$\left[(1 - M_\infty^2) - (\gamma + 1)M_\infty^2 \frac{\phi_x}{U_\infty}\right]\phi_{xx} + \phi_{yy} = 0$$

The above equation was solved on stretched grid, which had 80 grid points in the x-axis, with 40 of the grid points being over the airfoil and 20 grid points ahead of the airfoil and another 20 behind the airfoil. The stretching ahead and behind the airfoil was obtained using an exponential function, the 40 points above the airfoil however aren't stretched and are linearly distributed.
The y-axis consists of 40 points that are also stretched in an exponential manner with first grid spacing adjacent to the airfoil in the y-direction is $dy = \frac{t/c}{2}$
I solved the equations using the Gauss-Siedel method and obtained the following contour plot for the velocity potential, ϕ when the free-stream Mach number was chosen as $M_\infty = 0.86$,

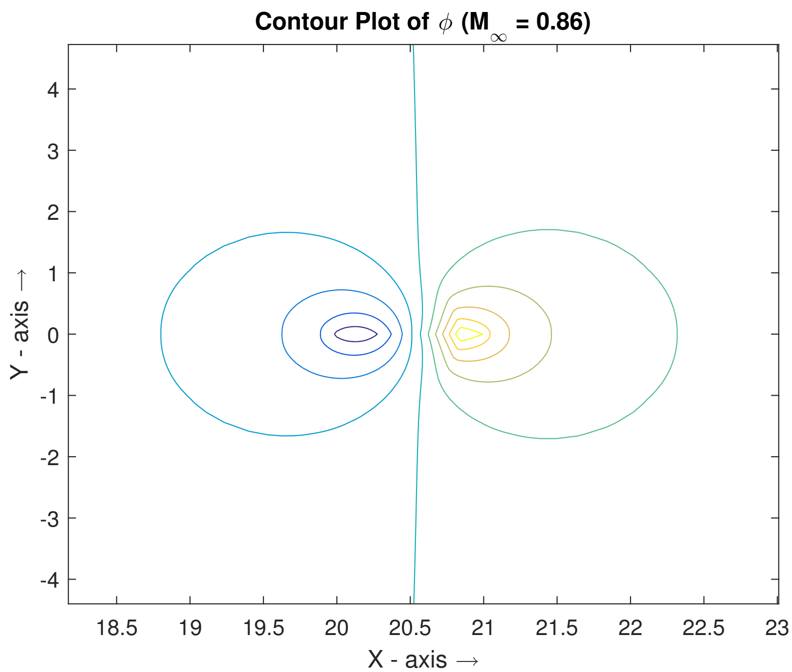

Figure 1: Solution for ϕ when $M_\infty = 0.86$

1.2 Question 2

In this question I vary the free stream Mach number, M_∞ from 0.80 to 0.90 with increments of 0.02, and for each case provide a plot for the convergence of the $L_\infty - norm$, surface pressure coefficient as a function of x, and the pressure contour directly over the airfoil with the y-axis limits being set from 0 to 1. The residual was reduced to 1×10^{-4}.
NOTE :The graphs of the Coefficients of Pressure are plotted upside down, i.e. the negative direction of the y-axis is facing upwards. The following plots were obtained;

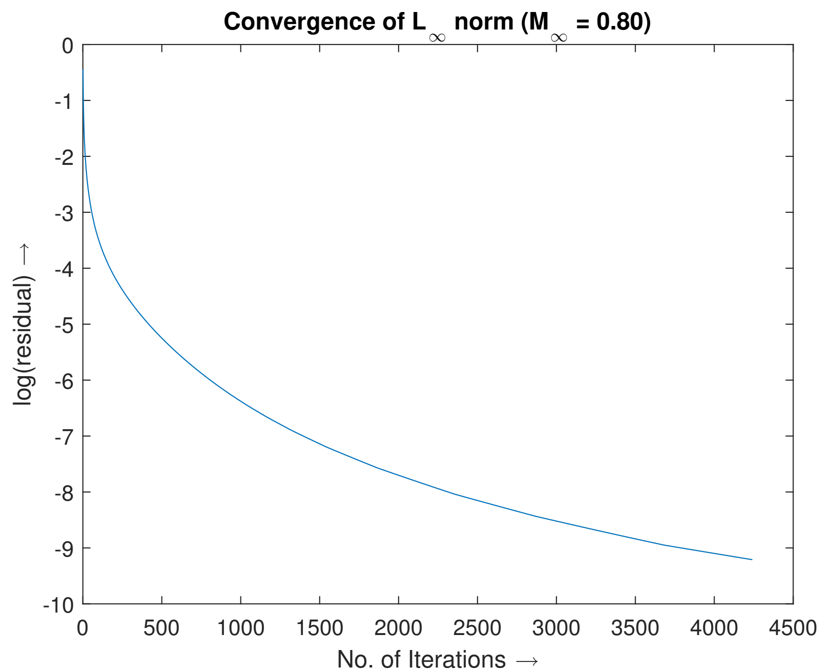

Figure 2: Convergence of the $L_\infty - norm$ when $M_\infty = 0.80$

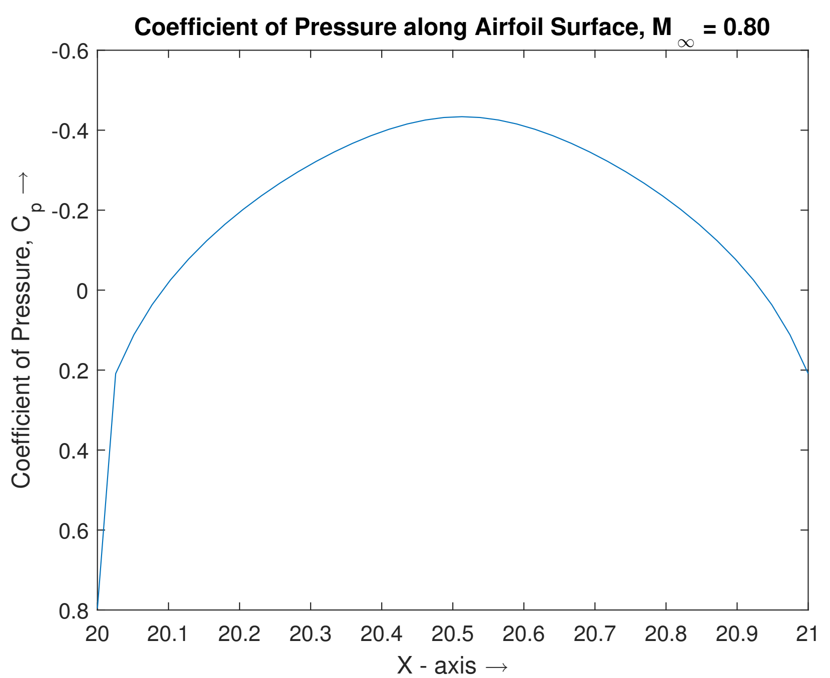

Figure 3: Surface Pressure Coefficient as a function of x when $M_\infty = 0.80$

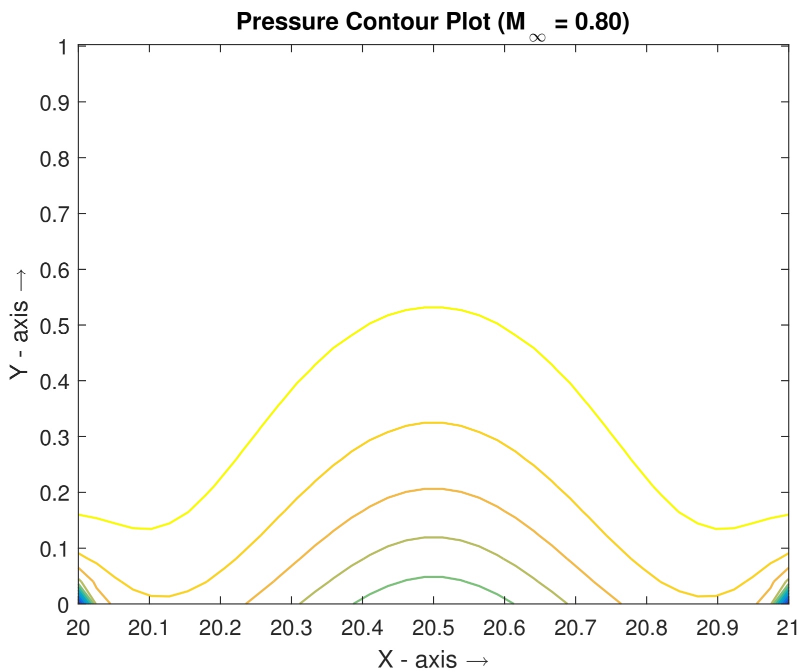

Figure 4: Pressure Contour above airfoil when $M_\infty = 0.80$

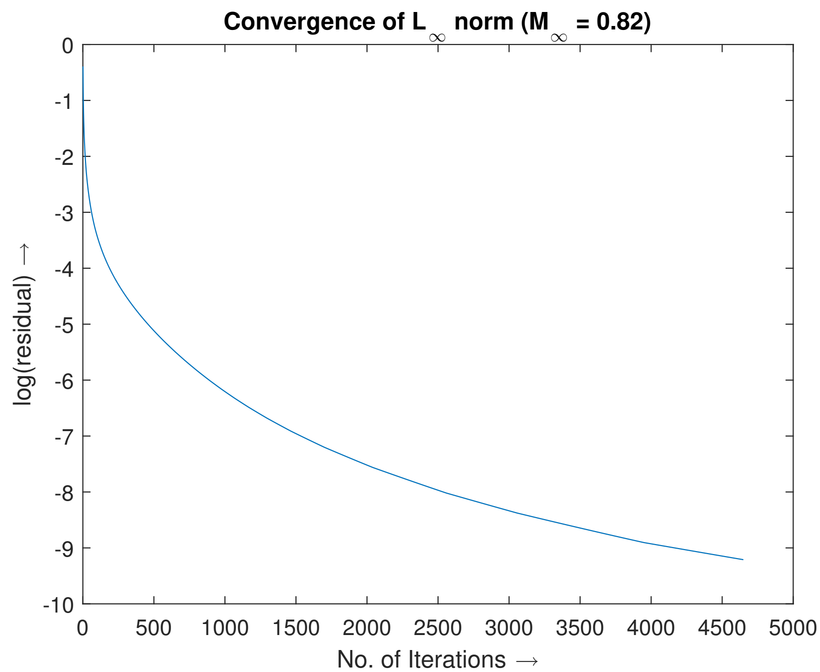

Figure 5: Convergence of the $L_\infty - norm$ when $M_\infty = 0.82$

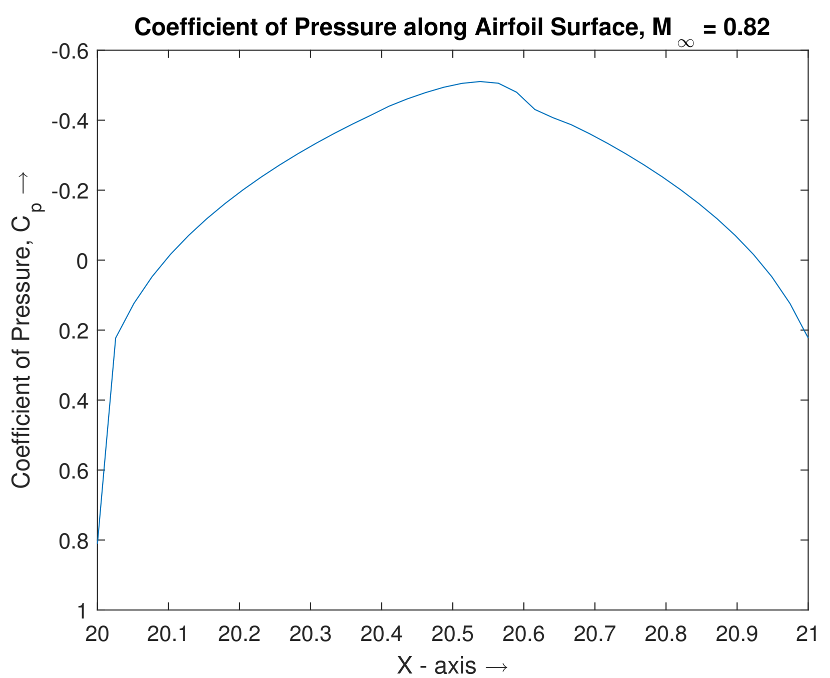

Figure 6: Surface Pressure Coefficient as a function of x when $M_\infty = 0.82$

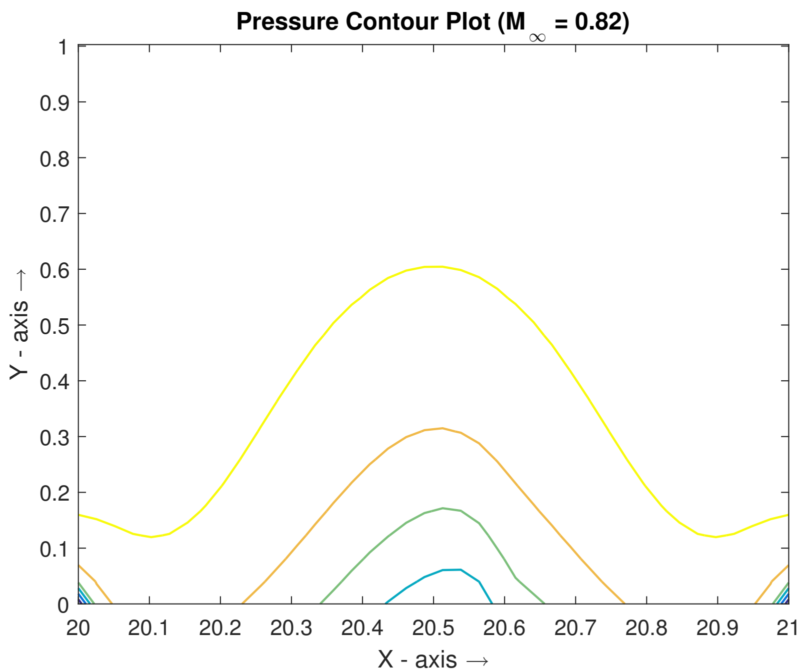

Figure 7: Pressure Contour above airfoil when $M_{\infty} = 0.82$

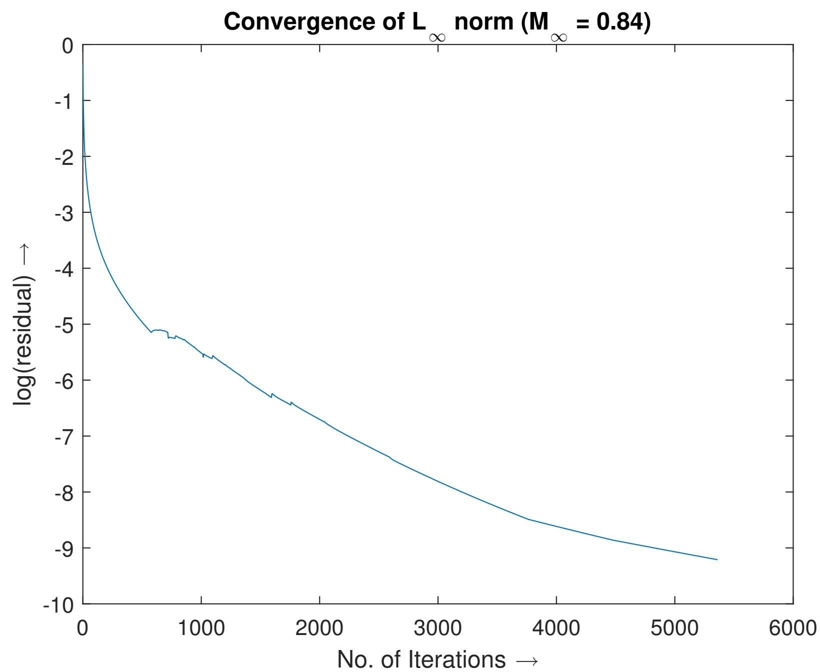

Figure 8: Convergence of the $L_\infty - norm$ when $M_\infty = 0.84$

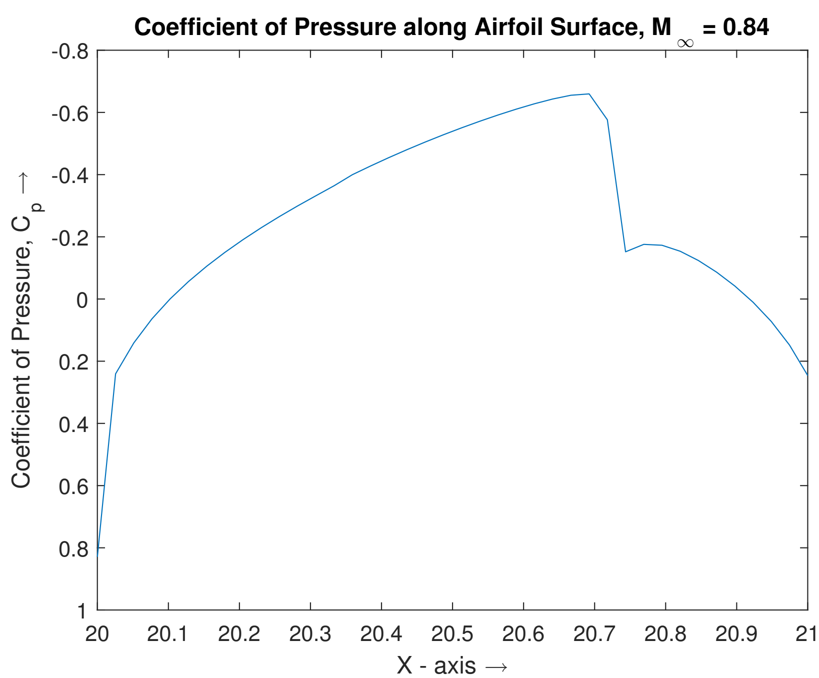

Figure 9: Surface Pressure Coefficient as a function of x when $M_\infty = 0.84$

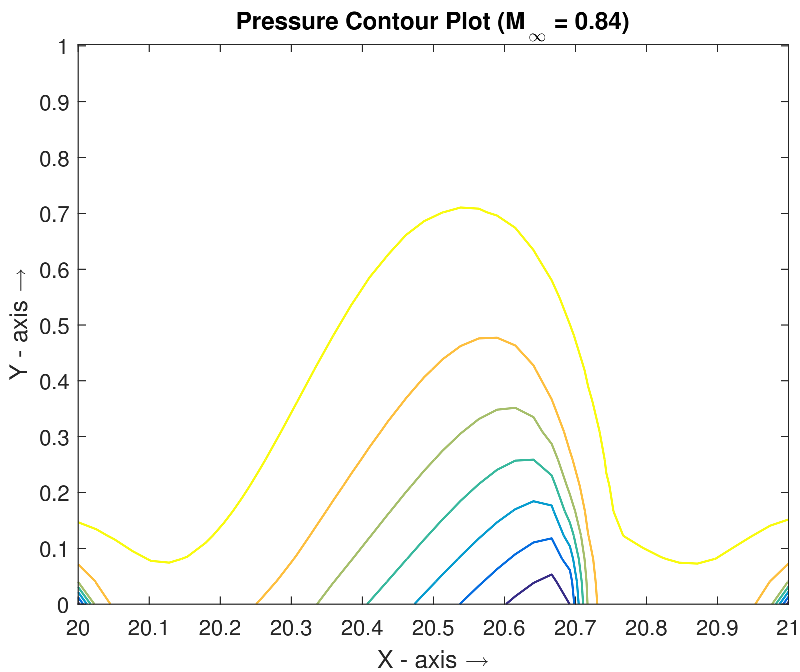

Figure 10: Pressure Contour above airfoil when $M_\infty = 0.84$

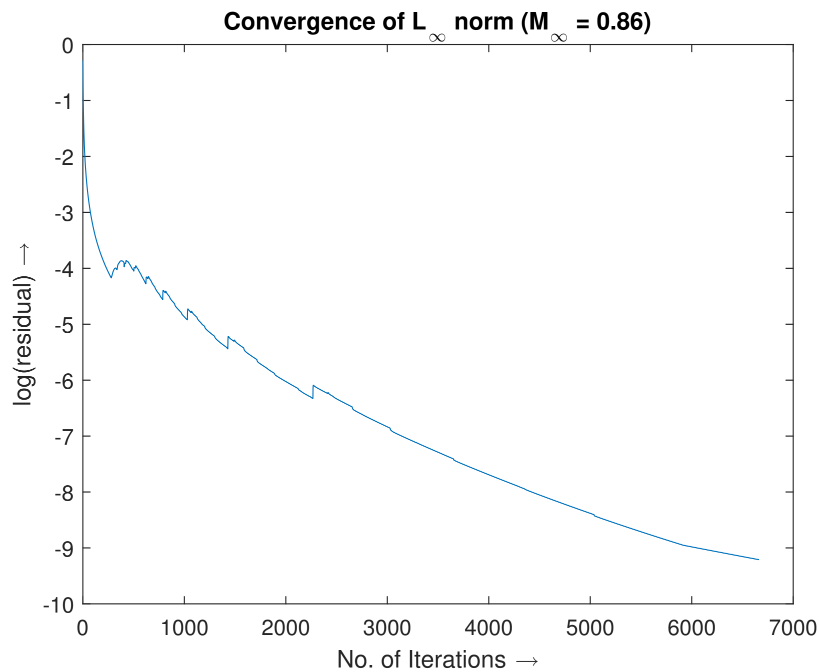

Figure 11: Convergence of the $L_\infty - norm$ when $M_\infty = 0.86$

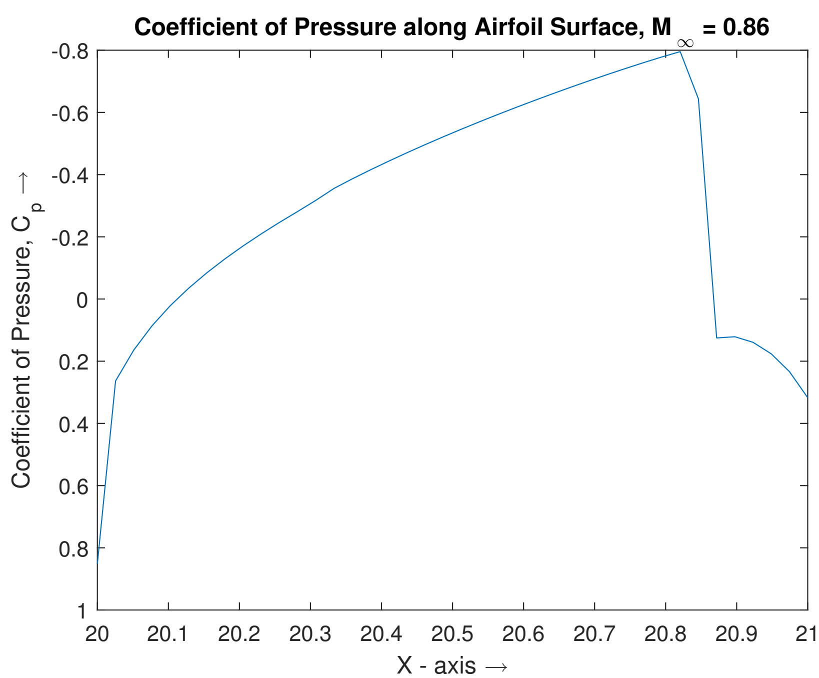

Figure 12: Surface Pressure Coefficient as a function of x when $M_\infty = 0.86$

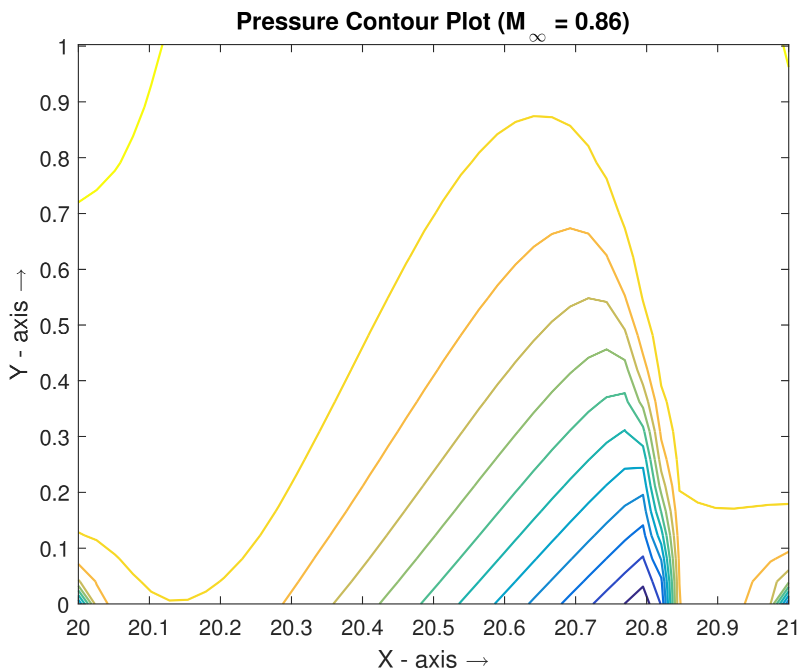

Figure 13: Pressure Contour above airfoil when $M_\infty = 0.86$

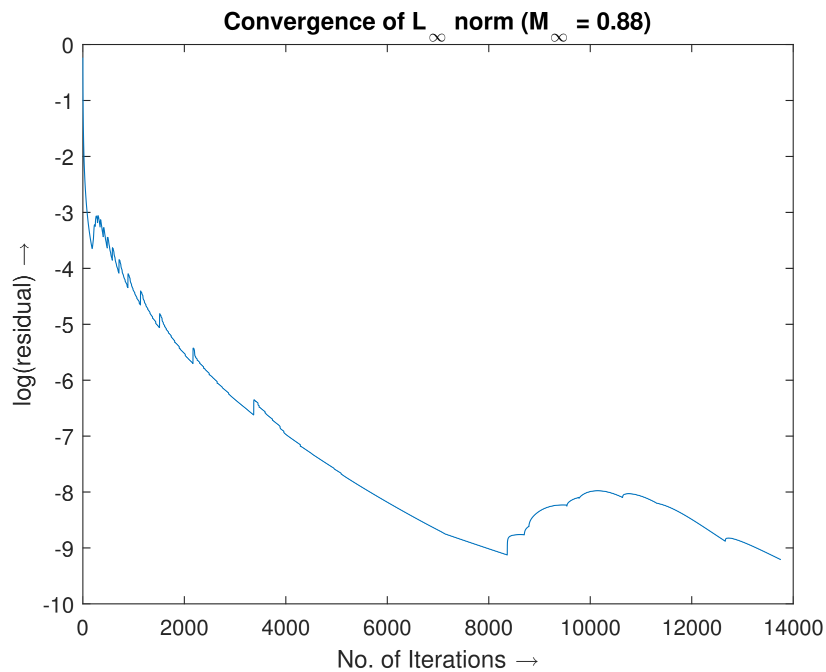

Figure 14: Convergence of the $L_\infty - norm$ when $M_\infty = 0.88$

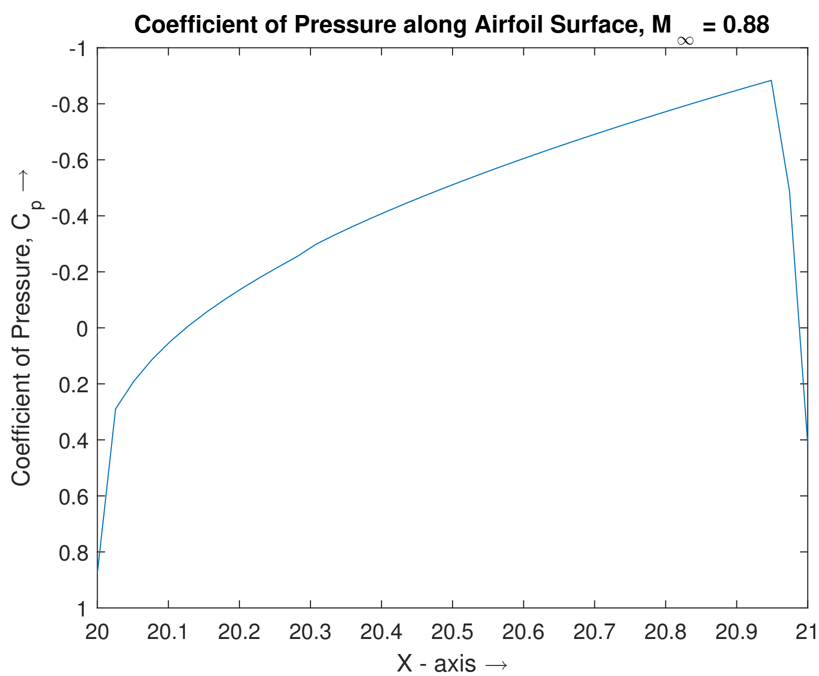

Figure 15: Surface Pressure Coefficient as a function of x when $M_\infty = 0.88$

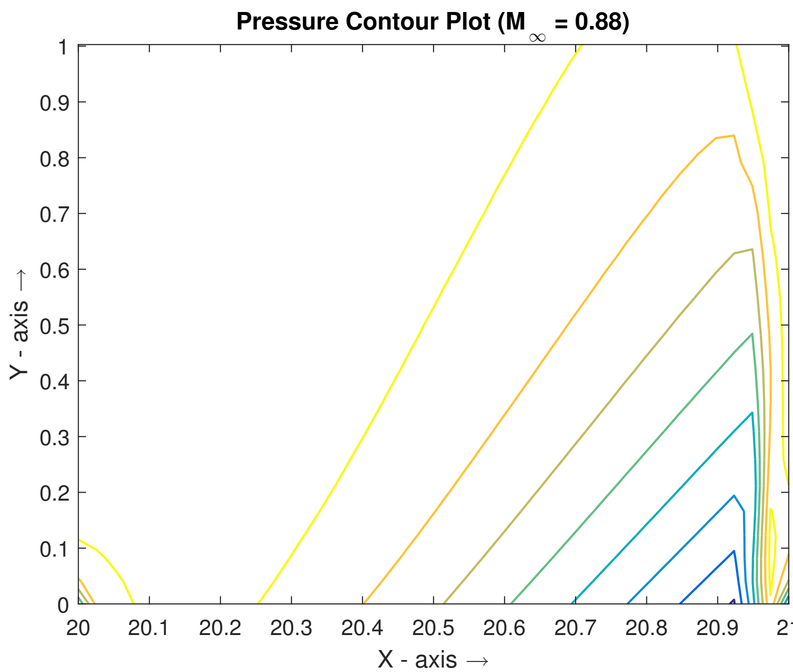

Figure 16: Pressure Contour above airfoil when $M_{\infty} = 0.88$

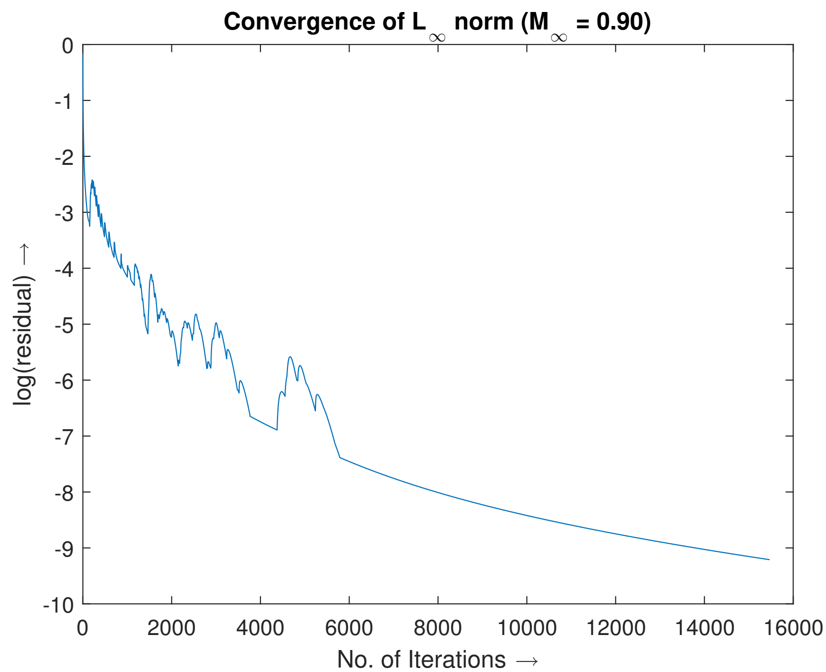

Figure 17: Convergence of the $L_\infty - norm$ when $M_\infty = 0.90$

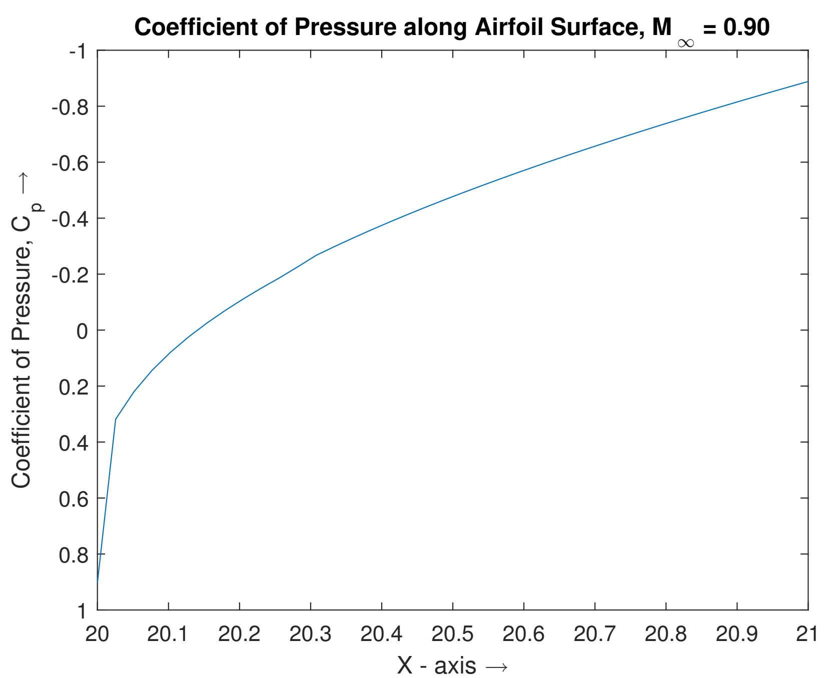

Figure 18: Surface Pressure Coefficient as a function of x when $M_\infty = 0.90$

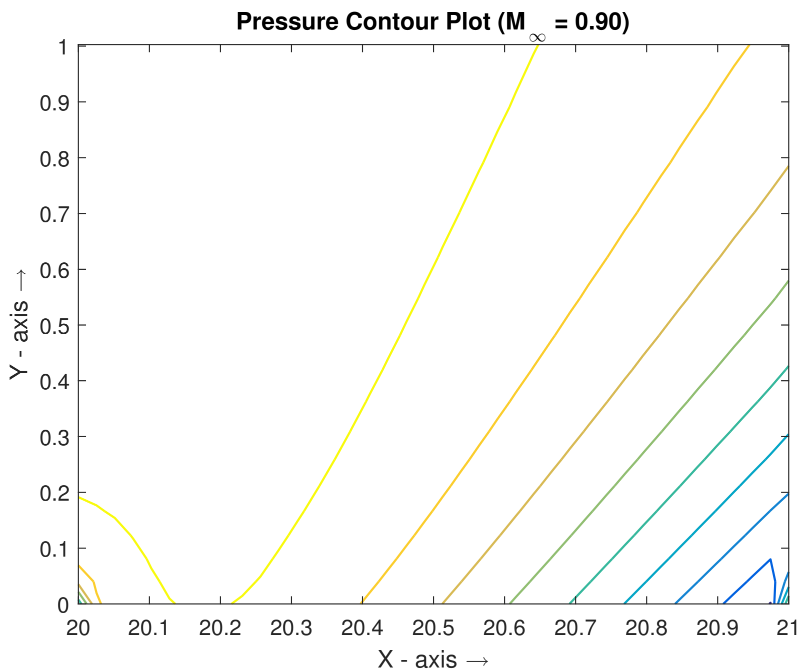

Figure 19: Pressure Contour above airfoil when $M_\infty = 0.90$

If we look at plots of the coefficients of pressures above the airfoil as the free stream Mach number, m_∞ is increased we see that there is no shock wave when $M_\infty = 0.80$ and the flow is completely subsonic. As the free stream Mach number is increased to 0.82, we see a sort of sudden kink beginning to develop in the plot of the coefficient of pressure, but this isn't big enough to suggest that a shock wave is taking place. When the free stream Mach number is made equal to 0.84, We see a sharp increase in the coefficient of pressure suddenly around $x = 20.6$, suggesting there is a shock wave at the that location. As the free stream Mach number is further increased to 0.86, we see the location of the sudden increase of the Coefficient of pressure move further back to $x = 20.8$ and now we also notice that The amount of increase in the value of C_p is also higher. The general trend of the Coefficient of Pressure we observe is that as the free stream Mach number is increased above a certain value (that is high enough to cause the flow to become supersonic), the C_p has sudden spikes and also as the free stream Mach number is increased, the location of the sudden spikes moves further back along the airfoil.
For the plots of the convergence of the $L_\infty - norm$, we observe that when the free stream Mach number is 0.80, the curve obtained is smooth, but as the free stream Mach number is increased the curve becomes increasingly 'wavy' and is no longer a smooth curve as it converges to the answer.
For the plots of the Pressure Contour above the airfoil, we see that when the free stream Mach number is 0.80, the curve is really smooth, but as the free stream Mach number is increased it is observed the lines in the pressure contour become steeper and steeper, indicating a sudden pressure increase due to the presence of shock waves.

1.3 Question 3

In this question, I vary the grid sizes by doubling the number of gridpoints in each direction consecutively to get a coarse(80×40), medium(160×80) and a fine (320×80 grid, and then plot the coefficient of pressure over the surface of the airfoil at a free stream Mach number of 0.88. The graph has been plotted below.
As we can see the location of the shock does move a bit as the grid is refined, the fine grid produces a shock that is slightly to the left of the coarse grid. I think this happens because I use an exponentially expanding grid, as I make the grid more dense, there are more nodes available which helps us more accurately predict the location of the shock on the surface of the airfoil.

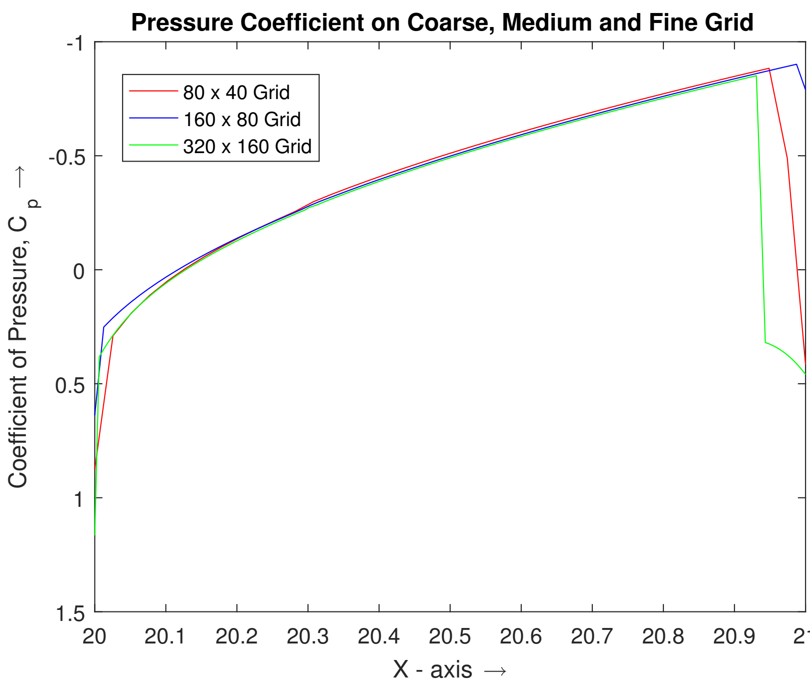

Figure 20: Effect of varying Grid sizes on C_p

1.4 Question 4

In this question I compare the convergence plot using the Gauss-Siedel method and the Line-Implicit Gauss Siedel method.The following graphs were obtained. I believe the reason for the inconsistency obtained in my graphs is because I have used the direct function to find the inverse of a matrix instead of using the Thomas algorithm, hence the longer computing times and the more number of iterations needed in order to converge to an answer.

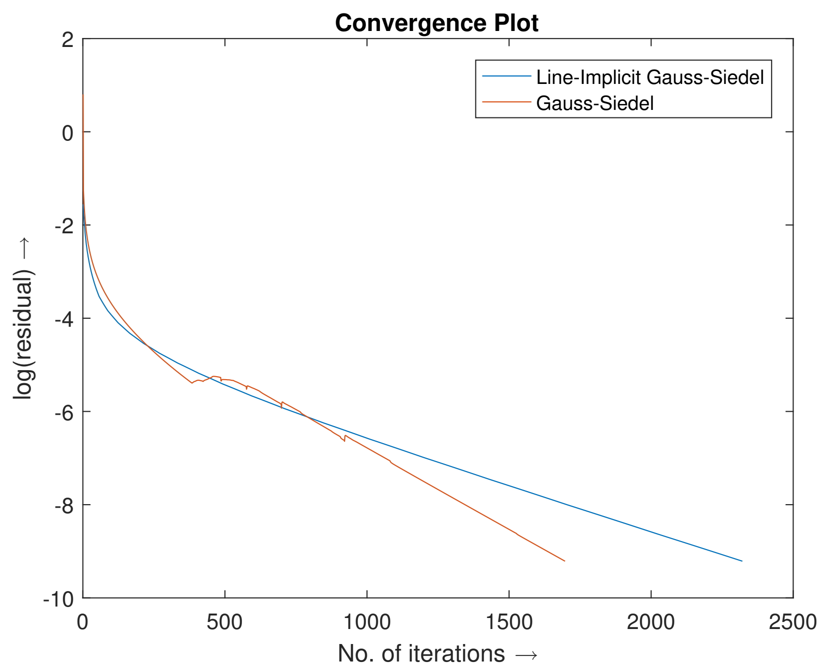

Figure 21: Convergence Plot

1.5 Question 5

In this question I am asked to vary the free stream Mach number between $[0.80, 0.90]$ with 0.02 increments and the plot the coefficient of pressure for each Mach number on the same plot.I have used a coarse grid (80×40) to carry out my calculations. The following plot was obtained.
As it is clearly visible from the plot below, as the free stream Mach number is increased, the location of the shock wave is pushed further back on the surface of the airfoil.
Also, we notice that when the free stream Mach number is equal 0.80, there are no sudden increases in the coefficient of pressure, hence indicating that the flow over the airfoil is completely subsonic.
We can also see that, as the free stream Mach number is increased the amounts by which C_p suddenly increases, also gets larger.
NOTE: In the plots below, the negative direction of the y-axis is facing upwards.

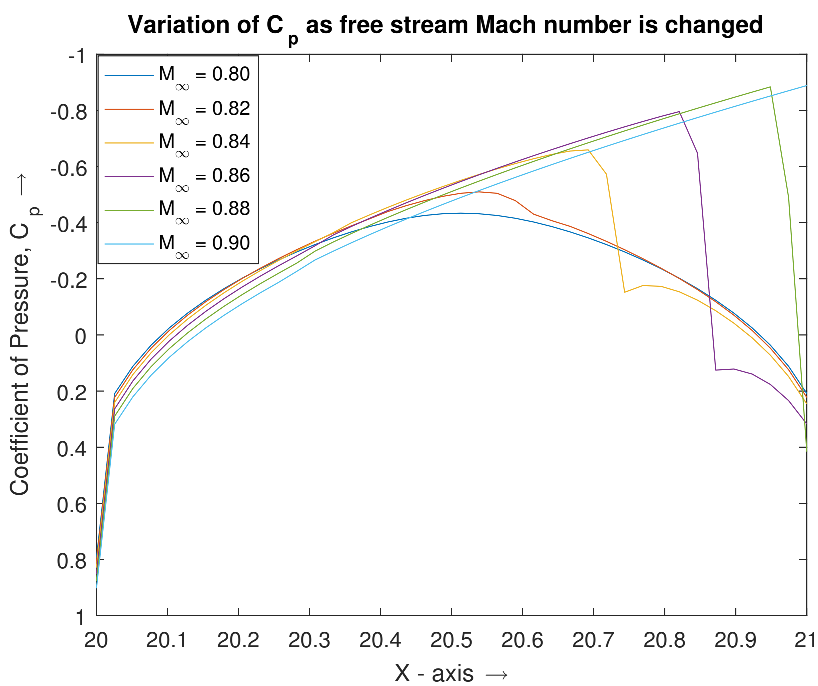

Figure 22: Coefficient of Pressure for increasing Mach numbers