Assignment 5:- Contours and Stream Lines

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05.01.2023

1. Scalar and vector field in Spherical Polar Coordinates

In[297]:=

```
Print["u = (k. r̂)/r^2"]
Print["u = (6 êz + êy) . r̂ / r^2"]
Print["Hence, the scalar field in spherical polar coordinates is:- "]
Print["(6 cosθ + sinθ sinϕ)/r^2"]
Print[]
scalField = (6 Cos[θ] + Sin[θ] Sin[ϕ] )/r^2;
vectField = -Grad[scalField, {r, θ, ϕ}, "Spherical"];
Print["The corresponding vector field in spherical polar coordinates is:- "]
Print[vectField]
Print[]
```

$$u = \frac{\left(k.\ \hat{r}\right)}{r^2}$$

$$u = \frac{\left(6\ \hat{e}_z + \hat{e}_y\right).\ \hat{r}}{r^2}$$

Hence, the scalar field in spherical polar coordinates is:-

$$\frac{6\cos\theta + \sin\theta\sin\phi}{r^2}$$

The corresponding vector field in spherical polar coordinates is:-

$$\left\{\frac{2\ (6\,\mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi])}{r^3},\ -\frac{-6\,\mathrm{Sin}[\theta] + \mathrm{Cos}[\theta]\,\mathrm{Sin}[\phi]}{r^3},\ -\frac{\mathrm{Cos}[\phi]}{r^3}\right\}$$

2. Scalar and Vector fields in Cartesian Coordinates

In[307]:=

```
scalCartes = TransformedField["Spherical" → "Cartesian",
scalField, {r, θ, ϕ} → {x, y, z}];
Print["The scalar field in Cartesian coordinates is:- "]
Print[scalCartes]
Print[]
vectCartes1 = TransformedField["Spherical" → "Cartesian",
vectField, {r, θ, ϕ} → {x, y, z}];
Print["The vector field in Cartesian coordinates is:- (using TransformField) "]
Print[vectCartes1]
Print[]
(*Note that finding the vector field using the Gradient of the
 Scalar Cartesian field will give a simplified version of the vector
 field found using TransformField of the Spherical Vector Field*)
vectCartes2 = -Grad[scalCartes, {x, y, z}, "Cartesian"];
Print["The vector field in Cartesian coordinates is:- (using Grad) "]
Print[vectCartes2]
Print[]
```

The scalar field in Cartesian coordinates is:-

$$\frac{y+6z}{\left(x^2+y^2+z^2\right)^{3/2}}$$

The vector field in Cartesian coordinates is:- (using TransformField)

$$\left\{\frac{x\,y}{\left(x^2+y^2\right)\left(x^2+y^2+z^2\right)^{3/2}}+\frac{2x\left(\frac{y}{\sqrt{x^2+y^2+z^2}}+\frac{6z}{\sqrt{x^2+y^2+z^2}}\right)}{\left(x^2+y^2+z^2\right)^2}-\frac{x\,z\left(-\frac{6\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}+\frac{y\,z}{\sqrt{x^2+y^2}\,\sqrt{x^2+y^2+z^2}}\right)}{\sqrt{x^2+y^2}\left(x^2+y^2+z^2\right)^2},\right.$$

$$-\frac{x^2}{\left(x^2+y^2\right)\left(x^2+y^2+z^2\right)^{3/2}}+\frac{2y\left(\frac{y}{\sqrt{x^2+y^2+z^2}}+\frac{6z}{\sqrt{x^2+y^2+z^2}}\right)}{\left(x^2+y^2+z^2\right)^2}-\frac{y\,z\left(-\frac{6\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}+\frac{y\,z}{\sqrt{x^2+y^2}\,\sqrt{x^2+y^2+z^2}}\right)}{\sqrt{x^2+y^2}\left(x^2+y^2+z^2\right)^2},$$

$$\left.\frac{2z\left(\frac{y}{\sqrt{x^2+y^2+z^2}}+\frac{6z}{\sqrt{x^2+y^2+z^2}}\right)}{\left(x^2+y^2+z^2\right)^2}+\frac{\sqrt{x^2+y^2}\left(-\frac{6\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}+\frac{y\,z}{\sqrt{x^2+y^2}\,\sqrt{x^2+y^2+z^2}}\right)}{\left(x^2+y^2+z^2\right)^2}\right\}$$

The vector field in Cartesian coordinates is:- (using Grad)

$$\left\{\frac{3x\,(y+6z)}{\left(x^2+y^2+z^2\right)^{5/2}},\ \frac{3y\,(y+6z)}{\left(x^2+y^2+z^2\right)^{5/2}}-\frac{1}{\left(x^2+y^2+z^2\right)^{3/2}},\ \frac{3z\,(y+6z)}{\left(x^2+y^2+z^2\right)^{5/2}}-\frac{6}{\left(x^2+y^2+z^2\right)^{3/2}}\right\}$$

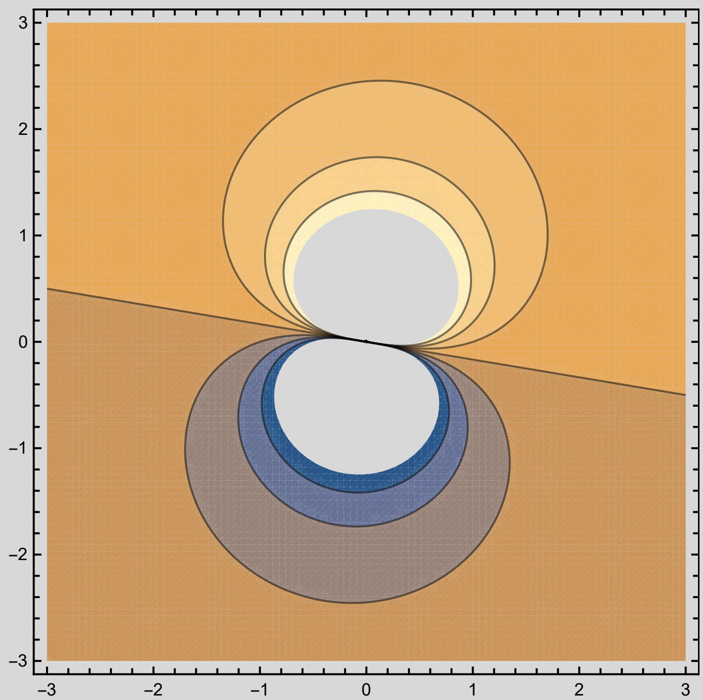
3. Contours of Scalar Field

In[319]:=

```
scalToPlot = scalCartes /. {x → 0} (*In the YZ plane*);
scalPlt = ContourPlot[scalToPlot , {y, -3, 3},
   {z, -3, 3}, PlotPoints → 100, ColorFunction → Automatic];
Print["On the Y-Z plane, the contour plot of the scalar field is:- "]
scalPlt
```

On the Y-Z plane, the contour plot of the scalar field is:-

Out[322]=



4. Stream Lines of the vector field

In[323]:=

```
Print["On the Y-Z plane:- "]
vecToPlot = vectCartes2 /. {x → 0}
a = StreamPlot[{3 y (y + 6 z)/(y^2 + z^2)^(5/2) - 1/(y^2 + z^2)^(3/2), 3 z (y + 6 z)/(y^2 + z^2)^(5/2) - 6/(y^2 + z^2)^(3/2)},
   {y, -3, 3}, {z, -3, 3}, StreamColorFunction → "Rainbow"];
b = VectorPlot[{3 y (y + 6 z)/(y^2 + z^2)^(5/2) - 1/(y^2 + z^2)^(3/2), 3 z (y + 6 z)/(y^2 + z^2)^(5/2) - 6/(y^2 + z^2)^(3/2)},
   {y, -3, 3}, {z, -3, 3}, VectorColorFunction → "Rainbow"];
c = VectorDensityPlot[{3 y (y + 6 z)/(y^2 + z^2)^(5/2) - 1/(y^2 + z^2)^(3/2), 3 z (y + 6 z)/(y^2 + z^2)^(5/2) - 6/(y^2 + z^2)^(3/2)}, {y, -3, 3},
   {z, -3, 3}, VectorColorFunction → "Rainbow", ColorFunction → "TemperatureMap"];
Print[
 "Showing the StreamPlot, VectorPlot and VectorDensityPlots for the force field:- "]
GraphicsRow[{a, b, c}]
GraphicsRow[{Show[scalPlt, a], Show[scalPlt, b]}]
```

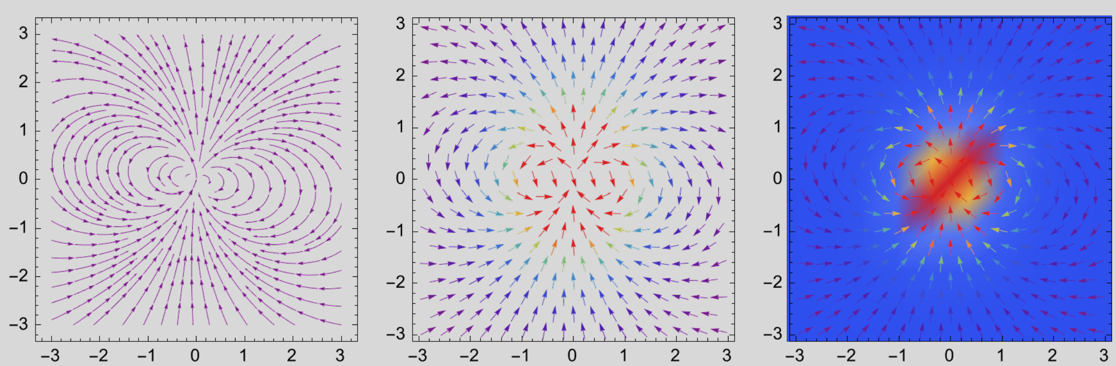
On the Y-Z plane:-

Out[324]=

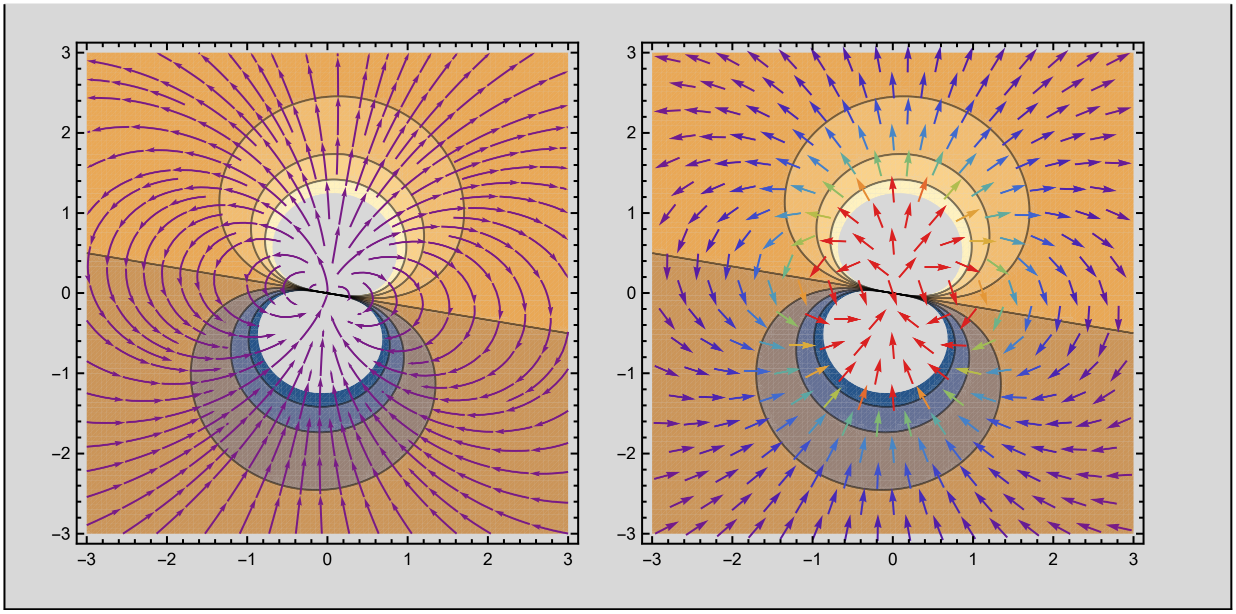
$$\left\{0, \frac{3 y (y + 6 z)}{(y^2 + z^2)^{5/2}} - \frac{1}{(y^2 + z^2)^{3/2}}, \frac{3 z (y + 6 z)}{(y^2 + z^2)^{5/2}} - \frac{6}{(y^2 + z^2)^{3/2}}\right\}$$

Showing the StreamPlot, VectorPlot and VectorDensityPlots for the force field:-

Out[329]=



Out[330]=



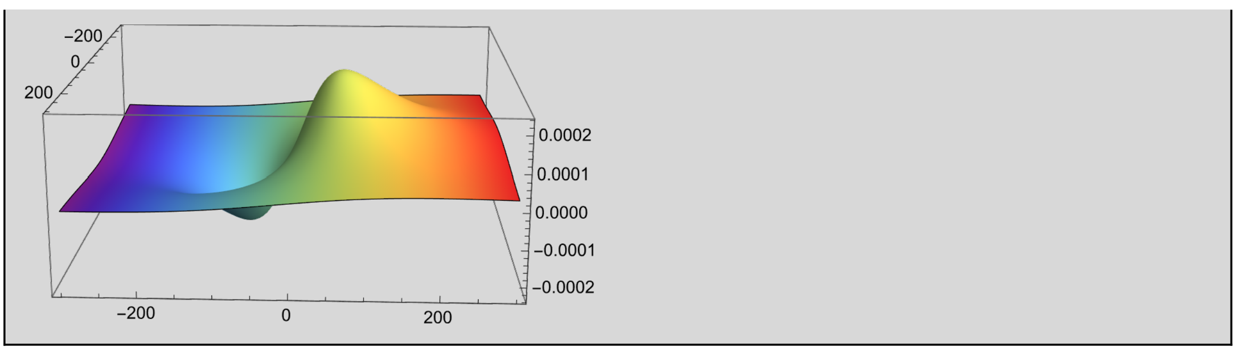
Extra:- 3D plots of the scalar and vector fields

In[331]:=

```
Print["3D plot of the potential:- "]
Plot3D[scalCartes /. {x → 100}, {y, -300, 300},
 {z, -300, 300}, PerformanceGoal → "Quality",
ColorFunction → (ColorData["Rainbow"][#2] &), PlotPoints → 100, Mesh → None]
```

3D plot of the potential:-

Out[332]=

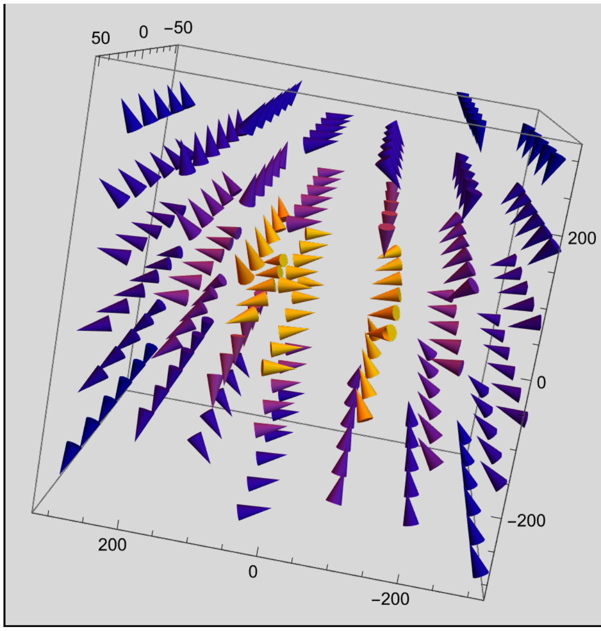


In[333]:=

```
Print["3D plot of the force field:- "]
VectorPlot3D[vectCartes2, {x, -50, 50},
 {y, -300, 300}, {z, -300, 300}, VectorStyle → "Arrow3D"]
```

3D plot of the force field:-

Out[334]=



5. Comments

In[335]:=

```
Print["By plotting the Regions of constant potential
   and the force field on the same line, we observe that the
   lines of force are always normal to the equipotential curves."]
Print["This is as expected, since the vector
   field is obtained by taking the gradient of the scalar field."]
```

By plotting the Regions of constant potential and the force field on the same line,
  we observe that the lines of force are always normal to the equipotential curves.

This is as expected, since the vector
  field is obtained by taking the gradient of the scalar field.