$R = (A, B, C, D, E)_F$

$F = \{ AB \to C,$
$BC \to AD,$
$D \to E,$
$CF \to B \}$

Q. $D \to A$ holds on R ?

Find D^+

$\{D\}^+ = (D)^+ = D$

$= DE$ from $D \to E$

$\{D\}^+ = \{D, E\}.$

$\therefore D \to A \notin F^+$

Q. $AB \to CDE$ holds ?

$(AB)^+ = AB$

$= ABC$, $AB \to C$

$= ABCD$, $BC \to AD$

$= ABCDE$, $D \to E$

$= ABCDE.$, $CF \to B$

$(AB)^+ = ABCDE.$

$AB \to CDE \in F^+$

Closure & Keys

$R = \{A_1, A_2, \dots A_n\}^+$ iff $A_1, A_2, \dots A_n$ is a super key.

Redundant Dependencies :

EX: $F = \{ A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C \}$

$$\frac{\begin{array}{c} A \rightarrow B \\ B \rightarrow C \end{array}}{A \rightarrow C}$$

$F' = \{ A \rightarrow B,\ B \rightarrow C \}$

Extraneous Attributes

Let $\alpha \to \beta \in F$

1). A is extraneous attribute in α

if $A \in \alpha$ and

F logically implies $(F - \{\alpha \to \beta\}) \cup \{(\alpha - A) \to \beta\}$

2). A is extraneous attribute in β

if $A \in \beta$ and

$(F - \{\alpha \to \beta\}) \cup \{\alpha \to (\beta - A)\}$ logically implies F.

Test Extraneous Attribute:

$\alpha \to \beta \in F$

Case (1): $A \in \alpha$ extraneous

1. compute $\gamma = \alpha - \{A\}$
2. check $\gamma \to \beta$ can be inferred from F

Ex: $F = \{ AB \to CD, A \to E, E \to C \}$

$\alpha \to \beta$

$AB \to CD$

$A \in \{A, B\} = \alpha$

1. $\gamma = \alpha - \{A\} = \{B\}$
2. $B \to CD \notin F^+$.

$B^+ = B$

$\Rightarrow$ A is not extraneous in $AB \to CD$ under F.

Q: C extraneous in $AB \to CD$?

$\alpha \to \beta$

Case(2): $A \in \beta$ extraneous

1. consider $F' = (F - \{\alpha \to \beta\}) \cup \{\alpha \to (\beta - A)\}$

2. check $\alpha \to A$ can be inferred from F'.

$F' = \{A \to E,\ E \to C,\ AB \to D\}$

$AB \to C \overset{?}{\in} F^+$.

$C \in (AB)^+$

$\Rightarrow AB \to C \in F^+$

$\therefore$ C is extraneous attribute in $AB \to CD$ under F.

$(AB)^+ = AB$
$= ABE$
$= ABCE$
$= ABCDE$

$AB \to CD$

$AB \to C$
$AB \to D$

Canonical cover F_c for F

A set of dependencies such that

F logically implies all dependencies in F_c

and vice versa.

and F_c must have the following properties.

1). No FD in F_c contains extraneous attribute

2). Each left-side of FD in F_c is unique, that is,

NO $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ in F_c with $\alpha_1 = \alpha_2$

$\boxed{F_c^+ = F^+}$ and F_c $\boxed{\text{minimal}}$

$F = \{ \underline{A \rightarrow BC}, B \rightarrow C, \underline{A \rightarrow B}, AB \rightarrow C \}$

Q: Find F_c for F?

1). $A \rightarrow BC$
$A \rightarrow B$

$\overline{A \rightarrow BC}$

$F_{1c} = \{ A \rightarrow BC, B \rightarrow C, \underline{A}B \rightarrow C \}$

$$\boxed{\alpha_1 \rightarrow \beta_1 \text{ and } \alpha_1 \rightarrow \beta_2}$$

2). Check A is extraneous in $AB \rightarrow C$ in F_{1c}?

<u>case(1)</u> $\gamma = AB - A = B$

$\cancel{B} \rightarrow C \overset{?}{\in} F_{1c}^{+}$

$F_{2c} = \{ A \rightarrow BC, B \rightarrow C \}$.

$B^{+} = B$
$= BC$, $B \rightarrow C$

$\therefore B \rightarrow C \in F_{1c}^{+}$.

3). check C is Extraneous in $A \rightarrow BC$ in F_{2c} ?

<u>case(2)</u>: 1. $F'_{2c} = \{A \rightarrow B, B \rightarrow C\}$

2. check $A \rightarrow C \underset{?}{\in} F'^{+}_{2c}$

$$A^+ = A$$
$$= AB \quad , A \rightarrow B$$
$$= ABC \quad , B \rightarrow C$$

$$C \in A^+ \Rightarrow A \rightarrow C \in F'^{+}_{2c}$$

$$\boxed{F_{3c} = \{A \rightarrow B, B \rightarrow C\}}$$