closure of F

F - Set of functional dependencies (FDs)

F^+ - closure of F

Set of all FDs logically implied by F

Ex $A \to B$, $B \to C$ $\in F$ holds in $r(R)$.

$$t_1[A] = t_2[A] \Rightarrow t_1[B] = t_2[B]$$

$$t_1[B] = t_2[B] \Rightarrow t_1[C] = t_2[C]$$

$$t_1[A] = t_2[A] \Rightarrow t_1[C] = t_2[C]$$

$A \to C$ holds.

Armstrong's Axioms: $\alpha, \beta, \gamma \subseteq R$

1) Reflexivity: if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ holds

2). Augmentation: $\gamma\alpha \rightarrow \gamma\beta$ hold if $\alpha \rightarrow \beta$ holds.

3). Transitivity: if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ holds.

Therom: Sound: generate FDs that holds in $r(R)$

Complete: generate all FDs that holds.

Additional rules :

1) Union : if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$

2). Decomposition : if $\alpha \to \beta\gamma$, then $\alpha \to \beta$ and $\alpha \to \gamma$.

3). Pseudotransitivity: $\alpha \to \beta$ & $\gamma\beta \to \delta$ holds

then $\alpha\gamma \to \delta$ holds.

Ex: $R(A, B, C, G, H, I)$

$$F = \{ A \to B,\ A \to C,\ CG \to H,\ CG \to I,\ B \to H \}$$

Q: check $\boxed{A \to H}$ holds ?

$$\frac{\begin{array}{c} A \to B \\ B \to H \end{array}}{A \to H}$$ infer — transitivity rule.

$A \to H$ holds in $r(R)$.

Q: $CG \to HI$ holds ?

$$\frac{\begin{array}{c} CG \to H \\ CG \to I \end{array}}{CG \to HI}$$ union

Q: $AG \to I$ hold ?

$$\frac{\begin{array}{c} A \to C \\ CG \to I \end{array}}{AG \to I}$$ pseudo transitivity.

Altnately.

$$\frac{A \to C}{AG \to CG}$$ augment.

$$\frac{\begin{array}{c} AG \to CG \\ CG \to I \end{array}}{AG \to I}$$ transitivity

Computing F^+

$F^+ = F$

repeat

  for each FD f in F^+

    apply (1) & (2) on f

    add result FD to F^+

  for each pair of f_1 and f_2 in F^+

    if f_1 and f_2 combined transitivity rule

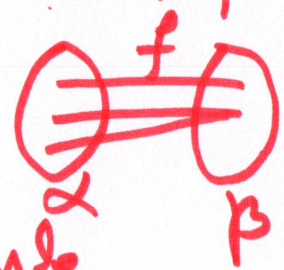
    then add the result FD to F^+.

until F^+ does not change further.

$|S| = n$

subset $- 2^n$ $\quad \alpha, \beta \subseteq R$

$f : \alpha \to \beta$



Complexity: $2^n \times 2^n$ possible FDs.

n - no. of attributes in R

Closure of attribute sets:

Given $\alpha \subseteq R$, R is a relation schema

$R = \{A, B, C\}$

$\alpha = \{A, B\}$

α^+ - closure of α under F.

: set of attributes that are functionally determined by α under F.

Note:

$$\boxed{\alpha \to \beta \in F^+ \iff \beta \subseteq \alpha^+}$$

Algorithm:

```
result = α
  while (changes to result) do
     for each β → γ in F do
       begin
          if β ⊆ result
          then result = result ∪ γ
       end
```

EX: $R = (A, B, C, D, E)$

$$F = \{ AB \to C,\ \boxed{BC \to AD},\ D \to E,\ CF \to B \}$$

$$\left.\begin{array}{l} BC \to A \\ BC \to D \end{array}\right\} \text{Decomposition rule.}$$

Note: possibly ensure, each FD of F has single attribute on the right.

Q: Find $\alpha^+ = \{A, B\}^+$.

$= AB^+$ (simplify)

$\boxed{\text{Left}}$ $\alpha = \{A, B\}$

1) AB in $\alpha^+ \Rightarrow$ ~~result =~~ $\boxed{\text{Right}}$

$\alpha^+ = \alpha \cup \{C\}$

$= \{A, B, C\}$

2) BC in $\alpha^+ \Rightarrow \alpha^+ = \alpha^+ \cup \{A, D\}$

$= \{A, B, C, D\}$

3). D in $\alpha^+ \Rightarrow \alpha^+ = \alpha^+ \cup \{E\}$.

$= \{A, B, C, D, E\}$

4) CF not in $\alpha^+ \Rightarrow$ cannot update.

$$\boxed{\alpha^+ = \{A, B, C, D, E\}}.$$

$AB \to D \in F^+$?

$\alpha \to D \in F^+ \Leftrightarrow \{D\} \subseteq \alpha^+$