Advanced Data Structures and Algorithms

B+ Tress

B+ Trees

- What are B+ Trees used for
- What is a B Tree
- What is a B+ Tree
- Searching
- Insertion
- Deletion

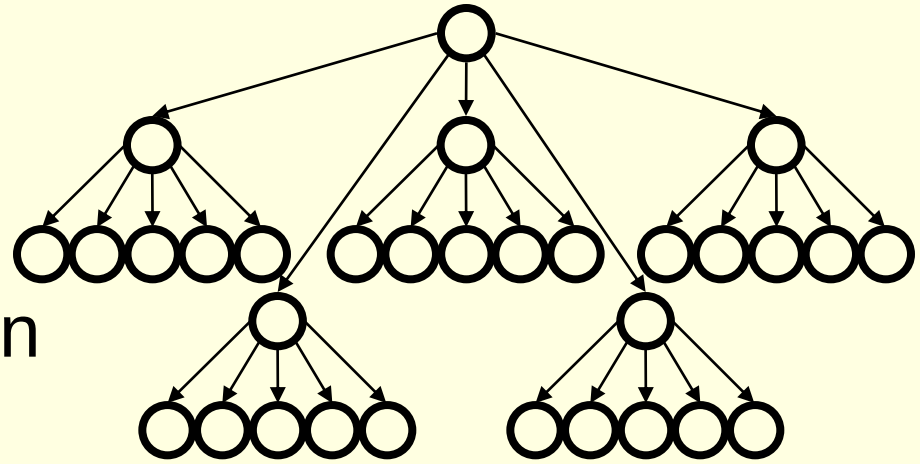
What are B+ Trees Used For?

- When we store data in a table in a DBMS we want
  - Fast lookup by primary key
    - Just this – hashtable O(c)
  - Ability to add/remove records on the fly
    - Some kind of dynamic tree **on disk**
  - Sequential access to records (physically sorted by primary key on disk)
    - Tree structured keys (hierarchical index for searching)
    - Records all at leaves in sorted order

What is an M-ary Search Tree?

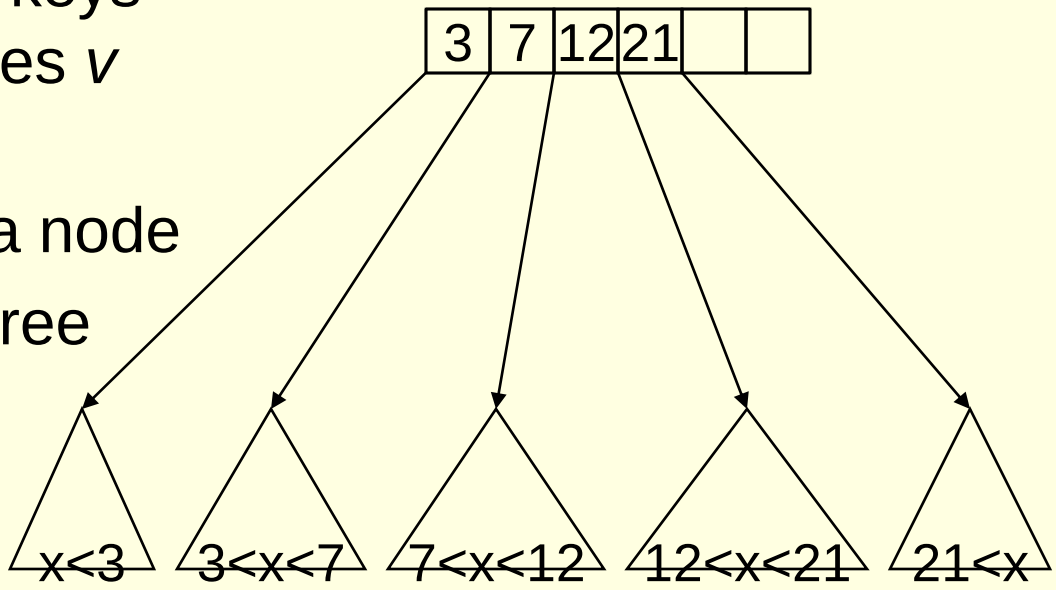
- Maximum branching factor of ***M***
- Complete tree has depth = $\log_M N$
- Each internal node in a complete tree has ***M*** – 1 keys

Binary search tree is a B Tree where M is 2



B Trees

- B-Trees are specialized *M*-ary search trees
- Each node has many keys
  - subtree between two keys *x* and *y* contains values *v* such that $x \leq v < y$
  - binary search within a node
  - to find correct subtree
- Each node takes one
- full {*page, block, line*}
- of memory (disk)



B-Tree Properties

- Properties
  - maximum branching factor of *M*
  - the root has between 2 and *M* children *or* at most *M-1* keys
  - All other nodes have between $\lceil M/2 \rceil$ and M records (children)

- Result
  - tree is O(log M) deep
  - all operations run in O(log M) time
  - operations pull in about *M* items at a time

What is a B+ Tree?

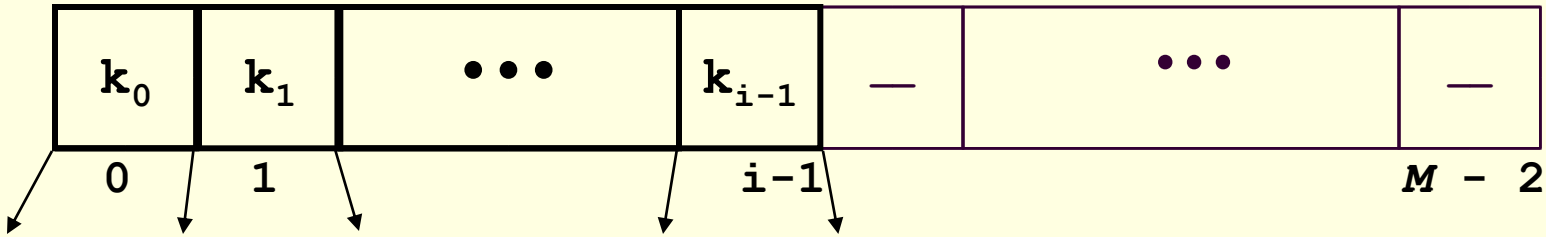
- A variation of B trees in which
  - Internal nodes contain only search keys (no data)
  - Leaf nodes contain pointers to data records
  - Data records are in sorted order by the search key
  - All leaves are at the same depth

Definition of a B+Tree

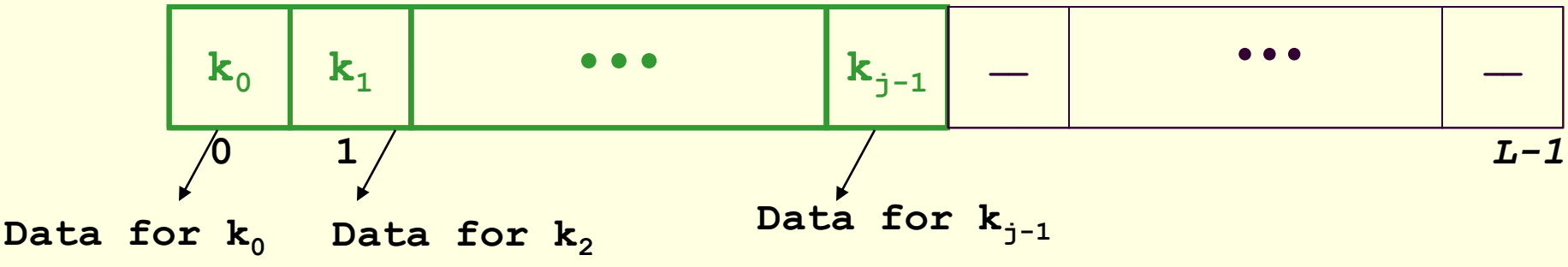
A B+ tree is a balanced tree in which every path from the root of the tree to a leaf is of the same length, and each non-leaf node of the tree has between [M/2] and [M] children, where n is fixed for a particular tree.

B+ Tree Nodes

- Internal node
  - Pointer (Key, NodePointer)*M-1 in each node
  - First i keys are currently in use



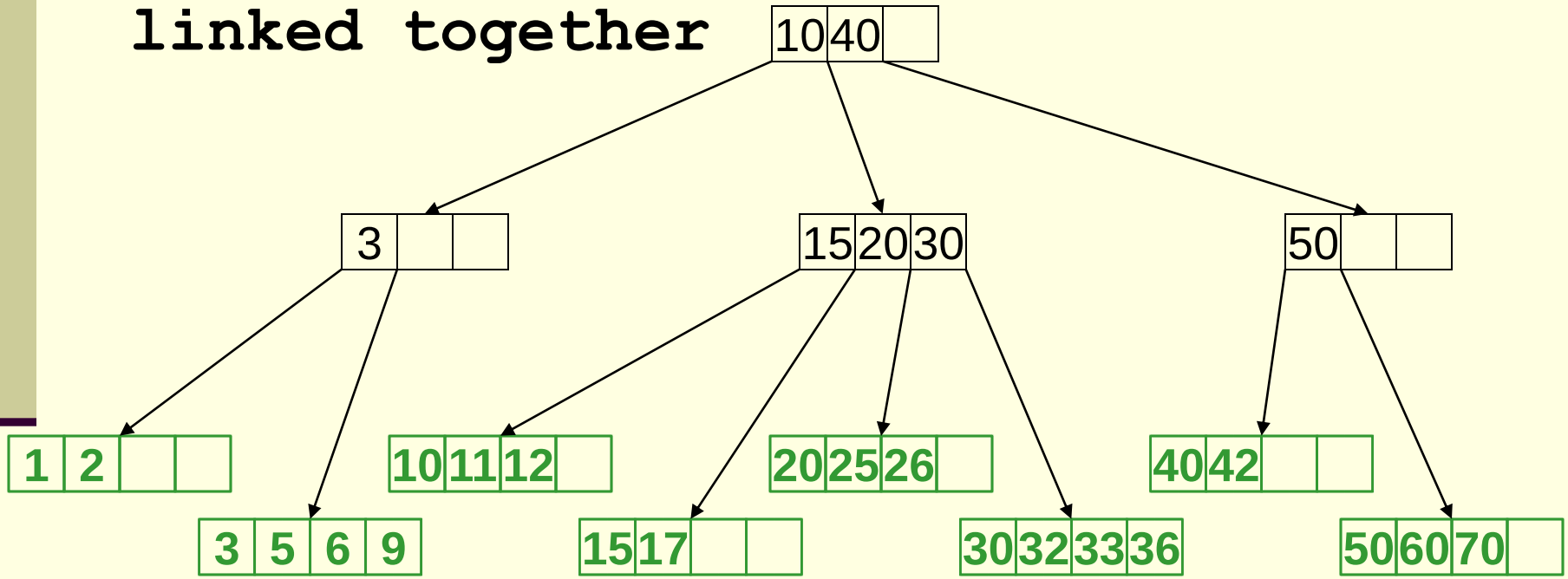
- Leaf
  - **(Key, DataPointer)* L in each node**
  - **first j Keys currently in use**



Example

B+ Tree with ***M = 4***

Often, leaf nodes linked together



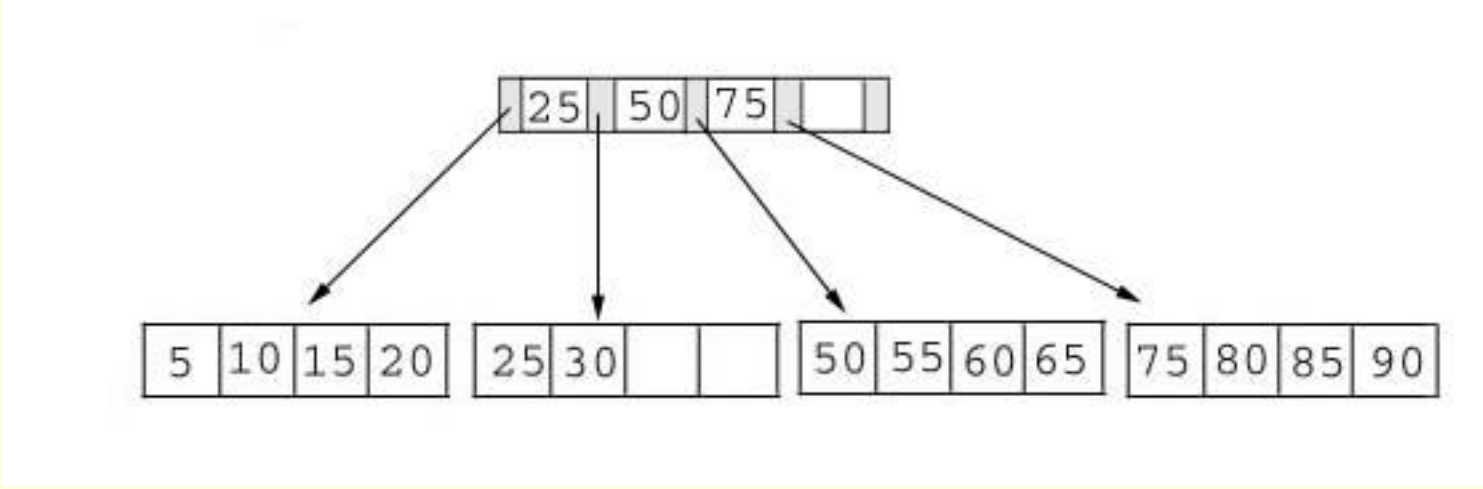
Advantages of B+ tree usage for databases

- keeps keys in sorted order for sequential traversing
- uses a hierarchical index to minimize the number of disk reads
- uses partially full blocks to speed insertions and deletions
- keeps the index balanced with a recursive algorithm
- In addition, a B+ tree minimizes waste by making sure the interior nodes are at least half full. A B+ tree can handle an arbitrary number of insertions and deletions.

Searching

- Just compare the key value with the data in the tree, then return the result.

  For example: find the value 45, and 15 in below tree.



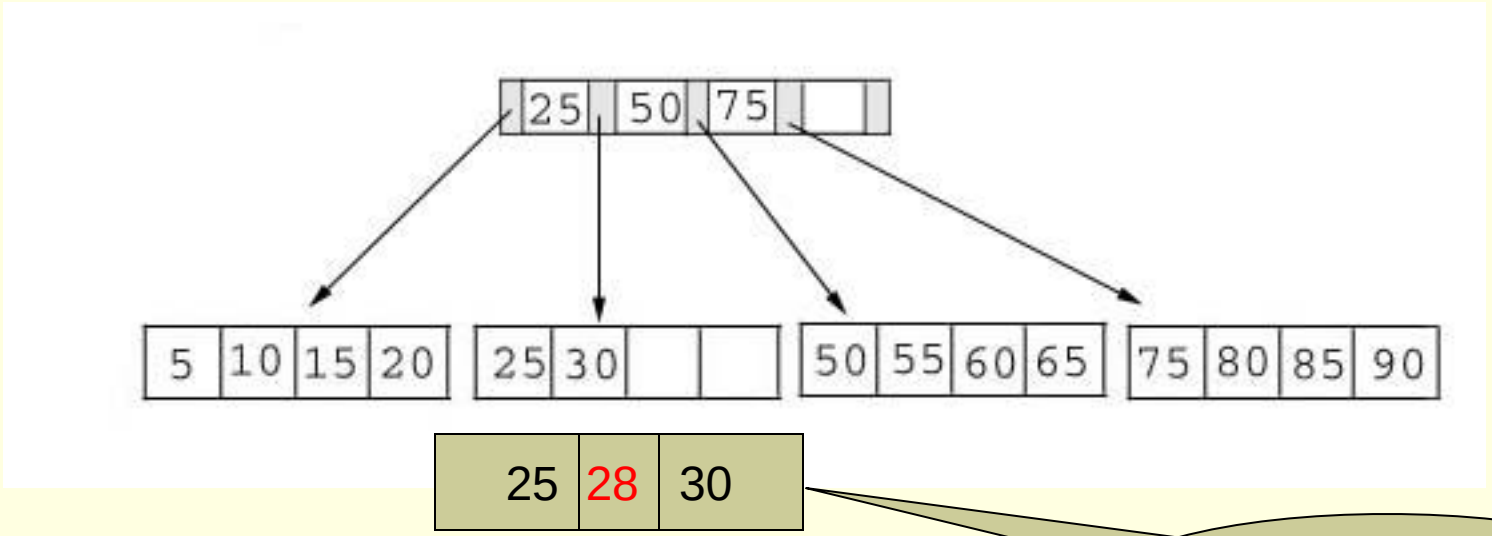
Searching

- Result:

  1. For the value of 45, not found.
  2. For the value of 15, return the position where the pointer located.

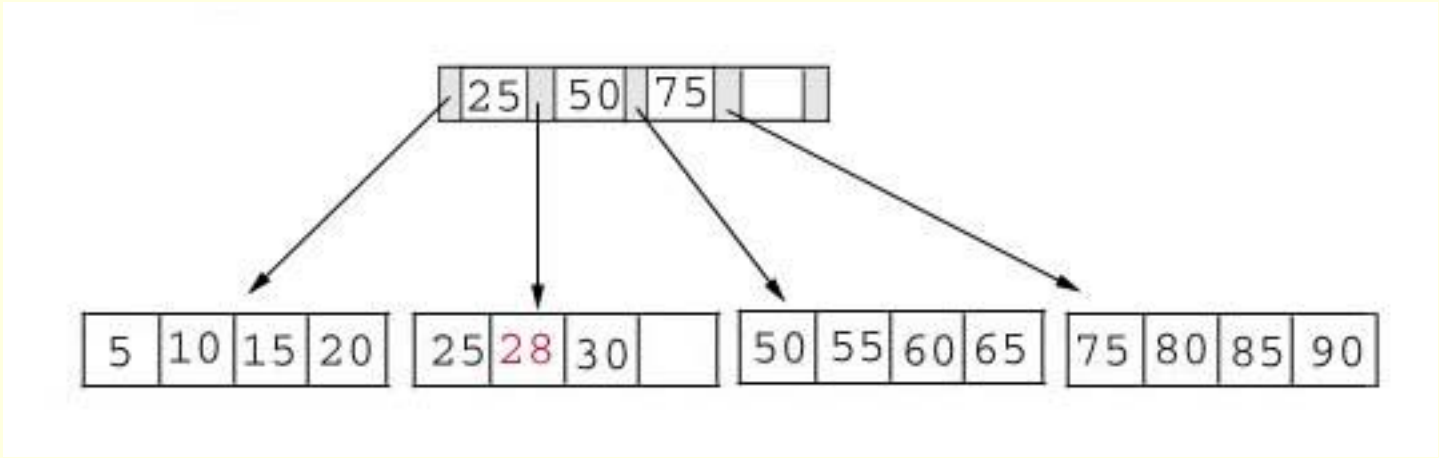
Insertion

- inserting a value into a B+ tree may unbalance the tree, so rearrange the tree if needed.
- Example #1: insert 28 into the below tree.



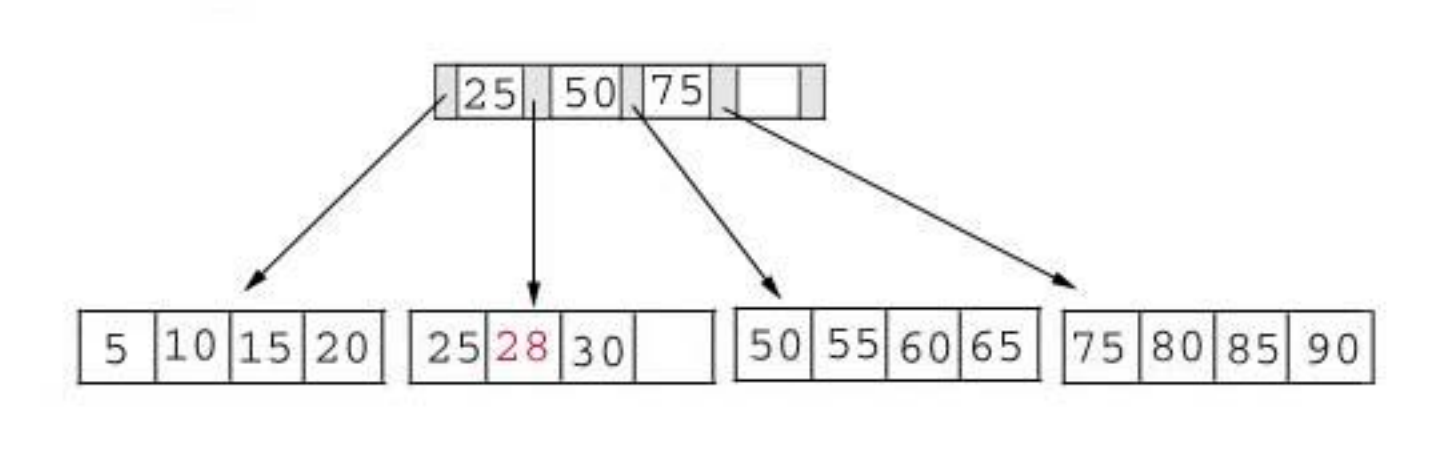
Insertion

- Result:



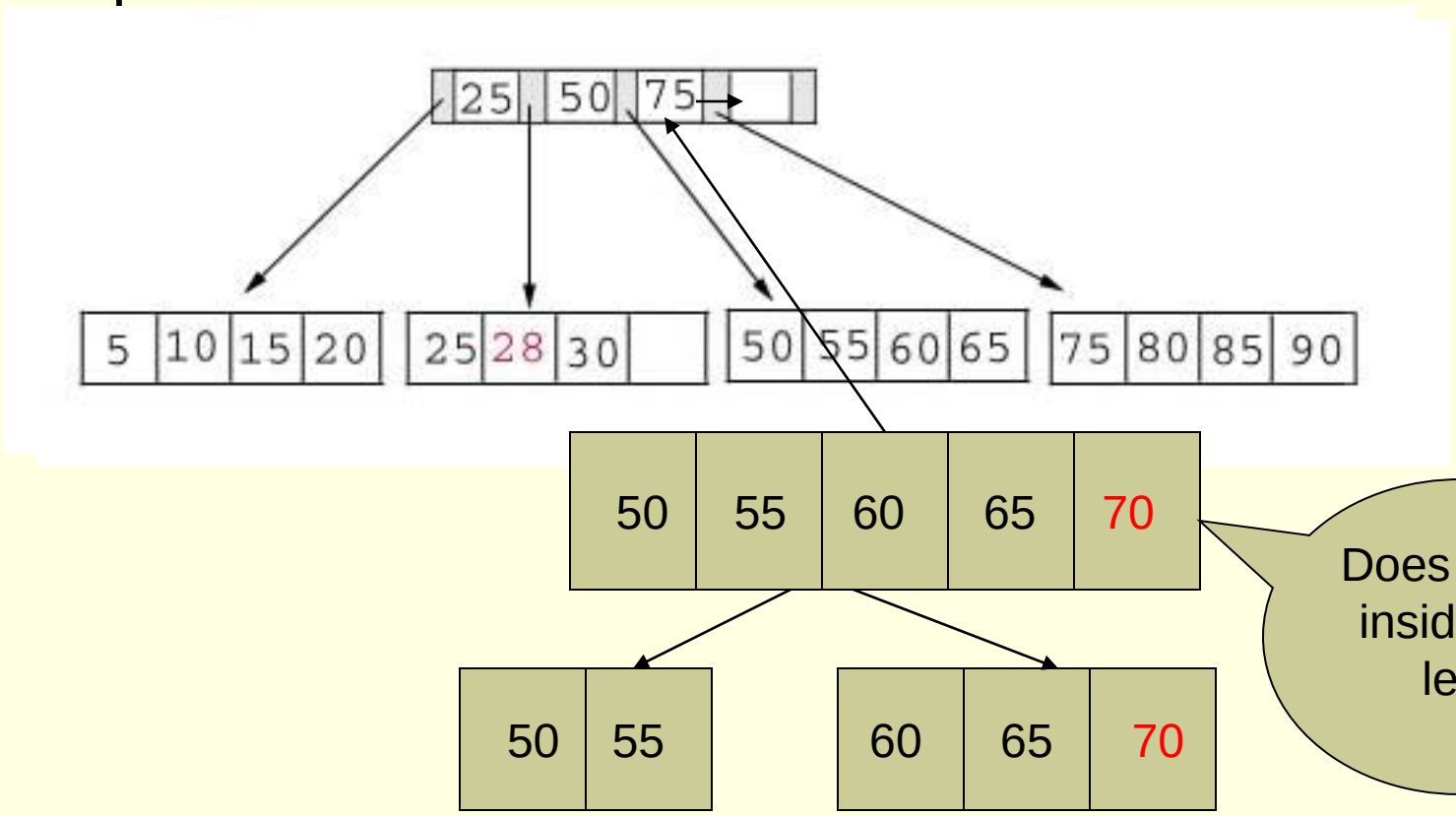
Insertion

- Example #2: insert 70 into below tree



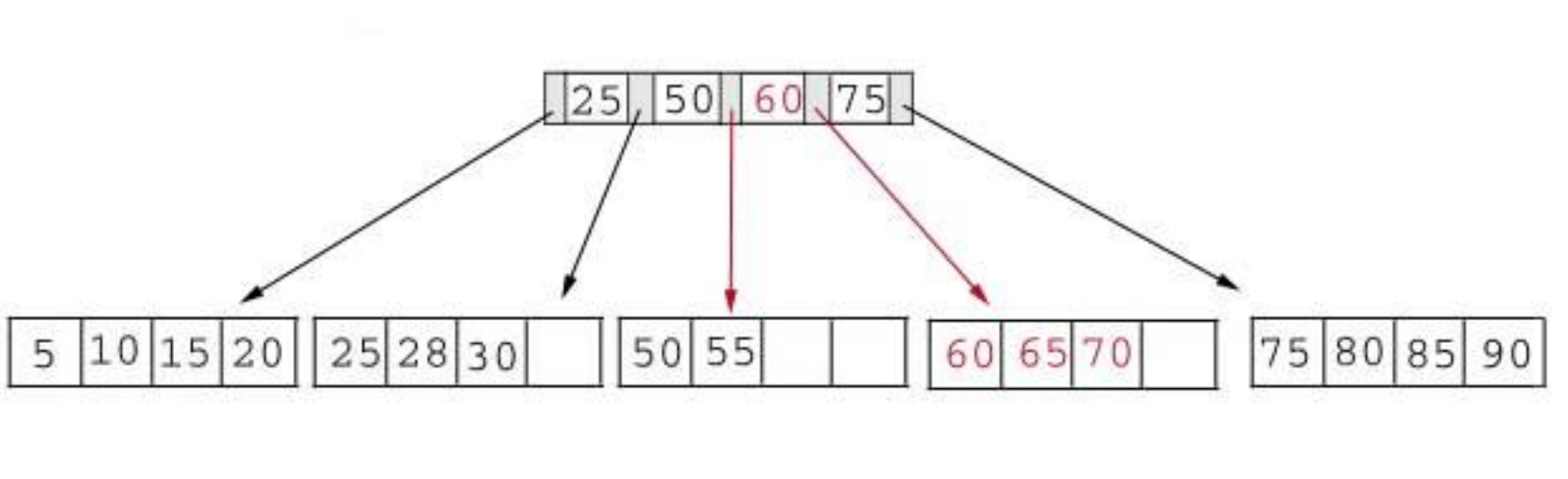
Insertion

- Process: split the leaf and propagate middle key up the tree



Insertion

- Result: chose the middle key 60, and place it in the index page between 50 and 75.



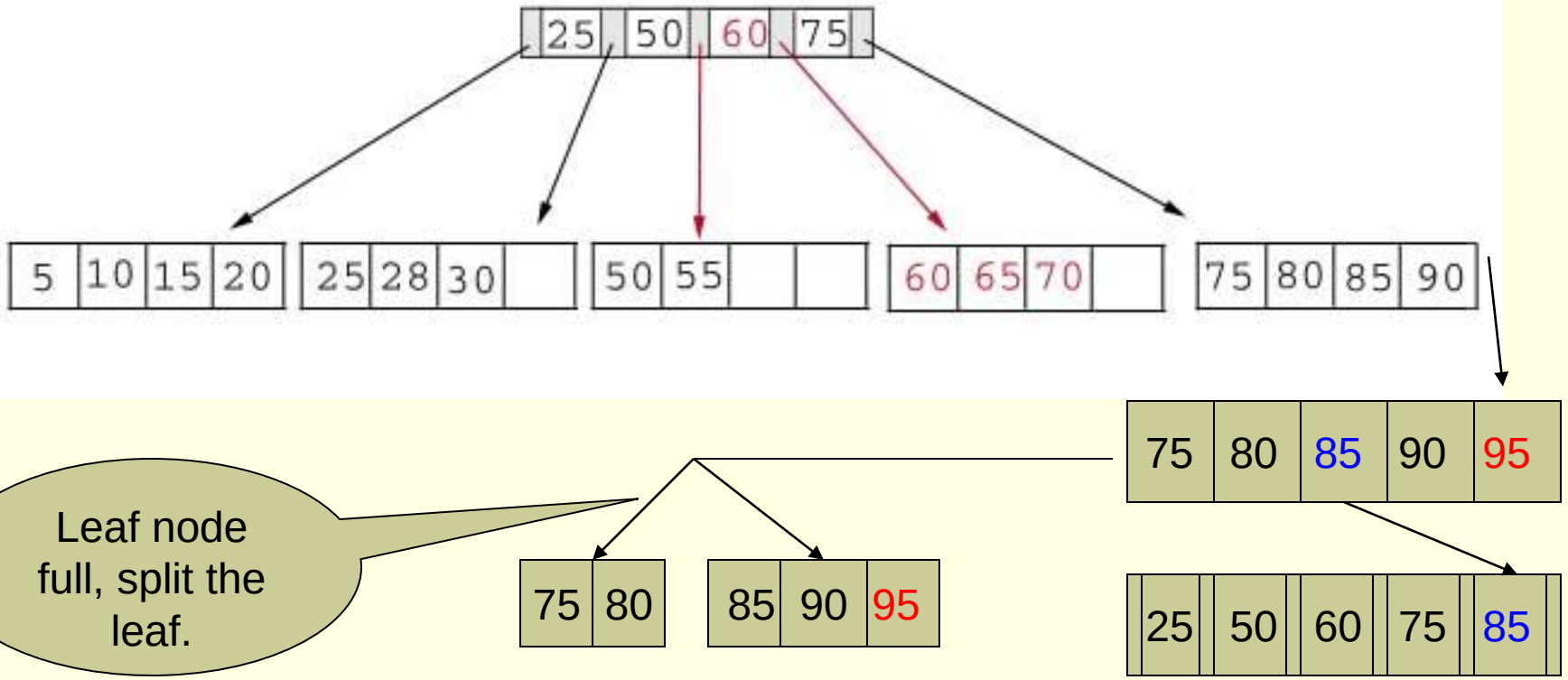
Insertion

The insert algorithm for B+ Tree

Leaf Node Full	Index Node Full	Action
NO	NO	Place the record in sorted position in the appropriate leaf page
YES	NO	1. Split the leaf node 2. Place Middle Key in the index node in sorted order. 3. Left leaf node contains records with keys below the middle key. 4. Right leaf node contains records with keys equal to or greater than the middle key.
YES	YES	1. Split the leaf node. 2. Records with keys < middle key go to the left leaf node. 3. Records with keys >= middle key go to the right leaf node. Split the index node. 4. Keys < middle key go to the left index node. 5. Keys > middle key go to the right index node. 6. The middle key goes to the next (higher level) index node. IF the next level index node is full, continue splitting the index nodes.

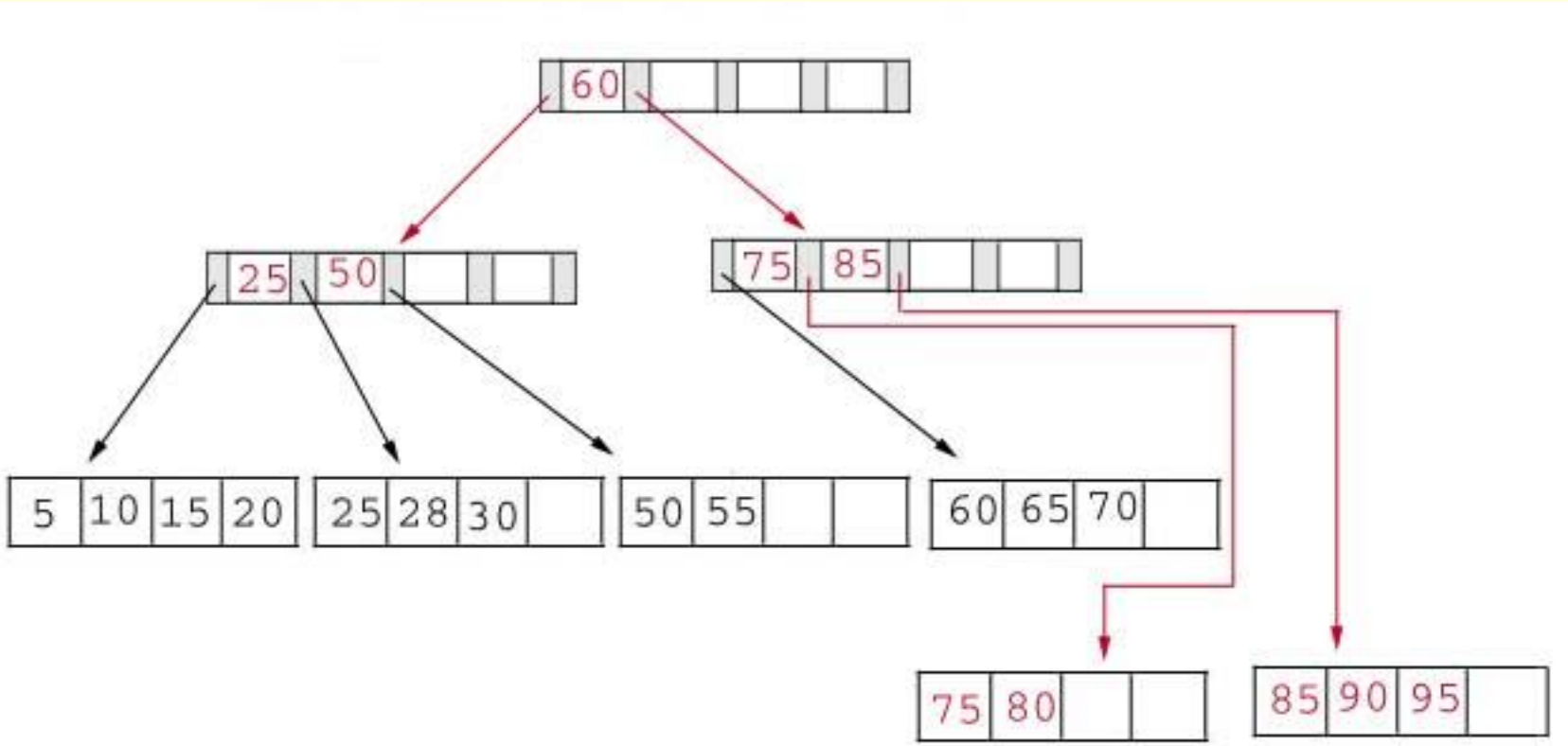
Insertion

- Exercise: add a key value 95 to the below tree.



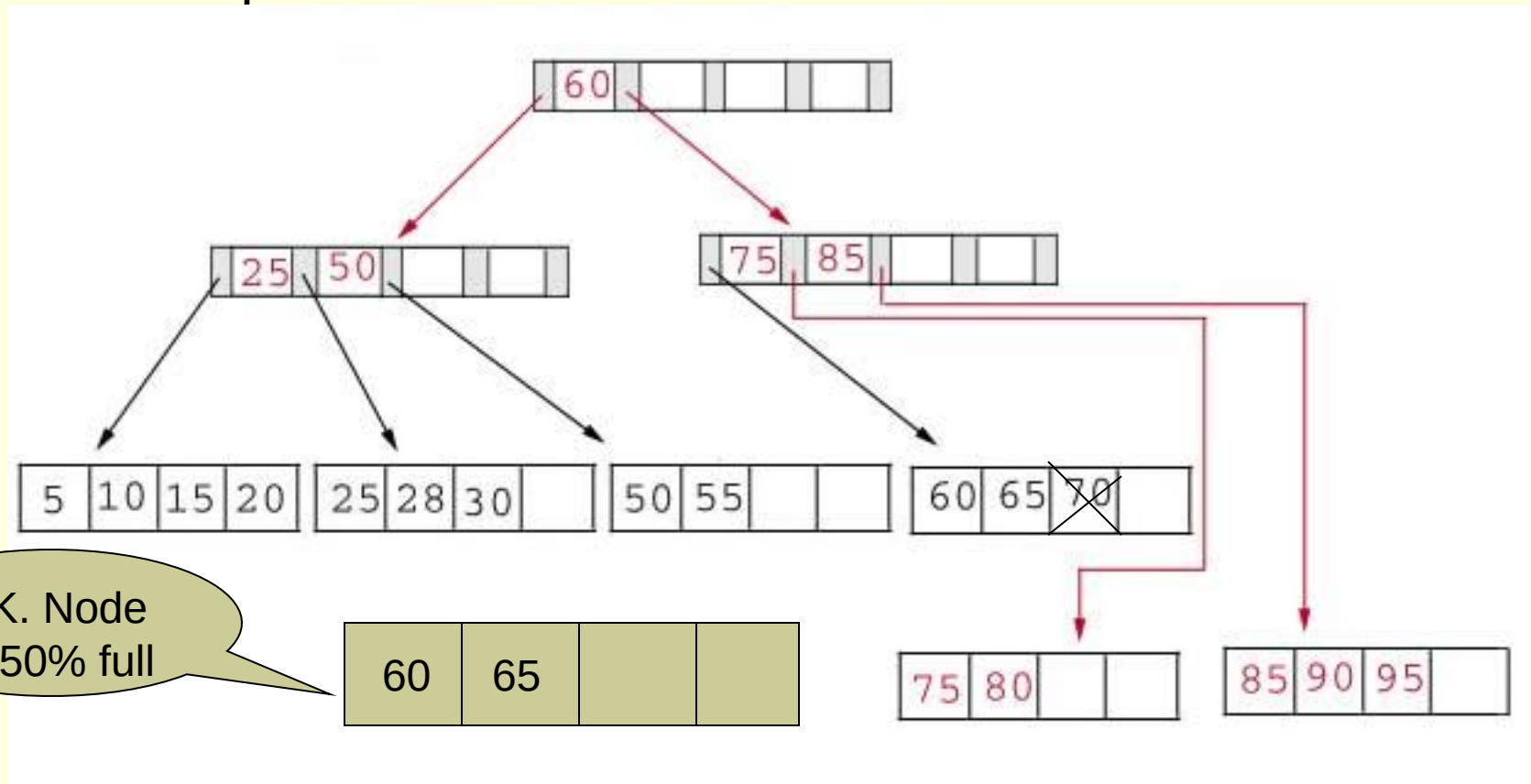
Insertion

- Result: again put the middle key 60 to the index page and rearrange the tree.



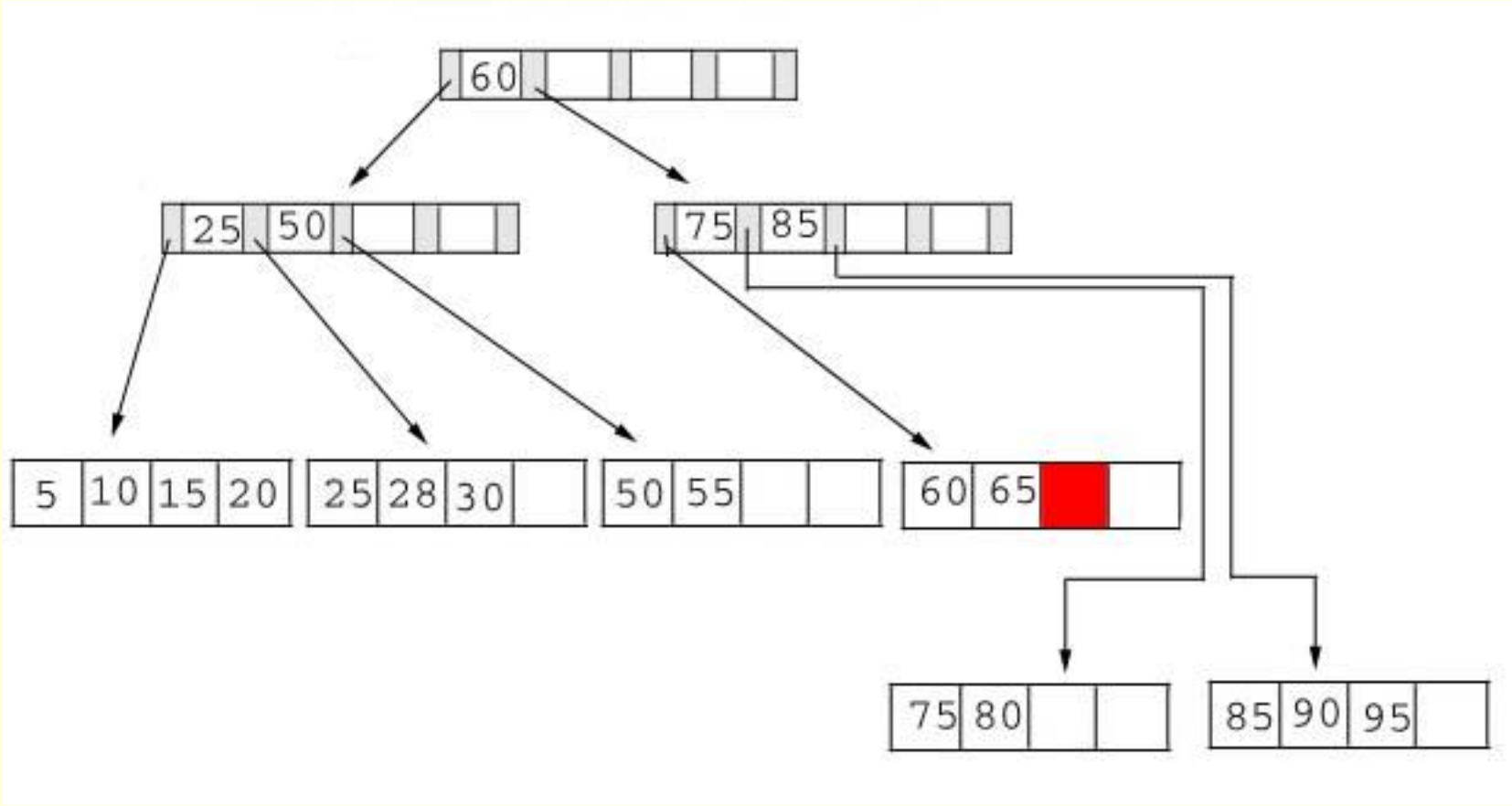
Deletion

- Same as insertion, the tree has to be rebuild if the deletion result violate the rule of B+ tree.
- Example #1: delete 70 from the tree



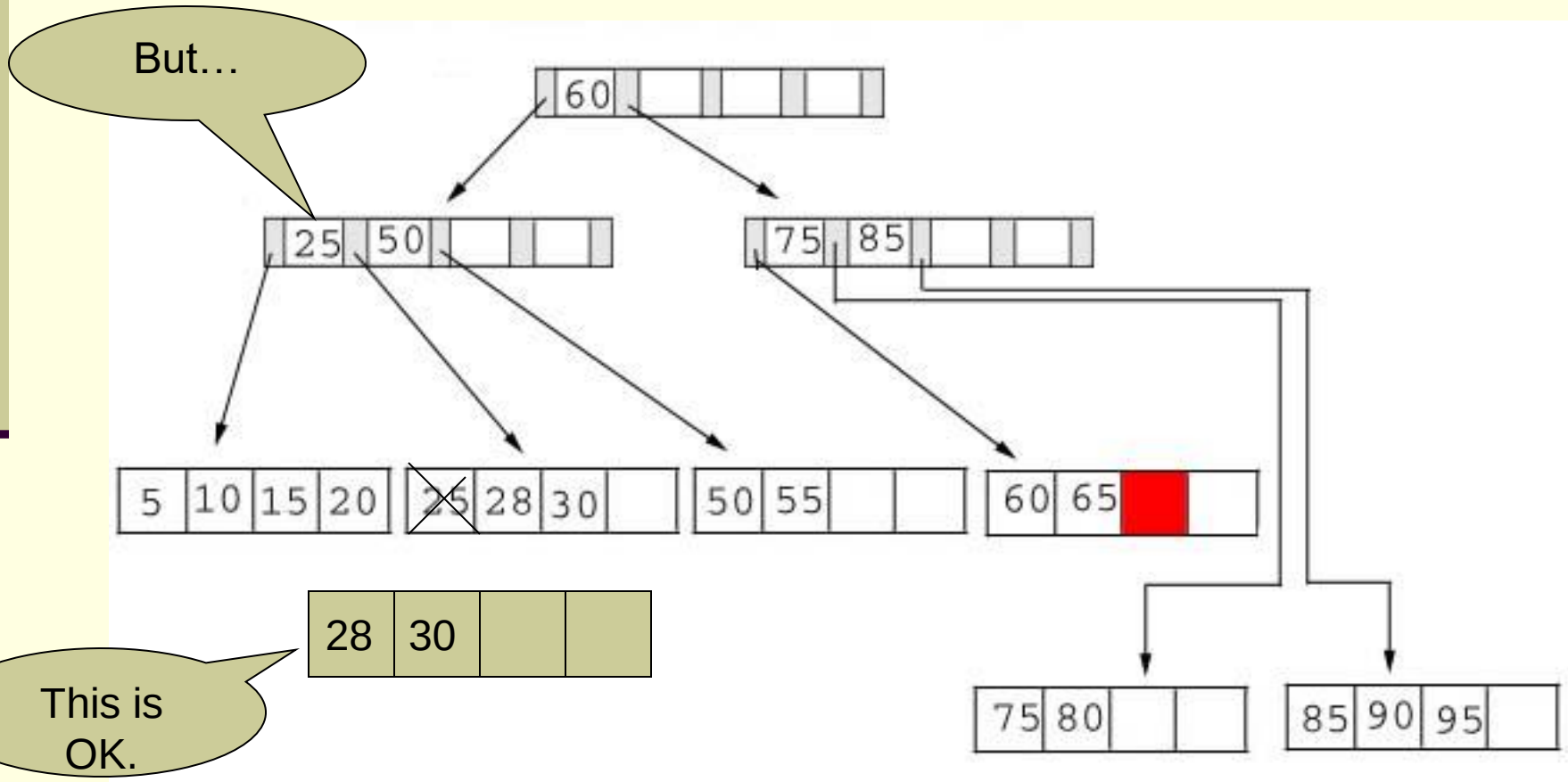
Deletion

- Result:



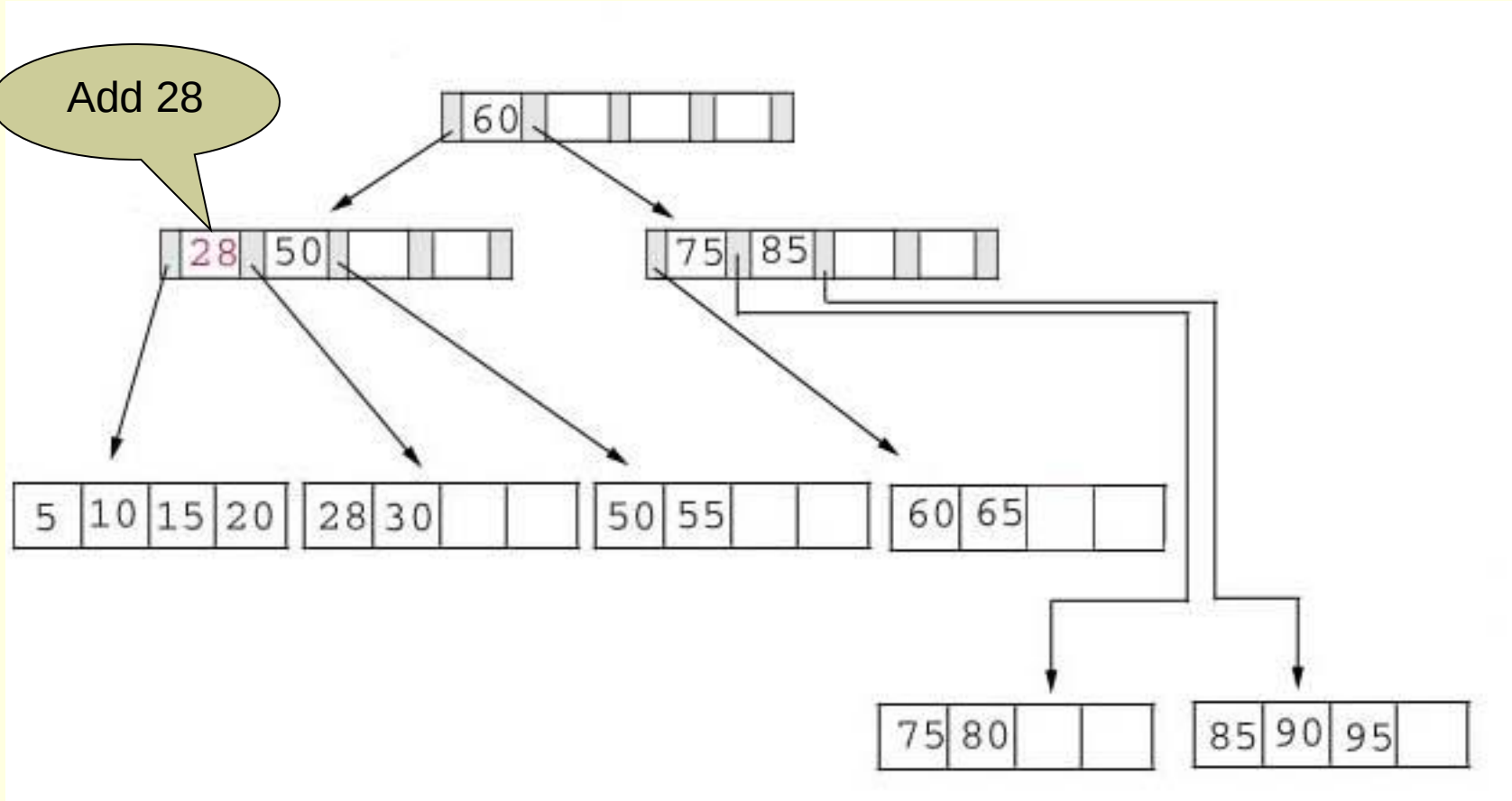
Deletion

Example #2: delete 25 from below tree, but 25 appears in the index page.



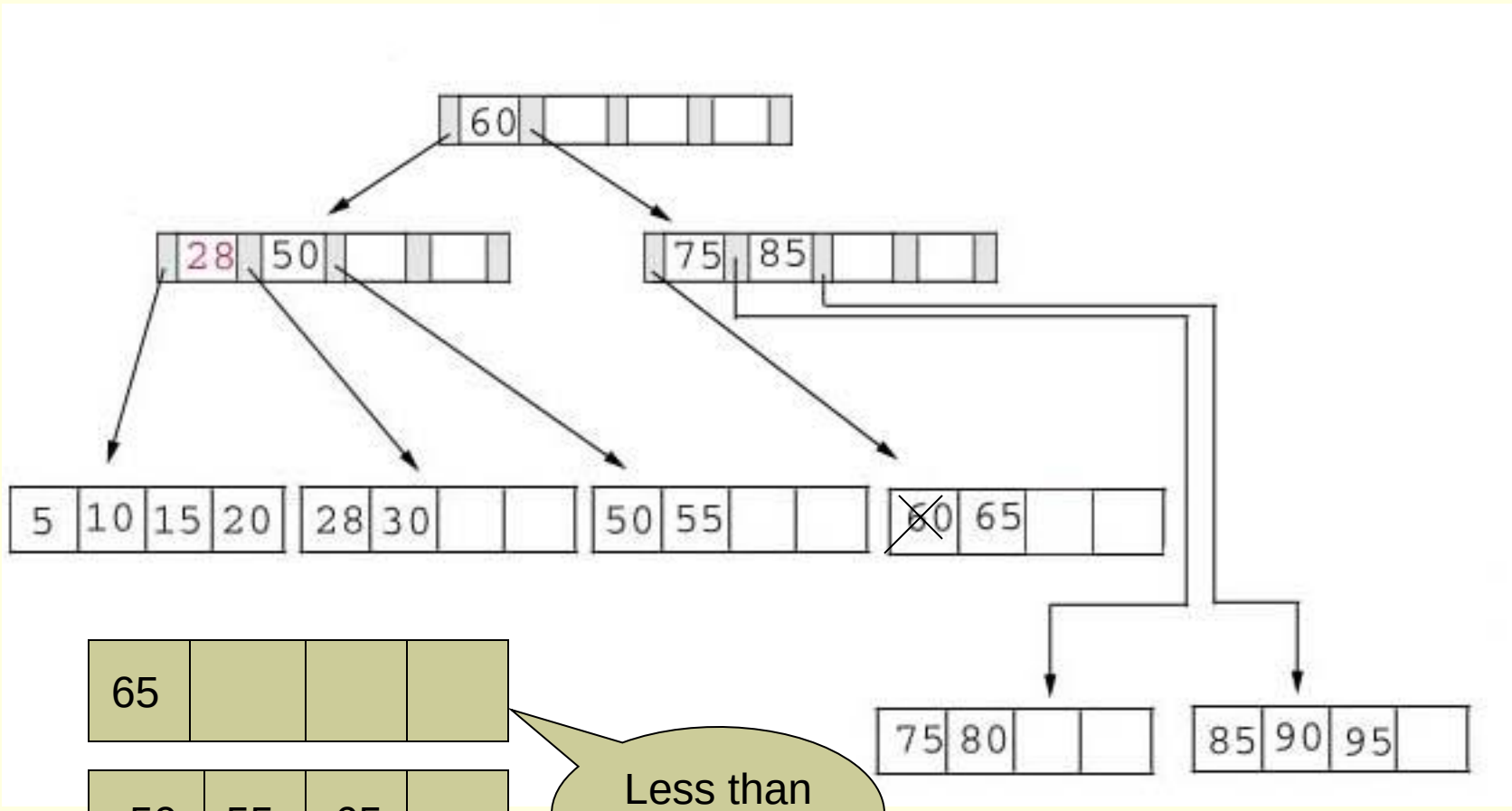
Deletion

- Result: replace 28 in the index page.



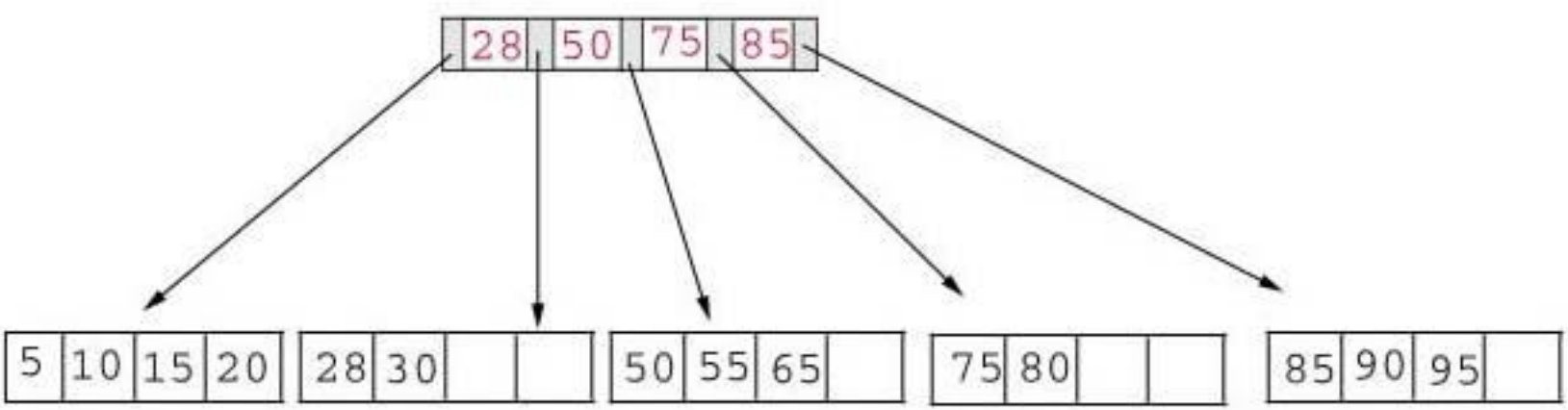
Deletion

- Example #3: delete 60 from the below tree



Deletion

- Result: delete 60 from the index page and combine the rest of index pages.



Deletion

- Delete algorithm for B+ trees

Data Page Below Fill Factor	Index Page Below Fill Factor	Action
NO	NO	Delete the record from the leaf page. Arrange keys in ascending order to fill void. If the key of the deleted record appears in the index page, use the next key to replace it.
YES	NO	Combine the leaf page and its sibling. Change the index page to reflect the change.
YES	YES	1. Combine the leaf page and its sibling. 2. Adjust the index page to reflect the change. 3. Combine the index page with its sibling. Continue combining index pages until you reach a page with the correct fill factor or you reach the root page.

Conclusion

- For a B+ Tree:
- It is “easy” to maintain its balance
  - Insert/Deletion complexity $O(\log_{M/2})$
- The searching time is shorter than most of other types of trees because branching factor is high

B+Trees and DBMS

- Used to index primary keys
- Can access records in $O(\log_{M/2})$ traversals (height of the tree)
- Interior nodes contain Keys only
  - Set node sizes so that the M-1 keys and M pointers fits inside a single block on disk
    - E.g., block size 4096B, keys 10B, pointers 8 bytes
    - (8+ (10+8)*M-1) = 4096
    - M = 228; 2.7 billion nodes in 4 levels
  - One block read per node visited

Reference

Li Wen & Sin-Min Lee, San Jose State University