Lab Assignment:-

1. Solve the following equation by Gauss Elimination method;

$$2x + 4y - 6z = -4;\ x + 5y + 3z = 10;\ x + 3y + 2z = 5$$

The given system of equation in matrix form.

$$Ax = B.$$

$$\begin{bmatrix} 2 & 4 & -6 \\ 1 & 5 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ 5 \end{bmatrix}$$

Augmanted matrix, $C = [A:B]$

$$\left[\begin{array}{ccc:c} 2 & 4 & -6 & -4 \\ 1 & 5 & 3 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$, $R_2 \longleftrightarrow R_3$
$R_3 \longleftrightarrow R_1$

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 5 \\ 2 & 4 & -6 & -4 \\ 1 & 5 & 3 & 10 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 5 \\ 0 & -2 & -10 & -14 \\ 0 & 2 & 1 & 5 \end{array}\right]$$

$R_2 \to [-1/2] R_2$

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 5 \\ 0 & 1 & 5 & 1 \\ 0 & 2 & 1 & 5 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 5 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & -9 & -9 \end{array}\right]$$

The corresponding of system of equation,

$x + 3y + 2z = 5$ —— ①
$y + 5z = 7$ —— ②
$-9z = -9$ —— ③

By solving eqn ③

$-9z = -9$
$z = 1$

Put $z = 1$ in eqn ②

$y + 5 = 7$
$y = 7 - 5 = \underline{y = 2}$

Put $y = 2$, $z = 1$ in eqn ①

$x + 6 + 2 = 5$
$x = 5 - 8$
$x = -3$

The values of $x = -3$, $y = 2$, $z = 1$ by Gauss Elimination method.

Lab Assignment:

1. Determine the positive root of x – cos x = 0 by bisection method

→ Here $x - \cos x = 0$

$\therefore x - \cos(x) = 0$

let $F(x) = x - \cos(x)$

Here

x	0	1
$F(x)$	-1	0.4597

Iteration-1:-

$F(0) = -1 < 0$ and $F(1) = 0.4597 > 0$

$\therefore$ Root lies betn 0 & 1

$x_0 = \frac{0+1}{2} = 0.5$

$F(x_0) = F(0.5) = 0.5 - \cos(0.5)$
$= -0.3776 < 0$

Iteration-2:-

$F(0.5) = -0.3776 < 0$ & $F(1) = 0.4597 > 0$

$\therefore$ Root lies betn 0.5 & 1

$x_1 = \frac{0.5+1}{2} = 0.75$

$F(x_1) = F(0.75) = 0.75 - \cos(0.75)$
$= 0.0183 > 0$

Iteration-3:-

$F(0.5) = -0.3776 < 0$ and

$F(0.75) = 0.0183 > 0$

$\therefore$ Root lies betn 0.5 & 0.75

$x_2 = \frac{0.5+0.75}{2} = 0.625$

$F(x_2) = F(0.625) = 0.625 - \cos(0.625)$
$= -0.186 < 0$

Iteration-4:-

$F(0.625) = -0.186 < 0$ &

$F(0.75) = 0.0183 > 0$

$\therefore$ Root lies betn 0.625 & 0.75

$x_3 = \frac{0.625+0.75}{2} = 0.687$

$F(x_3) = F(0.6875) = 0.6875 - \cos(0.6875)$
$= -0.0853 < 0$

Iteration-5:-

$F(0.6875) = -0.0853 < 0$ &

$F(0.75) = 0.0183 > 0$

$\therefore$ Root lies btn 0.6875 & 0.75

$x_4 = \frac{0.6875+0.75}{2} = 0.7188$

$F(x_4) = F(0.7188) = 0.7188 - \cos(0.7188)$
$= -0.0339 < 0$

Iteration-6:- $F(0.7188) = -0.0339 < 0$

& $F(0.75) = 0.0183 > 0$

$\therefore$ Root lies betn 0.7188 & 0.75

$x_5 = \frac{0.7188+0.75}{2} = 0.7344$

$F(x_5) = F(0.7344) = \cancel{0.7344} -$
$= 0.7344 - \cos(0.7344) = -0.0079 < 0$

Iteration-7:- $F(0.7344) = -0.0079 < 0$

& $F(0.75) = 0.0183 > 0$

$\therefore$ Root lies betn 0.7344 & 0.75

$x_6 = \frac{0.7344+0.75}{2} = 0.7422$

$F(x_6) = F(0.7422) = 0.7422 - \cos(0.7244) = 0.0052 > 0$

Iteration - 8 :- $F(0.7344) = -0.00790$ & $F(0.7422) = 0.0052 > 0$

$\therefore$ Root lies betn 0.7344 & 0.7422

$$x_7 = \frac{0.7344 + 0.7422}{2} = 0.7383$$

$$F(x_7) = F(0.7383) = 0.7383 - \cos(0.7383) = -0.0013 < 0$$

Iteration - 9 :- $F(0.7383) = -0.0013 < 0$ & $F(0.7422) = 0.0052 > 0$

$\therefore$ Root lies betn 0.7383 & 0.7422

$$x_8 = \frac{0.7383 + 0.7422}{2} = 0.7402$$

$$F(x_8) = F(0.7402) = 0.7402 - \cos(0.7402) = 0.0019 > 0$$

Iteration - 10 :- $F(0.7383) = -0.0013 < 0$ & $F(0.7402) = 0.0019 > 0$

$\therefore$ Root lies betn 0.7383 & 0.7402

$$x_9 = \frac{0.7383 + 0.7402}{2} = 0.7393$$

$$F(x_9) = F(0.7393) = 0.7393 - \cos(0.7393) = 0.0003 > 0$$

Iteration - 11 :- $F(0.7383) = -0.0013 < 0$ & $F(0.7393) = 0.0003 > 0$

$\therefore$ Root lies betn 0.7383 & 0.7393

$$x_{10} = \frac{0.7383 + 0.7393}{2} = 0.7388$$

$$F(x_{10}) = F(0.7388) = 0.7388 - \cos(0.7388) = -0.0005 < 0$$

Iteration - 12 :- $F(0.7388) = -0.0005 < 0$ & $F(0.7393) = 0.0003 > 0$

$\therefore$ Root lies betn 0.7388 & 0.7393

$$x_{11} = \frac{0.7388 + 0.7393}{2} = 0.739$$

$$F(x_{11}) = F(0.739) = 0.739 - \cos(0.739) = -0.0001 < 0$$

Iteration - 13 :- $F(0.739) = -0.0001 < 0$ & $F(0.7393) = 0.0003 > 0$

$\therefore$ Root lies betn 0.739 & 0.7393

$$x_{12} = \frac{0.739 + 0.7393}{2} = 0.7391$$

$$F(x_{12}) = F(0.7391) = 0.7391 - \cos(0.7391) = 0.0001 > 0$$

Iteration - 14 :- $F(0.7391) = -0.0001 < 0$ & $F(0.7391) = 0.0001 > 0$

$\therefore$ Root lies betn 0.739 & 0.7391

$$x_{13} = \frac{0.739 + 0.7391}{2} = 0.7391$$

$$F(x_{13}) = F(0.7391) = 0.7391 - \cos(0.7391) = 0 < 0$$

Approximate root of the eqn $x - \cos(x) = 0$ using Bisection method is 0.7391.

Lab Assignment:

1. Solve the positive root of $x^3 = 2x + 5$ by False Position Method.

$\rightarrow x^3 - 2x - 5 = 0$

$F(x) = x^3 - 2x - 5$

Put $x = 2$

$F(2) = 8 - 4 - 5 = -1$

Put $x = 3$

$F(3) = 27 - 6 - 5 = 16$

The root lies in between 2 & 3

$a = 2, \ b = 3$

$F(a) = -1, \ F(b) = 16$

$$x_1 = \frac{aF(b) - bF(a)}{F(b) - F(a)}$$

$$= \frac{32 + 3}{16 + 1} = \frac{35}{17}$$

$x_1 = 2.0588$

$F(x_1) = -0.3908 < 0$

2nd Iteration:-

$a = 2.0588, \ b = 3$

$F(a) = -0.3908, \ F(b) = 16$

$$x_2 = \frac{32.9408 + 1.1724}{16 + 0.3908}$$

$x_2 = 2.0812$

$F(x_2) = 9.0144 - 4.1624 - 5$

$= -0.1479 < 0$

3rd Iteration:-

$a = 2.0812, \ b = 3$

$F(a) = -0.1479, \ F(b) = 16$

$$x_3 = \frac{33.2992 + 0.4437}{16.1479}$$

$x_3 = 2.0896$

$F(x_3) = F(2.0896) = -0.0551 <$

4th Iteration:-

$a = 2.0896, \ b = 3$

$F(a) = -0.0551, \ F(b) = 16$

$$x_4 = \frac{16(2.0896) + 3(0.0551)}{16 + 0.0551}$$

$x_4 = 2.0927$

$F(x_4) = F(2.0927)$

$= -0.0207 < 0$

5th Iteration:-

$a = 2.0927, \ b = 3$

$F(a) = -0.0207, \ F(b) = 16$

$$x_5 = \frac{33.4832 + 0.0621}{16.0207}$$

$x_5 = 2.0939$

$F(x_5) = F(2.0939)$

$= -0.0073 < 0$

6^{th} Iteration :-

$a = 2.0939$, $b = 3$

$F(a) = -0.00839$, $F(b) = 16$

$$x_6 = \frac{33.5024 + 0.0219}{16.0073}$$

$x_6 = 2.0943$

$F(x_6) = F(2.0943)$

$= -0.0028 < 0$

7^{th} Iteration :-

$a = 2.0943$, $b = 3$

$F(a) = -0.0027$, $F(b) = 16$

$$x_7 = \frac{33.5088 + 0.0081}{16.0027}$$

$x_7 = 2.0944$

$\therefore F(x_7) = F(2.0944)$

$= -0.001 < 0$

8^{th} Iteration :-

$a = 2.0944$, $b = 3$

$F(a) = -0.001$, $F(b) = 16$

$$x_8 = \frac{33.5104 + 0.003}{16.001}$$

$x_8 = 2.0944$

$\therefore$ The root of the equation $x^3 - 2x - 5 = 0$ or $x^3 = 2x + 5$ is 2.0944 by False position method.

1)→ Advantages of secant methods:-

1) It coverages at faster than a linear rate. So that is more rapidly convergent than the bisec method
2) It does not require use of the derivative of the function. Something that is not available in a number of application.
3) It requires only one function evolution per itera as compared with Newton's method which require tu

* Disadvantages of secant method.

1) It may not converge
2) There is no guaranteed error bound for the iterates.
3) It is likely to have difficulty if $f_o(\alpha)=0$ This means the x-axis is tangent to the graph of $y=f(x)$ at $x=\alpha$.
4) Newton's method generalizes more easily to new meth for solving simultaneous system of non-linear equat

2] → $F(x) = x^3 - 5x + 1$

1st Iteration:-

Let $x_0 = 0$ & $x_1 = 1$

$F(x_0) = f(0) = 1$ & $F(x_1) = F(1) = 3$

$\therefore x_2 = x_0 - F(x_0) \cdot \dfrac{x_1 - x_0}{F(x_1) - F(x_0)} = 0 - 1 \times \dfrac{1-0}{-3-1} = 0.25$

$F(x_2) = F(0.25) = (0.25)^3 - 5(0.25) + 1$

$= -0.23437.$

2nd Iteration:-

$x_1 = 1$ and $x_2 = 0.25$

$F(x_1) = F(1) = -3$ & $F(x_2) = F(0.25) = -0.23437$

$x_3 = x_1 - F(x_1) \cdot \dfrac{x_2 - x_1}{F(x_2) - F(x_1)}$

$= 1 - (-3) \times \dfrac{0.25 - 1}{-0.23437 - (-3)}$

$= 0.18644$

$\therefore F(x_3) = F(0.18644)$

$= (0.18644)^3 - 5(0.18664) + 1$

$= 0.07428$

3rd Iteration:-

$x_2 = 0.25$ and $x_3 = 0.18644$

$F(0.25) = -0.23437$ & $F(0.18644) = 0.07428$

$x_4 = x_2 - F(x_2) \dfrac{x_3 - x_2}{F(x_3) - F(x_2)}$

$= 0.25 - (-0.23437) \times \dfrac{0.18644 - 0.25}{0.07428 + 0.23437}$

$x_4 = 0.20173$

$F(x_4) = F(0.20173)$

$= (0.20173)^3 - 5(0.20173) + 1$

$F(x_4) = 0.00044$

4th Iteration:-

$x_3 = 0.18644$ and $x_4 = 0.20173$

$F(x_3) = 0.07428$ and $F(x_4) = -0.00044$

$x_5 = x_3 - F(x_3) \cdot \dfrac{x_4 - x_3}{F(x_4) - F(x_3)}$

$= 0.186644 - 0.07428 \times \dfrac{0.20173 - 0.18644}{-0.00044 - 0.07428}$

$x_5 = 0.20162$

$F(x_5) = F(0.20162)$

$= (0.20162)^3 - 5(0.20162) + 1$

$= 0.000095 \approx 0$

$\therefore$ The required root for $F(x) = x^3 - 5x + 1$ by secant method is 0.20162.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12	[illegible]	-1	
1921	93	[illegible]	-4		
		8			
1931	101				

1

```
    6.0000
         0
Y =
    1.0000
    0.2000
    6.0000
    1.5000
Y =
    1.0000
    0.2000
    6.0000
    1.5000
X =
   -0.5000
   -5.5000
    1.5000
    1.5000
```

Conclusion:- This method is an LU Decompositia in which a matrix is Decomposed into lower triangular matrix, on upper triangular matrix. After decomposition, the method can be used to solve linear equation.

Lab Assignment:

1. Solve the following set of equations by Crout's Method

2x + 4y + 4z = 12; 8x -3y +2z = 20; 4x +11y –z = 33

$Ax = B$

$$\begin{bmatrix} 2 & 4 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$l = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}, \quad u = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

Now, $A = LU$

$$\begin{bmatrix} 2 & 4 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & l_{11}+u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12}+l_{22} & l_{21}u_{13}+l_{23}u_{23} \\ l_{31} & l_{31}u_{13}+l_{32} & l_{31}u_{13}+l_{32}l_{23}+l_{33} \end{bmatrix}$$

comparing we get.

$l_{11} = 2$, $l_{11}u_{12} = 4$, $u_{12} = 4/2$, $u_{12} = 2$; $l_{11}u_{13} = 4$, $u_{13} = 4/2$, $u_{13} = 2$

$l_{21} = 8$, $l_{21}u_{12} + l_{22} = -3$, $8 \times 2 + l_{22} = -3$, $l_{22} = -3-16$, $l_{22} = -19$; $l_{21}u_{13} + l_{22}u_{23} = 2$, $16 - 19u_{23} = 2$, $u_{23} = 14/19$

$l_{31} = 4$, $l_{31}u_{12} + l_{32} = 11$, $4 \times 2 + l_{32} = 11$, $8 + l_{32} = 11$, $l_{32} = 3$

$\therefore l_{31}u_{13} + l_{32}u_{23} + l_{33} = -1 \longrightarrow \frac{171+42}{19} = -l_{33}$

$4\times 2 + 3\times 14/19 + l_{33} = -1$ $\qquad l_{33} = -213/19$

$9 + 42/19 = -l_{33}$

$$l = \begin{bmatrix} 2 & 0 & 0 \\ 8 & -19 & 0 \\ 4 & 3 & \frac{-213}{19} \end{bmatrix}, \quad u = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 14/19 \\ 0 & 0 & 1 \end{bmatrix}$$

$LY = B$ But $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -19 & 0 \\ 4 & 3 & \frac{-213}{19} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$2y_1 = 12$

$y_1 = 12/2$

$y_1 = 6$

$\therefore 8y_1 - 19y_2 = 20$

$8\times 6 - 19y_2 = 20$

$48 - 19y_2 = 20$

$-19y_2 = 20 - 48$

$y_2 = +\frac{28}{19}$

$y_2 = \frac{28}{19}$

$4y_1 + 3y_2 - \frac{213}{19}y_3 = 33$

$4\times 6 + 3\left(\frac{28}{19}\right) - \frac{213}{19}y_3 = 33$

$24 - 33 + \frac{84}{19} = \frac{213}{19}y_3$

$-9 + \frac{84}{19} = \frac{213}{19}y_3$

$\frac{-171+84}{19} = \frac{213}{19}y_3$

$\frac{-87}{19} = \frac{213}{19}y_3$

$y_3 = \frac{-87}{213}$

$UX = Y$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 14/19 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 28/19 \\ -87/213 \end{bmatrix}$$

$x + 2y + 6z = 6$

$y + \frac{14}{19}z = \frac{28}{19}$

$z = \frac{-87}{213}$ $\qquad \therefore z = \underline{\underline{-0.4083}}$.

$y + \frac{14}{19}\times\left(\frac{-87}{213}\right) = \frac{28}{19}$

$y = \frac{26}{19} + \frac{14\times 87}{213\times 19}$

$= \frac{5964}{19} + \frac{1218}{19}$

$= \frac{7182}{19}$

$y = 1.7746$

$\therefore x + 2y + 6z = 6$

$x + 2\times 378 + 6\left(\frac{-87}{213}\right) = 6$

$x + 756 - \frac{522}{213} = 6$

$x = 3.2676$

$\therefore x = 3.2676,\ y = 1.7746.$

3541 .4583 5468 350 4409 ~~5560~~ 560 323 6441

```
      break;
    else
      k = k+1;
    end
end
fprintf('Solution vector after %d iterations is :\n', k-1);
for i = 1 : n
  fprintf(' %11.9f \n', x(i));
end
```

Output of Program:

Enter number of equation, n:- 3
Enter the tolerance, tol :- 0.0001
Enter maximum number of iterations, m: 1
Solution vector after 7 ~~int~~ iteration is:
= 0.99833921
3.00087719
2.00008625

Conclusion: Gauss - seidel method is an advantages approch to solving a system of Simultaneous linear equations.

Lab Assignment:

1. Using the Gauss Siedal Method solve the system of equations correct to three decimal places
$x+2y+z=0$; $3x+y-z=0$; $x-y+4z=3$

→ ∴ $3x+y-z=0$ —— ①
$x+2y+4z=0$ —— ②
$x-y+4z=3$ —— ③
condition of convergence,
$|3|>|1|+|-1|$, $|2|>|1|+|1|$ & $|4|>|1|+|-1|$

$x_1 = \frac{1}{a_1}(d_1 - b_1 y^0 - c_1 z^0)$

$y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z)$
$z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1)$

Iteration - 1 :- $x_1 = \frac{1}{3}(0-0-0) = \frac{1}{3} \times 0 = 0$
$y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z^0) = \frac{1}{2} \times 0 = 0$
$z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1) = \frac{1}{4}[3-0+0]$
$= \frac{3}{4} = 0.75$

Iteration - 2 :-
$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1) = \frac{1}{3}(0 - 1\times 0 + 1(0.75))$
$= \frac{1}{3}[0 - 0 + 0.75) = 0.25$
$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1) = \frac{1}{2}[0 - 1\times 0.25 - 1\times 0.75]$
$= -0.5$
$z_2 = \frac{1}{c_3}[d_3 - a_3 x_2 - b_3 y_2] = \frac{1}{4}[3 - 1\times 0.25 - 0.5]$
$= 0.5625$

Iteration - 3 :-
$x_3 = \frac{1}{a_1}[d_1 - b_1 y_2 - c_1 z_2] = \frac{1}{3}[0 - 1(-0.5) + 1(0.5625)] = 0.3541$
$y_3 = \frac{1}{b_2}[d_2 - a_2 x_3 - c_2 z_2] = \frac{1}{2}[0 - 0.35416 - 0.5625] = 0.4583$
$z_3 = \frac{1}{c_3}[d_3 - a_3 x_3 - b_3 y_3] = \frac{1}{4}[3 - 0.35416 - 0.4583]$
$= 0.5468$

Iteration 4 :-
$x_4 = \frac{1}{a_1}[d_1 - b_1 y_3 - c_1 z_3] = \frac{1}{3}[0 + 0.4583 + 0.547] = 0.3350$
$y_4 = \frac{1}{b_2}[d_2 - a_2 x_4 - c_2 z_3] = \frac{1}{2}[0 - 0.3350 - 0.5468] = 0.4409$ ~~0.5560~~
$z_3 = \frac{1}{c_3}[d_3 - a_3 x_4 - b_3 y_4] = \frac{1}{4}[3 - 0.3350 - 0.4409] = 0.5560$

Iteration 5 :-
$x_5 = \frac{1}{a_1}[d_1 - b_1 y_4 - c_1 z_4] = \frac{1}{3}[0 + 0.4409 + 0.5560] = 0.3323$
$y_5 = \frac{1}{b_2}[d_2 - a_2 x_5 - c_2 z_4] = \frac{1}{2}[0 - 0.3323 - 0.5560] = 0.4441$
$z_5 = \frac{1}{c_3}[d_3 - a_3 x_5 - b_3 y_5] = \frac{1}{4}[3 - 0.3323 - 0.4441]$
$= 0.5559$

Iteration 6 :-

$x_6 = \frac{1}{a_1}[d_1 - b_1 y_5 - c_1 z_5]$

$= \frac{1}{3}[0 + 0.444 + 0.5559] = 0.3333$

$y_6 = \frac{1}{b_2}[d_2 - a_2 x_6 - c_2 z_5]$

$= \frac{1}{2}[0 - 0.3333 + 0.5559] = 0.4446$

$z_6 = \frac{1}{c_3}[d_3 - a_3 x_6 - b_3 y_6]$

$= \frac{1}{4}[3 - 0.3333 - 0.4446] = 0.5555$

Iteration 7 :-

$x_7 = \frac{1}{a_1}[d_1 - b_1 y_6 - c_1 z_6]$

$= \frac{1}{3}[0 + 0.4446 + 0.5555] = 0.3333$

$y_7 = \frac{1}{b_2}[d_2 - a_2 x_7 - c_2 z_6]$

$= \frac{1}{2}[0 - 0.3333 - 0.5555] = -0.4444$

$z_7 = \frac{1}{c_3}[d_3 - a_3 x_7 - b_3 y_7)$

$= \frac{1}{4}[3 - 0.3333 - 0.4444]$

$= 0.5555$

The solution of Gauss Seidel method is

$x = 0.3333$

$y = -0.4444$

$z = 0.5555$.

Enter lower limit of integral = 0

Enter upper limit of integral = 1.2

Enter number of intervals = 6

Evaluated integral = 2.32784577

Conclusion: The trapezoidal rule also known as the trapezoidal rule of tapezium rule; see trapezoid for more information on terminology is a technique for approximating the definite integral.

Lab Assignment:

1. Evaluate the integral $\int_0^{1.2} e^x\, dx$, taking six intervals by using trapezoidal rule up to three significant figures.

2. Evaluate $\int_0^{12} \frac{dx}{1+x^2}$, by using trapezoidal rule, taking n=6, correct to give significant figures.

→ 1) Here,

$a=0,\ b=1.2$ and $n=6$

$n = \frac{b-a}{n} = \frac{1.2-0}{6} = \underline{\underline{0.2}}$

x	0	0.2	0.4	0.6	0.8	1.0	1.2
$Y=F(x)$	0	1.221	1.442	1.822	2.226	2.718	3.320

TR can be written as

$$I = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{0.2}{2}\left[(0+3.320)+2(1.221+1.442+1.823+2.226+2.718)\right]$$

$I = 2.3278 \simeq 2.328$

∴ The exact value is $\int_0^{1.2} e^x dx = 2.320$

Q.2. $\int_0^{12} \frac{dx}{1+x^2}$, $a=0,\ b=12,\ n=6$

$h = \frac{b-a}{n} = \frac{12}{6} = \underline{\underline{2}}$

x	$F(x)=\frac{1}{1+x^2}$	
0	1	y_0
2	0.2	y_1
4	0.0588	y_2
6	0.027	y_3
8	0.0153	y_4
10	0.0099	y_5
12	0.0068	y_6

$$I = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$I = \frac{2}{2}\left[(1+0.2)+2(0.2+0.0588+0.027+0.0153+0.0099)\right]$$

$\boxed{I = 1.6288}$

Enter lower limit of integral = 0

Enter upper limit of integral = 1.2

Enter number of intervals = 6

Evaluate value is 2.320137.

Conclusion: Simpson's 1/3 rule is an extension of the trapezoidal rule in which ~~fx~~ is approximated by a second order polynomial. it is a method or approximating area under the curve.

Lab Assignment:

1. Evaluate the integral $\int_0^{1.2} e^x\, dx$, taking six intervals by using Simpson's 1/3 rule.

2. Evaluate $\int_0^{12} \frac{dx}{1+x^2}$, by using Simpson's 1/3 rule, taking n=6.

→ 1) $a = 0,\ b = 1.2,\ n = 6$

$h = \frac{b-a}{n} = \frac{1.2-0}{6} = 0.2$

$\boxed{h = 0.2}$

x	0	0.2	0.4	0.6	0.8	1.0	1.2
F(x)	1	1.2214	1.4918	1.8221	2.2255	2.7182	3.3201
y	y_0	y_1	y_2	y_3	y_4	y_5	y_6

The Simpson's rule is,

$I_3 = \frac{h}{2}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$

$= \frac{0.2}{2}\left[(1+3.32012) + 4(1.2214+1.8221+2.7182) + 2(1.4918 + 2.2255)\right]$

$= \frac{0.2}{2}\left[4.32612 + 4(3.7618) + 2(3.7136)\right]$

$\boxed{I_3 = 2.320136}$ ∴ The exact value is = 2.3201.

2 → $a = 0,\ b = 1.2,\ n = 6$

$\therefore h = \frac{b-a}{n} = \frac{12-0}{6} = 2$

x	0	2	4	6	8	10	12
F(x)	1	1.2	0.058	0.0270	0.0153	0.0099	0.0069
y	y_0	y_1	y_2	y_3	y_4	y_5	y_6

The Simpson's rule is.

$I_{3\,1/3} = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$

$= \frac{2}{3}\left[(1+0.0069) + 4(1.2+0.0270+0.0099) + 2(0.058+0.0153)\right]$

$= \frac{2}{3}\left[1.0069 + 4(0.2369) + 2(0.0211)\right]$

$\boxed{I_3 = 1.40201}$