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MSc 2

MH AND GIBBS PRACTICAL

```
#Q1) A small company improved a product and wants to infer about the
# proportion of potential customers who will buy the product if the new produ
ct is preferred to the old one.
# The company is certain that this proportion will exceed 0.5, i.e. and uses
the uniform prior on [0.5, 1]. Out
# of 20 customers surveyed, 12 prefer the new product. Find the posterior for
p.


set.seed(2440)
theta_0 = 0
s = 10
x = c(theta_0)
m = c(theta_0)

p1 = c()
p=c()

for (j in 2:20000)
{
  x[j] = rnorm(1,m[j-1],s)   #Remember: m[j-1] and not x[j-1]
  p1[j-1] = exp(x[j] - m[j-1])*((0.5+exp(x[j]))/(0.5+exp(m[j-1])))^12 * ((1+e
xp(m[j-1]))/(1+exp(x[j])))^22
  if (p1[j-1] > 1) {p[j-1]=1}
  else {p[j-1] = p1[j-1]}
  u1 = runif(1,0,1)
  if (u1 < p[j-1]) {m[j] = x[j]}
  else {m[j] = x[j-1]}
}
m[seq(18000,20000,1)]

hist(m[seq(18000,20000,1)])
```

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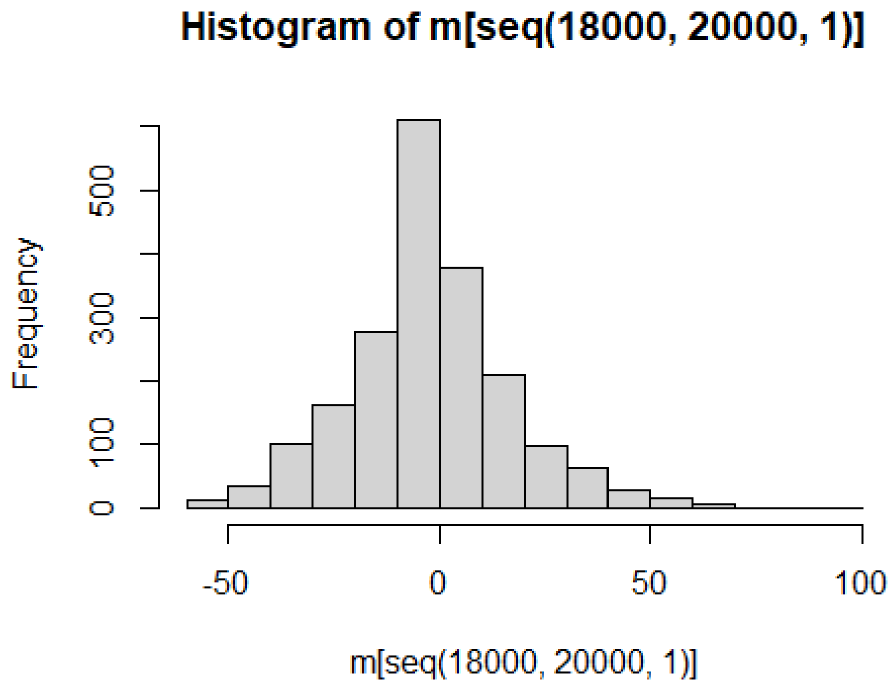


```r
## Q2)
set.seed(2440)
x = matrix(rep(0,500*400), nrow = 400, ncol = 500)
x0 = runif(1,0,1)
c = 0.2
m = matrix(rep(0,500*400),nrow = 400,ncol = 500)
m[,1] = x0
for (i in 1:400)
{
  for (j in 2:500)
  {
    x[i,j] = runif(1,x0-c,x0+c)
    p1 = (x[i,j]/x0)^5
    if (p1 > 1){p=1}
    else if (x[i,j]<0){p=0}
    else if (x[i,j]>1){p=0}
    else {p = p1}
    u = runif(1,0,1)
    if (u<p)
    {m[i,j] = x[i,j]}
    else
    {m[i,j] = x[i,j-1]}

  }
}
m[,500]
```

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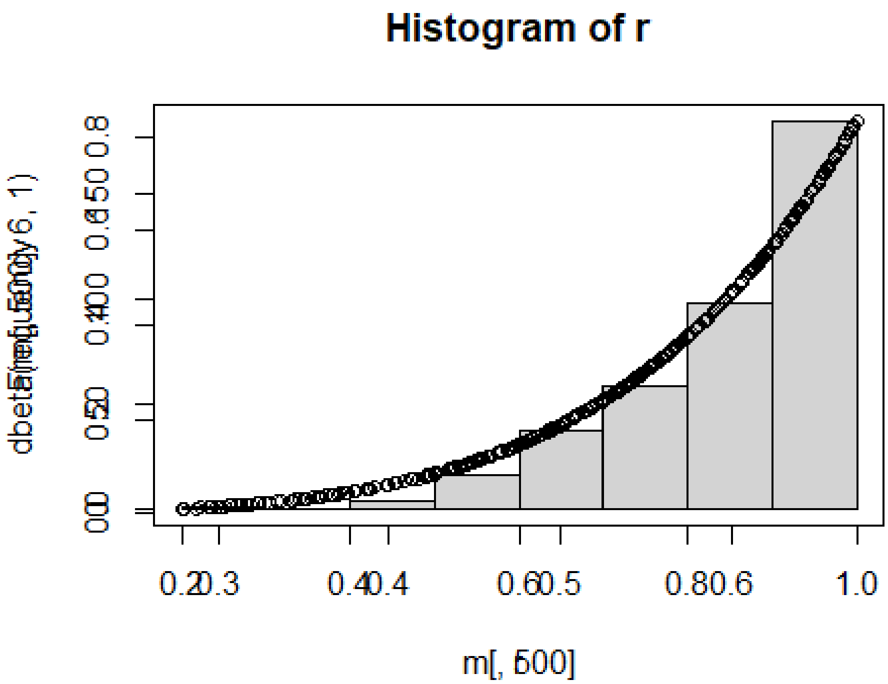
```
# Visualization to check goodness of fit
r = rbeta(400,6,1)
hist(r)
par(new = TRUE)
plot(m[,500],dbeta(m[,500],6,1))
```




```
## GIBBS:
#Q1)
x_0 = runif(1,0,1)
y_0 = runif(1,0,1)
x = c()
y = c()
x[1] = x_0
y[1] = y_0

for (i in 2:1000)
{
  u = runif(1,0,1)
  x[i] = (-1/y[i-1])*log(1-u+u*exp(-y[i-1]))
  y[i] = (-1/x[i])*log(1-u+u*exp(-x[i]))
}
x[1:20]

##  [1] 0.754858673 0.362064850 0.743774995 0.622959544 0.556781667 0.1061421
98
##  [7] 0.575407922 0.987991759 0.160506777 0.035057668 0.681534520 0.3997657
```

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```
76
## [13] 0.651332176 0.726294141 0.219421262 0.320951016 0.978917869 0.5762040
76
## [19] 0.561246324 0.003580853

y[1:20]

##  [1] 0.947210330 0.429634118 0.711490691 0.633499710 0.566271693 0.1288847
34
##  [7] 0.520049415 0.984449399 0.219403369 0.038246055 0.607084665 0.4243725
97
## [13] 0.624640360 0.715645415 0.262712102 0.314758789 0.969899389 0.6240047
84
## [19] 0.569000949 0.004682571

#Q2)
x_0 = 5
y_0 = 0.5
x = c()
y = c()
x[1] = x_0
y[1] = y_0

for (i in 2:1000)
{
  u = runif(1,0,1)
  x[i] = rbinom(1,16,y[i-1])
  y[i] = rbeta(1,x[i]+2,16-x[i]+4)
}
x[1:20]

##  [1]  5 10  8  6  6  2  1  0  0  4  4  6  7  1  1  4  2  4  6  8

y[1:20]

##  [1] 0.50000000 0.52172757 0.39042101 0.28703581 0.29789565 0.10512241
##  [7] 0.06787151 0.04290157 0.22115765 0.29959999 0.28033279 0.50749852
## [13] 0.22791449 0.11293814 0.05345059 0.22547433 0.16000498 0.20706431
## [19] 0.40405134 0.29398256
```

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MH Algorithm & Gibbs Algorithm.

Q3) Gibbs Sampler :-

Consider a conditional density

$f(x|Y=y) = ye^{-xy}$, $0 < x < 1$

$f(Y|x=x) = xe^{-xy}$, $0 < y < 1$

Proof :-

$$f(x|Y=y) = ye^{-xy}$$

$$g(x|Y=y) = \frac{1}{k} f(x|Y=y)$$

$$k = \int_{x=0}^{1} ye^{-xy}\, dx$$

$$= y\left[\frac{e^{-xy}}{y}\right]_{1}^{0}$$

$$k = 1-e^{-y}$$

$$g(x|Y=y) = \frac{ye^{-xy}}{1-e^{-y}}$$

$$F(x|Y=y) = \int_0^x g(x|Y=y)\, dx$$

$$= \int_0^x \frac{ye^{-xy}}{1-e^{-y}}\, dx$$

$$= \frac{y}{1-e^{-y}} \int_0^x e^{-xy}\, dx$$

$$= \frac{y}{1-e^{-y}} \left[\frac{1-e^{-xy}}{y} \right]$$

let, $F[x \mid Y=y] = U$

$$\frac{1}{1-e^{-y}} [1-e^{-xy}] = U$$

$$1-e^{-xy} = U[1-e^{-y}]$$

$$1-e^{-xy} = U[1-e^{-y}]$$

$$e^{-xy} = 1-U(1-e^{-y})$$

$$-xy = \ln[1-U(1-e^{-y})]$$

$$x = \frac{-1}{y} \ln[1-U(1-e^{-y})]$$

lll'y, $y = \frac{-1}{x} \ln[1-U(1-e^{-y})]$

Q2) For the joint distn of x & y, $F(x,y) \propto {}^nC_x\, y^{x+\alpha-1}(1-y)^{n-x+\beta-}$

$n = 16,\ \alpha = 2,\ \beta = 4$

Ans.

$$F(x,y) \propto \binom{n}{2} y^{x+\alpha-1} (1-y)^{n-x+\beta-1}$$

$$F(x,y) = {}^nC_x\, y^{x+\alpha-1} (1-y)^{n-x+\beta-1}$$

$$f(x) = \int_0^1 f(x,y)\, dy$$

$$= \int_0^1 {}^nC_x\, y^{x+\alpha-1} (1-y)^{n-x+\beta-1}\, dy$$

$$f(x) = {}^nC_x\, \beta(x+\alpha, n-x+\beta)$$

Also,

$$f(y) = \int_0^1 f(x,y)\,dx$$

$$= \sum_{x=0}^{n} \binom{n}{x} y^{x+\alpha-1}(1-y)^{n-x+\beta-1}$$

$$= y^{\alpha-1}(1-y)^{\beta-1} \sum_{x=0}^{n} \binom{n}{x} y^{x}(1-y)^{n-x}$$

$$= y^{\alpha-1}(1-y)^{\beta-1}$$

$$F(X|Y=y) = \frac{f(x,y)}{f(y)}$$

$$= \binom{n}{x} y^{x}(1-y)^{n-x}$$

$$X|Y = y \sim \text{Binom}(n,y)$$

$$F(Y|X=x) = \frac{f(x,y)}{f(x)}$$

$$= \frac{{}^nC_x\, y^{x+\alpha-1}(1-y)^{n-x+\beta-1}}{{}^nC_x\, \beta(x+\alpha, n-x+\beta)}$$

$$= \frac{1}{\beta(x+\alpha, n-x+\beta)}\, y^{x+\alpha-1}(1-y)^{n-x+\beta-1}$$

$$Y|X=x \longrightarrow \beta_1(x+\alpha, n-x+\beta)$$