The bigintcalc package

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Abstract

This package provides expandable arithmetic operations with big integers that can exceed TeX's number limits.

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1 Documentation

1.1 Introduction

Package bigintcalc defines arithmetic operations that deal with big integers. Big integers can be given either as explicit integer number or as macro code that expands to an explicit number. Big means that there is no limit on the size of the number. Big integers may exceed $T_{\rm E}X$'s range limitation of -2147483647 and 2147483647. Only memory issues will limit the usable range.

In opposite to package intcalc unexpandable command tokens are not supported, even if they are valid TeX numbers like count registers or commands created by \chardef. Nevertheless they may be used, if they are prefixed by \number.

Also ε -TeX's \numexpr expressions are not supported directly in the manner of package intcalc. However they can be given if \the\numexpr or \numexpr are used.

The operations have the form of macros that take one or two integers as parameter and return the integer result. The macro name is a three letter operation name prefixed by the package name, e.g. \bigintcalcAdd{10}{43} returns 53.

The macros are fully expandable, exactly two expansion steps generate the result. Therefore the operations may be used nearly everywhere in TEX, even inside \csname, file names, or other expandable contexts.

1.2 Conditions

1.2.1 Preconditions

- Arguments can be anything that expands to a number that consists of optional signs and digits.
- The arguments and return values must be sound. Zero as divisor or factorials of negative numbers will cause errors.

1.2.2 Postconditions

Additional properties of the macros apart from calculating a correct result (of course ©):

- The macros are fully expandable. Thus they can be used inside \edef, \csname, for example.
- Furthermore exactly two expansion steps calculate the result.
- The number consists of one optional minus sign and one or more digits. The first digit is larger than zero for numbers that consists of more than one digit.

In short, the number format is exactly the same as \number generates, but without its range limitation. And the tokens (minus sign, digits) have catcode 12 (other).

• Call by value is simulated. First the arguments are converted to numbers. Then these numbers are used in the calculations.

Remember that arguments may contain expensive macros or ε -TeX expressions. This strategy avoids multiple evaluations of such arguments.

1.3 Error handling

Some errors are detected by the macros, example: division by zero. In this cases an undefined control sequence is called and causes a TeX error message, example: \BigIntCalcError:DivisionByZero. The name of the control sequence contains the reason for the error. The TEX error may be ignored. Then the operation returns zero as result. Because the macros are supposed to work in expandible contexts. An traditional error message, however, is not expandable and would break these contexts.

1.4 Operations

Some definition equations below use the function Int that converts a real number to an integer. The number is truncated that means rounding to zero:

$$Int(x) := \begin{cases} \lfloor x \rfloor & \text{if } x \ge 0 \\ \lceil x \rceil & \text{otherwise} \end{cases}$$

1.4.1 Num

\bigintcalcNum $\{\langle x \rangle\}$

Macro \bigintcalcNum converts its argument to a normalized integer number without unnecessary leading zeros or signs. The result matches the regular expression:

3

1.4.2 Inv, Abs, Sgn

\bigintcalcInv $\{\langle x \rangle\}$

Macro \bigintcalcInv switches the sign.

$$Inv(x) := -x$$

\bigintcalcAbs $\{\langle x \rangle\}$

Macro \bigintcalcAbs returns the absolute value of integer $\langle x \rangle$.

$$Abs(x) := |x|$$

\bigintcalcSgn $\{\langle x \rangle\}$

Macro \bigintcalcSgn encodes the sign of $\langle x \rangle$ as number.

$$Sgn(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

These return values can easily be distinguished by \ifcase:

```
\ifcase\bigintcalcSgn{<x>}
    $x=0$
\or
    $x>0$
\else
    $x<0$
\fi</pre>
```

1.4.3 Min, Max, Cmp

\bigintcalcMin $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMin returns the smaller of the two integers.

$$Min(x, y) := \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

\bigintcalcMax $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMax returns the larger of the two integers.

$$Max(x, y) := \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

\bigintcalcCmp $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcCmp encodes the comparison result as number:

$$\operatorname{Cmp}(x,y) := \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

These values can be distinguished by \c

1.4.4 Odd

\bigintcalcOdd $\{\langle x \rangle\}$

$$Odd(x) := \begin{cases} 1 & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

1.4.5 Inc, Dec, Add, Sub

\bigintcalcInc $\{\langle x \rangle\}$

Macro \bigintcalcInc increments $\langle x \rangle$ by one.

$$Inc(x) := x + 1$$

\bigintcalcDec $\{\langle x angle\}$

Macro \bigintcalcDec decrements $\langle x \rangle$ by one.

$$Dec(x) := x - 1$$

\bigintcalcAdd $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcAdd adds the two numbers.

$$Add(x, y) := x + y$$

\bigintcalcSub $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcSub calculates the difference.

$$Sub(x, y) := x - y$$

1.4.6 Shl, Shr

\bigintcalcShl $\{\langle x \rangle\}$

Macro \bigintcalcShl implements shifting to the left that means the number is multiplied by two. The sign is preserved.

$$Shl(x) := x * 2$$

\bigintcalcShr $\{\langle x \rangle\}$

Macro \bigintcalcShr implements shifting to the right. That is equivalent to an integer division by two. The sign is preserved.

$$Shr(x) := Int(x/2)$$

1.4.7 Mul, Sqr, Fac, Pow

\bigintcalcMul $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMul calculates the product of $\langle x \rangle$ and $\langle y \rangle$.

$$Mul(x, y) := x * y$$

\bigintcalcSqr $\{\langle x \rangle\}$

Macro $\begin{tabular}{l} \begin{tabular}{l} \begi$

$$Sqr(x) := x^2$$

\bigintcalcFac $\{\langle x \rangle\}$

Macro \bigintcalcFac returns the factorial of $\langle x \rangle$. Negative numbers are not permitted.

$$\operatorname{Fac}(x) := x! \qquad \text{ for } x \ge 0$$

$$(0! = 1)$$

$\$ \bigintcalcPow $Mx\ My$

Macro \bigintcalcPow calculates the value of $\langle x \rangle$ to the power of $\langle y \rangle$. The error "division by zero" is thrown if $\langle x \rangle$ is zero and $\langle y \rangle$ is negative. permitted:

$$\operatorname{Pow}(x,y) := \operatorname{Int}(x^y) \qquad \text{for } x \neq 0 \text{ or } y \geq 0$$

$$(0^0 = 1)$$

1.4.8 Div, Mul

\bigintcalcDiv $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcDiv performs an integer division. Argument $\langle y \rangle$ must not be zero.

$$\mathrm{Div}(x,y) := \mathrm{Int}(x/y)$$
 for $y \neq 0$

\bigintcalcMod $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMod gets the remainder of the integer division. The sign follows the divisor $\langle y \rangle$. Argument $\langle y \rangle$ must not be zero.

$$Mod(x, y) := x \% y$$
 for $y \neq 0$

The result ranges:

$$-|y| < \operatorname{Mod}(x, y) \le 0 \quad \text{for } y < 0$$

$$0 \le \operatorname{Mod}(x, y) < y \quad \text{for } y \ge 0$$

1.5 Interface for programmers

If the programmer can ensure some more properties about the arguments of the operations, then the following macros are a little more efficient.

In general numbers must obey the following constraints:

- Plain number: digit tokens only, no command tokens.
- Non-negative. Signs are forbidden.
- Delimited by exclamation mark. Curly braces around the number are not allowed and will break the code.

```
\BigIntCalcOdd \langle number \rangle !
```

1/0 is returned if $\langle number \rangle$ is odd/even.

```
\BigIntCalcInc \langle number \rangle !
```

Incrementation.

```
\BigIntCalcDec \langle number \rangle !
```

Decrementation, positive number without zero.

```
\BigIntCalcAdd \langle number \ A \rangle ! \langle number \ B \rangle !
```

Addition, $A \geq B$.

```
\BigIntCalcSub \langle number A \rangle ! \langle number B \rangle !
```

Subtraction, $A \geq B$.

```
\BigIntCalcShl \langle number \rangle!
```

Left shift (multiplication with two).

```
\ \BigIntCalcShr \langle number \rangle !
```

Right shift (integer division by two).

```
\BigIntCalcMul \langle number A \rangle ! \langle number B \rangle !
```

Multiplication, $A \geq B$.

```
\BigIntCalcDiv \langle number A \rangle ! \langle number B \rangle !
```

Division operation.

```
\BigIntCalcMod\ \langle number\ A \rangle \ ! \ \langle number\ B \rangle \ !
```

Modulo operation.

2 Implementation

```
1 (*package)
```

2.1 Reload check and package identification

Reload check, especially if the package is not used with LATEX.

```
2 \begingroup\catcode61\catcode48\catcode32=10\relax%
      \catcode13=5 % ^^M
      \endlinechar=13 %
     \catcode35=6 % #
     \catcode39=12 % '
     \catcode44=12 % ,
     \catcode45=12 % -
  9
     \catcode46=12 % .
 10
     \catcode58=12 % :
      \catcode64=11 % @
 11
      \catcode123=1 % {
 12
      \catcode125=2 % }
 13
      \expandafter\let\expandafter\x\csname ver@bigintcalc.sty\endcsname
 14
      \ifx\x\relax % plain-TeX, first loading
 15
 16
      \else
 17
        \def\empty{}%
 18
        \ifx\x\empty % LaTeX, first loading,
 19
          % variable is initialized, but \ProvidesPackage not yet seen
 20
        \else
          \expandafter\ifx\csname PackageInfo\endcsname\relax
 21
            \def\x#1#2{%}
 22
              \immediate\write-1{Package #1 Info: #2.}%
 23
            }%
 24
          \else
 25
            \def\x#1#2{\PackageInfo{#1}{#2, stopped}}%
 26
 27
          \x{bigintcalc}{The package is already loaded}%
 28
 29
          \aftergroup\endinput
 30
        \fi
      \fi
 31
 32 \endgroup%
Package identification:
 33 \begingroup\catcode61\catcode48\catcode32=10\relax%
      \catcode13=5 % ^^M
 35
      \endlinechar=13 %
      \catcode35=6 % #
 36
 37
      \catcode39=12 % '
 38
      \catcode40=12 % (
 39
      \catcode41=12 % )
      \catcode44=12 % ,
 41
      \catcode45=12 % -
 42
      \catcode46=12 % .
      \catcode47=12 % /
 43
      \catcode58=12 % :
 44
      \catcode64=11 % @
 45
      \catcode91=12 % [
 46
 47
      \catcode93=12 % ]
 48
      \catcode123=1 % {
 49
      \catcode125=2 % }
 50
      \expandafter\ifx\csname ProvidesPackage\endcsname\relax
 51
        \def\x#1#2#3[#4]{\endgroup
 52
          \immediate\write-1{Package: #3 #4}%
 53
          \xdef#1{#4}%
        }%
 54
 55
      \else
        \def \x#1#2[#3] \endgroup
 56
```

```
#2[{#3}]%
57
        \ifx#1\@undefined
58
          \xdef#1{#3}%
59
        \fi
60
        \int x#1\relax
61
62
          \xdef#1{#3}%
63
        \fi
64
      }%
   \fi
65
66 \expandafter\x\csname ver@bigintcalc.sty\endcsname
67 \ProvidesPackage{bigintcalc}%
    [2012/04/08 v1.3 Expandable calculations on big integers (HO)]%
```

2.2 Catcodes

```
69 \begingroup\catcode61\catcode48\catcode32=10\relax%
    \catcode13=5 % ^^M
70
71
     \endlinechar=13 %
72
    \catcode123=1 % {
    \catcode125=2 % }
73
    \catcode64=11 % @
74
75
     \def\x{\endgroup
76
       \expandafter\edef\csname BIC@AtEnd\endcsname{%
77
         \endlinechar=\the\endlinechar\relax
78
         \catcode13=\the\catcode13\relax
79
         \catcode32=\the\catcode32\relax
         \catcode35=\the\catcode35\relax
80
         \catcode61=\the\catcode61\relax
81
         \catcode64=\the\catcode64\relax
82
         \catcode123=\the\catcode123\relax
         \catcode125=\the\catcode125\relax
85
       }%
86 }%
87 \x\catcode61\catcode48\catcode32=10\relax%
88 \catcode13=5 % ^^M
89 \endlinechar=13 %
90 \catcode35=6 % #
91 \catcode64=11 % @
92 \catcode123=1 % {
93 \catcode125=2 % }
94 \def\TMP@EnsureCode#1#2{%
95 \edef\BIC@AtEnd{%
96
       \BIC@AtEnd
      \catcode#1=\the\catcode#1\relax
97
   }%
98
    \catcode#1=#2\relax
99
100 }
101 \TMP@EnsureCode{33}{12}%!
102 \TMP@EnsureCode{36}{14}% $ (comment!)
103 \TMP@EnsureCode{38}{14}% & (comment!)
104 \TMP@EnsureCode{40}{12}% (
105 \TMP@EnsureCode{41}{12}%)
106 \TMP@EnsureCode{42}{12}% *
107 \TMP@EnsureCode{43}{12}% +
108 \TMP@EnsureCode{45}{12}% -
109 \TMP@EnsureCode\{46\}\{12\}\% .
110 \TMP@EnsureCode{47}{12}% /
111 \TMP@EnsureCode{58}{11}% : (letter!)
112 \TMP@EnsureCode{60}{12}% <
113 \TMP@EnsureCode{62}{12}% >
114 \TMP@EnsureCode{63}{14}% ? (comment!)
115 \TMP@EnsureCode{91}{12}% [
```

```
116 \TMP@EnsureCode{93}{12}% ]
                   117 \edef\BIC@AtEnd{\BIC@AtEnd\noexpand\endinput}
                   118 \begingroup\expandafter\expandafter\expandafter\endgroup
                   119 \expandafter\ifx\csname BIC@TestMode\endcsname\relax
                   120 \else
                   121 \catcode63=9 % ? (ignore)
                   122 \fi
                   123 ? \let\BIC@@TestMode\BIC@TestMode
                  2.3 -T<sub>E</sub>X detection
                   124 \begingroup\expandafter\expandafter\expandafter\endgroup
                   125 \expandafter\ifx\csname numexpr\endcsname\relax
                   126 \catcode36=9 % $ (ignore)
                   127 \else
                   128 \catcode38=9 % & (ignore)
                   129 \fi
                  2.4 Help macros
         \BIC@Fi
                   130 \let\BIC@Fi\fi
    \BIC@AfterFi
                   131 \def\BIC@AfterFi#1#2\BIC@Fi{\fi#1}%
  \BIC@AfterFiFi
                   132 \def\BIC@AfterFiFi#1#2\BIC@Fi{\fi\fi#1}%
\BIC@AfterFiFiFi
                   133 \def\BIC@AfterFiFiFi#1#2\BIC@Fi{\fi\fi\fi#1}%
      \BIC@Space
                   134 \begingroup
                       \def\x#1{\endgroup
                   136
                          \let\BIC@Space= #1%
                   137 }%
                   138 \x{ }
                        Expand number
                  2.5
                   139 \begingroup\expandafter\expandafter\expandafter\endgroup
                   140 \expandafter\ifx\csname RequirePackage\endcsname\relax
                        \def\TMP@RequirePackage#1[#2]{%
                   141
                          \begingroup\expandafter\expandafter\expandafter\endgroup
                   142
                          \expandafter\ifx\csname ver@#1.sty\endcsname\relax
                   143
                            \input #1.sty\relax
                   144
                          \fi
                   145
                        }%
                   146
                        \TMP@RequirePackage{pdftexcmds}[2007/11/11]%
                   147
                        \RequirePackage{pdftexcmds}[2007/11/11]%
                   150 \fi
                   151 \begingroup\expandafter\expandafter\expandafter\endgroup
                   152 \expandafter\ifx\csname pdf@escapehex\endcsname\relax
     \BIC@Expand
                        \def\BIC@Expand#1{%
                   153
                          \romannumeral0%
                   154
                   155
                          \BIC@@Expand#1!\@nil{}%
                   156
```

```
\BIC@@Expand
                            \def\BIC@@Expand#1#2\@ni1#3{%
                      157
                              \expandafter\ifcat\noexpand#1\relax
                      158
                                \expandafter\@firstoftwo
                      159
                      160
                      161
                                \expandafter\@secondoftwo
                              \fi
                      162
                      163
                                \expandafter\BIC@@Expand#1#2\@ni1{#3}%
                      164
                              }{%
                      165
                                \ifx#1!%
                      166
                                  \expandafter\@firstoftwo
                      167
                      168
                                \else
                      169
                                  \expandafter\@secondoftwo
                      170
                                \fi
                                { #3}{%
                      171
                                  \BIC@@Expand #2\\@nil{#3#1}%
                      172
                                }%
                      173
                              }%
                      174
                            }%
                      175
      \@firstoftwo
                            \expandafter\ifx\csname @firstoftwo\endcsname\relax
                      176
                              \long\def\@firstoftwo#1#2{#1}%
                      177
                      178
     \@secondoftwo
                      179
                            \expandafter\ifx\csname @secondoftwo\endcsname\relax
                      180
                              \long\def\@secondoftwo#1#2{#2}%
                      181
                      182 \ensuremath{\setminus} else
       \BIC@Expand
                      183
                            \def\BIC@Expand#1{%
                      184
                              \romannumeral0\expandafter\expandafter\expandafter\BIC@Space
                      185
                              \pdf@unescapehex{%
                                \expandafter\expandafter\expandafter
                      186
                                \BIC@StripHexSpace\pdf@escapehex{#1}20\@nil
                      187
                      188
                              }%
                            }%
                      189
\BIC@StripHexSpace
                            \def\BIC@StripHexSpace#120#2\@nil{%
                      190
                      191
                              #1%
                              \int x^{\#2}\
                      192
                              \else
                      193
                                \BIC@AfterFi{%
                      194
                                  \BIC@StripHexSpace#2\@nil
                      195
                                }%
                      196
                      197
                              \BIC@Fi
                      198
                            }%
                      199 \fi
                            Normalize expanded number
                     2.6
                     #1: result sign
    \BIC@Normalize
                     #2: first token of number
                      200 \def\BIC@Normalize#1#2{%
                           \ifx#2-%
                      201
                      202
                              \ifx\\#1\\%
```

```
\BIC@AfterFiFi{%
                       203
                       204
                                   \BIC@Normalize-%
                                }%
                       205
                       206
                               \else
                       207
                                 \BIC@AfterFiFi{%
                       208
                                   \BIC@Normalize{}%
                       209
                                }%
                              \fi
                       210
                            \else
                       211
                              \ifx#2+%
                       212
                                 \BIC@AfterFiFi{%
                       213
                                   \BIC@Normalize{#1}%
                       214
                       215
                       216
                               \else
                       217
                                 \ifx#20%
                       218
                                   \BIC@AfterFiFiFi{%
                                     219
                                   }%
                       220
                       221
                                 \else
                                   \BIC@AfterFiFiFi{%
                       222
                                     \BIC@NormalizeDigits#1#2%
                       223
                       224
                                   }%
                       225
                                 \fi
                       226
                               \fi
                       227
                            \BIC@Fi
                       228 }
 \BIC@NormalizeZero
                       229 \def\BIC@NormalizeZero#1#2{%
                            \ifx#2!%
                       230
                              \BIC@AfterFi{ 0}%
                       231
                       232
                            \else
                       233
                              \ifx#20%
                                 \BIC@AfterFiFi{%
                       234
                       235
                                   \BIC@NormalizeZero{#1}%
                                }%
                       236
                               \else
                       237
                                 \BIC@AfterFiFi{%
                       238
                                   \verb|\BIC@NormalizeDigits#1#2||
                       239
                                }%
                       240
                              \fi
                       241
                       242
                            \BIC@Fi
                       243 }
\BIC@NormalizeDigits
                       244 \def\BIC@NormalizeDigits#1!{ #1}
                      2.7
                            Num
      \bigintcalcNum
                       245 \def\bigintcalcNum#1{\%
                       246
                            \romannumeral0%
                       247
                             \expandafter\expandafter\BIC@Normalize
                       248
                            \expandafter\expandafter\expandafter{%
                            \expandafter\expandafter\%
                       249
                       250
                            \verb|\BIC@Expand{#1}!%|
                       251 }
                             Inv, Abs, Sgn
                      2.8
```

\bigintcalcInv

```
252 \def\bigintcalcInv#1{%
                                                                          254 \bigintcalcNum{-#1}%
                                                                         255 }
\bigintcalcAbs
                                                                         256 \def\bigintcalcAbs#1{%
                                                                         257 \romannumeral0%
                                                                         {\tt 258} \quad \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ } \texttt{\ \ } \texttt{\ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ } \texttt{\ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ } \texttt{\ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \  \ } \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ }} \texttt{\ \ \ } \texttt{\ \ \ }} \texttt{\ \ \ \ 
                                                                         259 \bigintcalcNum{#1}%
                                                                          260 }
                          \BIC@Abs
                                                                         261 \def\BIC@Abs#1{%
                                                                         262 \ifx#1-%
                                                                         263
                                                                                                 \expandafter\BIC@Space
                                                                         264 \else
                                                                                                       \expandafter\BIC@Space
                                                                         265
                                                                          266
                                                                                                       \expandafter#1%
                                                                          267 \fi
                                                                         268 }
\bigintcalcSgn
                                                                         269 \def\bigintcalcSgn#1{%
                                                                         270 \number
                                                                                               \expandafter\expandafter\expandafter\BIC@Sgn
                                                                        272 \bigintcalcNum{#1}! %
                                                                         273 }
                          \BIC@Sgn
                                                                         274 \def\BIC@Sgn#1#2!{%
                                                                         275 \ifx#1-%
                                                                         276
                                                                                                       -1%
                                                                         277 \else
                                                                                                  \ifx#10%
                                                                          278
                                                                          279
                                                                                                              0%
                                                                                                 \else
                                                                         280
                                                                                                            1%
                                                                         281
                                                                                                 \fi
                                                                         282
                                                                        283 \fi
                                                                        284 }
                                                                                                Cmp, Min, Max
                                                                      2.9
\bigintcalcCmp
                                                                         285 \def\bigintcalcCmp#1#2{%
                                                                         286 \number
                                                                         {\tt 287} \quad \verb|\expandafter\expandafter\expandafter\expandafter\arguments| 287}
                                                                         288 \bigintcalcNum{#2}!{#1}%
                                                                         289 }
                          \BIC@Cmp
                                                                          290 \def\BIC@Cmp#1!#2{%
                                                                          291 \expandafter\expandafter\BIC@Cmp
                                                                        292 \bigintcalcNum{#2}!#1!%
                                                                         293 }
                      \BIC@@Cmp
                                                                          294 \def\BIC@@Cmp#1#2!#3#4!{%
                                                                          295 \ifx#1-%
                                                                         296 \ifx#3-%
```

```
\BIC@AfterFiFi{%
                  297
                  298
                              \BIC@@Cmp#4!#2!%
                  299
                  300
                         \else
                  301
                           \BIC@AfterFiFi{%
                  302
                             -1 %
                  303
                           }%
                         \fi
                  304
                       \else
                  305
                         \ifx#3-%
                  306
                           \BIC@AfterFiFi{%
                  307
                  308
                             1 %
                  309
                           }%
                  310
                         \else
                  311
                            \BIC@AfterFiFi{%
                  312
                              \BIC@CmpLength#1#2!#3#4!#1#2!#3#4!%
                           }%
                  313
                         \fi
                  314
                       \BIC@Fi
                 315
                 316 }
   \BIC@PosCmp
                  317 \def\BIC@PosCmp#1!#2!{%
                      \BIC@CmpLength#1!#2!#1!#2!%
                  319 }
\BIC@CmpLength
                  320 \def\BIC@CmpLength#1#2!#3#4!{%
                       \ifx\\#2\\%
                  322
                         \int x^{\#4}\
                  323
                           \BIC@AfterFiFi\BIC@CmpDiff
                  324
                         \else
                           \BIC@AfterFiFi{%
                  325
                              \BIC@CmpResult{-1}%
                  326
                           }%
                  327
                         \fi
                  328
                       \else
                  329
                         \ifx\\#4\\%
                  330
                  331
                            \BIC@AfterFiFi{%
                  332
                              \BIC@CmpResult1%
                           }%
                  333
                  334
                         \else
                           \BIC@AfterFiFi{%
                  335
                              \BIC@CmpLength#2!#4!%
                 336
                           }%
                  337
                         \fi
                  338
                       \BIC@Fi
                  339
                 340 }
\BIC@CmpResult
                  341 \def\BIC@CmpResult#1#2!#3!{#1 }
  \BIC@CmpDiff
                  342 \def\BIC@CmpDiff#1#2!#3#4!{%
                      \ifnum#1<#3 %
                  343
                         \BIC@AfterFi{%
                  344
                           -1 %
                  345
                         }%
                  346
                  347
                       \else
                         \ifnum#1>#3 %
                  348
                           \BIC@AfterFiFi{%
                  349
                              1 %
                  350
```

```
}%
                 351
                 352
                        \else
                          \ifx\\#2\\%
                 353
                            \BIC@AfterFiFiFi{%
                 354
                 355
                              0 %
                            }%
                 356
                 357
                          \else
                            \BIC@AfterFiFiFi{%
                 358
                              \BIC@CmpDiff#2!#4!%
                 359
                            }%
                 360
                 361
                          \fi
                        \fi
                 362
                      \BIC@Fi
                 363
                 364 }
\bigintcalcMin
                 365 \def\bigintcalcMin#1{%
                     \romannumeral0%
                      \expandafter\expandafter\BIC@MinMax
                 367
                 368
                      \bigintcalcNum{#1}!-!%
                 369 }
\bigintcalcMax
                 370 \def\bigintcalcMax#1{%
                 371
                      \romannumeral0%
                      \expandafter\expandafter\BIC@MinMax
                 372
                 373
                      \bigintcalcNum{#1}!!%
                 374 }
   \BIC@MinMax
               #1: x
                #2: sign for comparison
                #3: y
                 375 \def\BIC@MinMax#1!#2!#3{%
                 376 \expandafter\expandafter\expandafter\BIC@@MinMax
                      \bigintcalcNum{#3}!#1!#2!%
                 377
                 378 }
 \BIC@@MinMax #1: y
                #2: x
                #3: sign for comparison
                 379 \def\BIC@@MinMax#1!#2!#3!{%
                     \ifnum\BIC@@Cmp#1!#2!=#31 %
                 380
                 381
                        \BIC@AfterFi{ #1}%
                 382
                      \else
                        \BIC@AfterFi{ #2}%
                 383
                 384
                      \BIC@Fi
                 385 }
                2.10
                       Odd
\bigintcalcOdd
                 386 \def\bigintcalcOdd#1{%
                      \romannumeral0%
                      \expandafter\expandafter\BIC@Odd
                 389
                      \bigintcalcAbs{#1}!%
                 390 }
\BigIntCalcOdd
                 391 \def\BigIntCalcOdd#1!{%
                 392
                     \romannumeral0%
                      \BIC@Odd#1!%
                 393
                 394 }
```

```
\BIC@Odd #1: x
                395 \def\BIC@Odd#1#2{%
                396 \ifx#2!%
                397
                      \ifodd#1 %
                398
                         \BIC@AfterFiFi{ 1}%
                399
                      \else
                        \BIC@AfterFiFi{ 0}%
                400
                       \fi
                401
                402 \else
                       \expandafter\BIC@Odd\expandafter#2%
                403
                404 \BIC@Fi
                405 }
               2.11 Inc, Dec
\bigintcalcInc
                406 \def\bigintcalcInc#1{%
                    \romannumeral0%
                407
                     \expandafter\expandafter\expandafter\BIC@IncSwitch
                408
                409
                     \bigintcalcNum{#1}!%
                410 }
\BIC@IncSwitch
                411 \def\BIC@IncSwitch#1#2!{%
                412 \ifcase\BIC@@Cmp#1#2!-1!%
                      \BIC@AfterFi{ 0}%
                413
                414 \or
                       \BIC@AfterFi{%
                415
                416
                        \BIC@Inc#1#2!{}%
                      }%
                417
                418 \else
                419
                       \BIC@AfterFi{%
                420
                         \expandafter-\romannumeral0%
                421
                         \BIC@Dec#2!{}%
                       }%
                422
                423 \BIC@Fi
                424 }
\bigintcalcDec
                425 \def\bigintcalcDec#1{%
                426 \romannumeral0%
                427 \expandafter\expandafter\expandafter\BIC@DecSwitch
                     \bigintcalcNum{#1}!%
                428
                429 }
\BIC@DecSwitch
                430 \def\BIC@DecSwitch#1#2!{%
                431 \ifcase\BIC@Sgn#1#2! %
                432
                      \BIC@AfterFi{ -1}%
                433 \or
                434 \BIC@AfterFi{%
                        \BIC@Dec#1#2!{}%
                435
                       }%
                436
                    \else
                437
                       \BIC@AfterFi{%
                438
                         \expandafter-\romannumeral0%
                439
                440
                         \BIC@Inc#2!{}%
                       }%
                441
                     \BIC@Fi
                442
                443 }
```

```
\BigIntCalcInc
                 444 \def\BigIntCalcInc#1!{%
                      \romannumeral0\BIC@Inc#1!{}%
                 445
                 446 }
\BigIntCalcDec
                 447 \def\BigIntCalcDec#1!{%
                 448 \romannumeral0\BIC@Dec#1!{}%
                 449 }
      \BIC@Inc
                 450 \def\BIC@Inc#1#2!#3{%
                 451 \ifx\\#2\\%
                        \BIC@AfterFi{%
                 452
                 453
                           \BIC@@Inc1#1#3!{}%
                        }%
                 454
                 455
                     \else
                        \BIC@AfterFi{%
                 456
                 457
                           \BIC@Inc#2!{#1#3}%
                 458
                        }%
                      \BIC@Fi
                 459
                 460 }
     \BIC@@Inc
                 461 \def\BIC@@Inc#1#2#3!#4{%
                 462 \ifcase#1 %
                 463
                        \ifx\\#3\\%
                          \BIC@AfterFiFi{ #2#4}%
                 464
                 465
                        \else
                          \BIC@AfterFiFi{%
                 466
                            \BIC@@IncO#3!{#2#4}%
                 467
                          }%
                 468
                        \fi
                 469
                 470
                      \else
                 471
                        \ifnum#2<9 %
                 472
                          \BIC@AfterFiFi{%
                 473 &
                             \expandafter\BIC@@@Inc\the\numexpr#2+1\relax
                 474 $
                             \expandafter\expandafter\BIC@@@Inc
                             \ifcase#2 \expandafter1%
                 475 $
                             \or\expandafter2%
                 476 $
                 477 $
                             \or\expandafter3%
                             \or\expandafter4%
                 478 $
                 479 $
                             \or\expandafter5%
                 480 $
                             \or\expandafter6%
                 481 $
                             \or\expandafter7%
                 482 $
                             \or\expandafter8%
                 483 $
                             \or\expandafter9%
                 484 $?
                             \else\BigIntCalcError:ThisCannotHappen%
                 485 $
                             \fi
                 486
                             0#3!{#4}%
                 487
                          }%
                 488
                         \else
                           \BIC@AfterFiFi{%
                 489
                             \BIC@@@IncO1#3!{#4}%
                 490
                          }%
                 491
                 492
                         \fi
                      \BIC@Fi
                 493
                 494 }
    \BIC@@@Inc
                 495 \def\BIC@@@Inc#1#2#3!#4{%
                 496 \ifx\\#3\\%
```

```
\ifnum#2=1 %
            497
                      \BIC@AfterFiFi{ 1#1#4}%
            498
                    \else
            499
            500
                      \BIC@AfterFiFi{ #1#4}%
            501
                    \fi
            502
                 \else
            503
                    \BIC@AfterFi{%
                      \BIC@@Inc#2#3!{#1#4}%
            504
                    }%
            505
                 \BIC@Fi
            506
            507 }
 \BIC@Dec
            508 \def\BIC@Dec#1#2!#3{%
            509 \ifx\\#2\\%
                    \BIC@AfterFi{%
            510
                      \BIC@@Dec1#1#3!{}%
            511
                    }%
            512
            513
                \else
                    \BIC@AfterFi{%
            514
                      \BIC@Dec#2!{#1#3}%
            515
            516
                    }%
            517
                 \BIC@Fi
            518 }
\BIC@@Dec
            519 \def\BIC@@Dec#1#2#3!#4{%
                 \ifcase#1 %
            521
                    \ifx\\#3\\%
            522
                      \BIC@AfterFiFi{ #2#4}%
            523
                    \else
            524
                      \BIC@AfterFiFi{%
                        \BIC@@Dec0#3!{#2#4}%
            525
                      }%
            526
                    \fi
            527
                 \else
            528
                    \ifnum#2>0 %
            529
                      \BIC@AfterFiFi{%
            530
            531 &
                        \expandafter\BIC@@Dec\the\numexpr#2-1\relax
            532 $
                        \expandafter\expandafter\expandafter\BIC@@@Dec
            533 $
                          \BigIntCalcError:ThisCannotHappen%
            534 $?
            535 $
                        \or\expandafter0%
            536 $
                        \or\expandafter1%
            537 $
                        \or\expandafter2%
            538 $
                        \or\expandafter3%
            539 $
                        \or\expandafter4%
                        \or\expandafter5%
            540 $
            541 $
                        \or\expandafter6%
            542 $
                        \or\expandafter7%
            543 $
                        \or\expandafter8%
            544 $?
                        \else\BigIntCalcError:ThisCannotHappen%
            545 $
                        \fi
                        0#3!{#4}%
            546
                      }%
            547
                    \else
            548
            549
                      \BIC@AfterFiFi{%
                        \BIC@@@Dec91#3!{#4}%
            550
                      }%
            551
            552
                    \fi
            553
                 \BIC@Fi
            554 }
```

```
\BIC@@@Dec
```

```
555 \def\BIC@@@Dec#1#2#3!#4{%
556 \ifx\\#3\\%
557
      \ifcase#1 %
558
         \ifx\\#4\\%
          \BIC@AfterFiFiFi{ 0}%
559
         \else
560
           \BIC@AfterFiFiFi{ #4}%
561
         \fi
562
563
      \else
564
        \BIC@AfterFiFi{ #1#4}%
565
       \fi
566
    \else
567
       \BIC@AfterFi{%
568
         \BIC@@Dec#2#3!{#1#4}%
       }%
569
    \BIC@Fi
570
571 }
```

2.12 Add, Sub

\bigintcalcAdd

```
572 \def\bigintcalcAdd#1{%
573 \romannumeral0%
574 \expandafter\expandafter\BIC@Add
575 \bigintcalcNum{#1}!%
576 }
```

\BIC@Add

```
577 \def\BIC@Add#1!#2{%

578 \expandafter\expandafter\expandafter

579 \BIC@AddSwitch\bigintcalcNum{#2}!#1!%

580 }
```

\bigintcalcSub

```
581 \def\bigintcalcSub#1#2{%
582 \romannumeral0%
583 \expandafter\expandafter\expandafter\BIC@Add
584 \bigintcalcNum{-#2}!{#1}%
585 }
```

\BIC@AddSwitch Decision table for \BIC@AddSwitch.

x < 0	y < 0	-x > -y	_	Add $(-x, -y)$
		else		Add(-y, -x)
	else	-x > y	_	Sub(-x,y)
		-x = y		0
		else	+	Sub(y, -x)
else	y < 0	x > -y	+	Sub(x, -y)
		x = -y		0
		else	_	Sub(-y,x)
	else	x > y	+	Add(x,y)
		else		Add(y,x)

```
586 \def\BIC@AddSwitch#1#2!#3#4!{%
587 \ifx#1-% x < 0
588 \ifx#3-% y < 0
589 \expandafter-\romannumeral0%
590 \ifnum\BIC@PosCmp#2!#4!=1 % -x > -y
591 \BIC@AfterFiFiFi{%
592 \BIC@AddXY#2!#4!!!%
```

```
593
                               }%
                             \else % -x <= -y
                  594
                               \BIC@AfterFiFiFi{%
                  595
                                 \BIC@AddXY#4!#2!!!%
                  596
                  597
                               }%
                  598
                            \fi
                  599
                           \else % y >= 0
                            \label{eq:condition} $$ \left( \frac{y}{x} - x = y \right) $$
                  600
                               \BIC@AfterFiFiFi{ 0}%
                  601
                             602
                               \expandafter-\romannumeral0%
                  603
                               \BIC@AfterFiFiFi{%
                  604
                                 \BIC@SubXY#2!#3#4!!!%
                  605
                               }%
                  606
                  607
                             \else % -x <= y
                  608
                               \BIC@AfterFiFiFi{%
                                 \BIC@SubXY#3#4!#2!!!%
                  609
                               }%
                  610
                  611
                            \fi
                          \fi
                  612
                        \leq % x >= 0
                  613
                          \int x#3-% y < 0
                  614
                             \label{eq:linear_problem} \label{eq:linear_problem} $$ \left( \frac{BIC@PosCmp#1#2!#4!}{x} = -y \right) $$
                  615
                               \BIC@AfterFiFiFi{ 0}%
                  616
                             \or % x > -y
                  617
                               \BIC@AfterFiFiFi{%
                  618
                  619
                                 \BIC@SubXY#1#2!#4!!!%
                               }%
                  620
                            \else % x \le -y
                  621
                               \expandafter-\romannumeral0%
                  622
                               \BIC@AfterFiFiFi{%
                  623
                  624
                                 \BIC@SubXY#4!#1#2!!!%
                               }%
                  625
                            \fi
                  626
                  627
                           \else % y >= 0
                  628
                             629
                               \BIC@AfterFiFiFi{%
                                 \BIC@AddXY#1#2!#3#4!!!%
                  630
                               }%
                  631
                             \else % x <= y
                  632
                               \BIC@AfterFiFiFi{%
                  633
                                 \BIC@AddXY#3#4!#1#2!!!%
                  634
                  635
                               }%
                  636
                             \fi
                  637
                           \fi
                  638
                        \BIC@Fi
                  639 }
\BigIntCalcAdd
                  640 \def\BigIntCalcAdd#1!#2!{%
                       \romannumeral0\BIC@AddXY#1!#2!!!%
                  641
                  642 }
\BigIntCalcSub
                  643 \def\BigIntCalcSub#1!#2!{%
                  644 \romannumeral0\BIC@SubXY#1!#2!!!%
                  645 }
    \BIC@AddXY
                  646 \def\BIC@AddXY#1#2!#3#4!#5!#6!{%
                  647 \ifx\\#2\\%
                          \ifx\\#3\\%
                  648
```

```
\BIC@AfterFiFi{%
                  649
                               \BIC@DoAdd0!#1#5!#60!%
                  650
                            }%
                  651
                  652
                          \else
                  653
                             \BIC@AfterFiFi{%
                  654
                               \BIC@DoAdd0!#1#5!#3#6!%
                  655
                            }%
                          \fi
                  656
                  657
                        \else
                          \int \frac{\pi}{\pi} \frac{4}{\pi}
                  658
                            \ifx\\#3\\%
                  659
                               \BIC@AfterFiFiFi{%
                  660
                                 \BIC@AddXY#2!{}!#1#5!#60!%
                  661
                               }%
                  662
                  663
                             \else
                  664
                               \BIC@AfterFiFiFi{%
                                 \BIC@AddXY#2!{}!#1#5!#3#6!%
                  665
                               }%
                  666
                             \fi
                  667
                  668
                          \else
                             \BIC@AfterFiFi{%
                  669
                               \BIC@AddXY#2!#4!#1#5!#3#6!%
                  670
                            }%
                  671
                          \fi
                  672
                  673
                        \BIC@Fi
                  674 }
    \BIC@DoAdd
                 #1: carry
                 #2: reverted result
                  #3#4: reverted x
                 #5#6: reverted y
                  675 \def\BIC@DoAdd#1#2!#3#4!#5#6!{%
                        \int \frac{\pi}{\pi} \frac{4}{\pi}
                  677
                          \BIC@AfterFi{%
                  678 &
                             \expandafter\BIC@Space
                  679 &
                             \theta = 1+#3+#5\relax#2%
                             \expandafter\expandafter\expandafter\BIC@AddResult
                  680 $
                  681 $
                             \BIC@AddDigit#1#3#5#2%
                  682
                          }%
                        \else
                  683
                  684
                          \BIC@AfterFi{%
                             \expandafter\expandafter\BIC@DoAdd
                  685
                  686
                             \BIC@AddDigit#1#3#5#2!#4!#6!%
                  687
                          }%
                  688
                        \BIC@Fi
                  689 }
\BIC@AddResult
                  690 $ \def\BIC@AddResult#1{%
                  691 $
                          \ifx#10%
                  692 $
                             \expandafter\BIC@Space
                  693 $
                             \expandafter\BIC@Space\expandafter#1%
                  694 $
                  695 $
                          \fi
                  696 $ }%
 \BIC@AddDigit #1: carry
                  #2: digit of x
                 #3: digit of y
                  697 \def\BIC@AddDigit#1#2#3{\%
                       \romannumeral0%
                  699 & \expandafter\BIC@@AddDigit\the\numexpr#1+#2+#3!\%
```

```
700 $ \expandafter\BIC@@AddDigit\number%
                                                701 $ \csname
                                                               BIC@AddCarry%
                                                702 $
                                                703 $
                                                                \ifcase#1 %
                                                704 $
                                                                    #2%
                                                705 $
                                                                \else
                                                706 $
                                                                    \footnote{Moreover the continuous of the conti
                                                707 $
                                                               \fi
                                               708 $ \endcsname#3!%
                                                709 }
         \BIC@@AddDigit
                                                710 \def\BIC@@AddDigit#1!{%
                                                711 \ifnum#1<10 %
                                                                \BIC@AfterFi{ 0#1}%
                                                712
                                                         \else
                                               713
                                                               \BIC@AfterFi{ #1}%
                                                714
                                                715 \BIC@Fi
                                               716 }
         \BIC@AddCarry0
                                                \BIC@AddCarry10
                                                718 $\expandafter\def\csname BIC@AddCarry10\endcsname#1{1#1}%
\BIC@AddCarry[1-9]
                                               719 $ \def\BIC@Temp#1#2{%
                                                                \expandafter\def\csname BIC@AddCarry#1\endcsname##1{%
                                                720 $
                                                721 $
                                                                    \ifcase##1 #1\or
                                                722 $
                                                723 $?
                                                                     \else\BigIntCalcError:ThisCannotHappen%
                                                724 $
                                                                     \fi
                                                725 $
                                                               }%
                                                726 $ }%
                                                727 $ \BIC@Temp 0{1\or2\or3\or4\or5\or6\or7\or8\or9}%
                                               729 $ \BIC@Temp 2{3\or4\or5\or6\or7\or8\or9\or10\or11}%
                                               730 $ \BIC@Temp 3{4\or5\or6\or7\or8\or9\or10\or11\or12}%
                                               731 $ \BIC@Temp 4{5\or6\or7\or8\or9\or10\or11\or12\or13}%
                                                732 $ \BIC@Temp 5{6\or7\or8\or9\or10\or11\or12\or13\or14}%
                                                733 $ \BIC@Temp 6{7\or8\or9\or10\or11\or12\or13\or14\or15}%
                                                734 $ \BIC@Temp 7{8\or9\or10\or11\or12\or13\or14\or15\or16}%
                                                735 $ \BIC@Temp 8{9\or10\or11\or12\or13\or14\or15\or16\or17}%
                                                736 $ \BIC@Temp 9{10\or11\or12\or13\or14\or15\or16\or17\or18}%
                  \BIC@SubXY Preconditions:
                                                    • x > y, x \ge 0, \text{ and } y >= 0
                                                    • digits(x) = digits(y)
                                                737 \def\BIC@SubXY#1#2!#3#4!#5!#6!{%
                                                         \ifx\\#2\\%
                                                738
                                                                \ifx\\#3\\%
                                                739
                                                740
                                                                     \BIC@AfterFiFi{%
                                                741
                                                                         \BIC@DoSub0!#1#5!#60!%
                                                742
                                                                    }%
                                                743
                                                                \else
                                                                    \BIC@AfterFiFi{%
                                                744
                                                                         \BIC@DoSub0!#1#5!#3#6!%
                                               745
                                                                    }%
                                                746
                                                747
                                                                \fi
```

```
\else
                                                 748
                                                                     \ifx\\#4\\%
                                                 749
                                                                           \ifx\\#3\\%
                                                 750
                                                                                 \BIC@AfterFiFiFi{%
                                                 751
                                                 752
                                                                                       \BIC@SubXY#2!{}!#1#5!#60!%
                                                                                 }%
                                                 753
                                                 754
                                                                            \else
                                                                                 \BIC@AfterFiFiFi{%
                                                 755
                                                                                       \BIC@SubXY#2!{}!#1#5!#3#6!%
                                                 756
                                                                                 }%
                                                 757
                                                                           \fi
                                                 758
                                                                      \else
                                                 759
                                                                            \BIC@AfterFiFi{%
                                                 760
                                                 761
                                                                                 \BIC@SubXY#2!#4!#1#5!#3#6!%
                                                 762
                                                 763
                                                                      \fi
                                                                \BIC@Fi
                                                 764
                                                 765 }
           \BIC@DoSub #1: carry
                                              #2: reverted result
                                              #3#4: reverted x
                                              #5#6: reverted y
                                                 766 \def\BIC@DoSub#1#2!#3#4!#5#6!{%
                                                               \ifx\\#4\\%
                                                 767
                                                                     \BIC@AfterFi{%
                                                 768
                                                                            \verb|\expandafter| expandafter| BIC@SubResult|
                                                 769
                                                                            \BIC@SubDigit#1#3#5#2%
                                                 770
                                                                     }%
                                                 771
                                                 772
                                                               \else
                                                 773
                                                                      \BIC@AfterFi{%
                                                                            \expandafter\expandafter\BIC@DoSub
                                                 774
                                                 775
                                                                            \BIC@SubDigit#1#3#5#2!#4!#6!%
                                                 776
                                                                      }%
                                                 777
                                                                \BIC@Fi
                                                 778 }
\BIC@SubResult
                                                 779 \def\BIC@SubResult#1{%
                                                              \ifx#10%
                                                 781
                                                                      \expandafter\BIC@SubResult
                                                 782
                                                 783
                                                                      \expandafter\BIC@Space\expandafter#1%
                                                 784
                                                                \fi
                                                 785 }
  \BIC@SubDigit #1: carry
                                              #2: digit of x
                                              #3: digit of y
                                                 786 \ensuremath{\mbox{\sc NBIC@SubDigit#1#2#3}}\%
                                                 787 \romannumeral0%
                                                 788 & \expandafter\BIC@@SubDigit\the\numexpr#2-#3-#1!%
                                                 789 $ \expandafter\BIC@@AddDigit\number
                                                                     \csname
                                                 790 $
                                                 791 $
                                                                           BIC@SubCarry%
                                                 792 $
                                                                           \ifcase#1 %
                                                 793 $
                                                                                 #3%
                                                 794 $
                                                                            \else
                                                                                 \footnote{Missing the continuous of the contin
                                                 795 $
                                                 796 $
                                                                            \fi
                                                                     \endcsname#2!%
                                                 797 $
                                                 798 }
```

```
\BIC@@SubDigit
                     799 & \def\BIC@@SubDigit#1!{%
                     800 &
                           \ifnum#1<0 %
                              \BIC@AfterFi{%
                     801 &
                     802 &
                                \expandafter\BIC@Space
                     803 &
                                \verb|\expandafter1\the\numexpr#1+10\relax| \\
                     804 &
                              }%
                     805 &
                           \else
                              \BIC@AfterFi{ 0#1}%
                     806 &
                           \BIC@Fi
                     807 &
                     808 & }%
    \BIC@SubCarry0
                     809 $\expandafter\def\csname BIC@SubCarryO\endcsname#1{#1}%
   \BIC@SubCarry10
                     810 $\expandafter\def\csname BIC@SubCarry10\endcsname#1{1#1}%
\BIC@SubCarry[1-9]
                     811 $ \def\BIC@Temp#1#2{%
                            \expandafter\def\csname BIC@SubCarry#1\endcsname##1{%
                     812 $
                               \ifcase##1 #2%
                     813 $
                     814 $?
                               \else\BigIntCalcError:ThisCannotHappen%
                     815 $
                               \fi
                            }%
                     816 $
                     817 $ }%
                     818 $\BIC@Temp 1{19\or0\or1\or2\or3\or4\or5\or6\or7\or8}%
                     819 $ \BIC@Temp 2{18\or19\or0\or1\or2\or3\or4\or5\or6\or7}%
                     820 $ \BIC@Temp 3{17\or18\or19\or0\or1\or2\or3\or4\or5\or6}%
                     821 $ \BIC@Temp 4{16\or17\or18\or19\or0\or1\or2\or3\or4\or5}%
                     822 $ \BIC@Temp 5{15\or16\or17\or18\or19\or0\or1\or2\or3\or4}%
                     \$23 \ \BIC@Temp 6{14\or15\or16\or17\or18\or19\or0\or1\or2\or3}%
                     \$24 \ \BIC@Temp 7{13\or14\or15\or16\or17\or18\or19\or0\or1\or2}%
                     \$25 \ \BIC@Temp \$\{12\or13\or14\or15\or16\or17\or18\or19\or0\or1\}\%
                     826 $ \BIC@Temp 9{11\or12\or13\or14\or15\or16\or17\or18\or19\or0}%
                    2.13 Shl, Shr
    \bigintcalcShl
                     827 \def\bigintcalcShl#1{%
                     828 \romannumeral0%
                          \expandafter\expandafter\BIC@Shl
                     829
                     830
                          \bigintcalcNum{#1}!%
                     831 }
          \BIC@Shl
                     832 \def\BIC@Shl#1#2!{%
                     833 \ifx#1-%
                            \BIC@AfterFi{%
                     834
                              \expandafter-\romannumeral0%
                     835
                              \BIC@@Sh1#2!!%
                     836 &
                     837 $
                              \BIC@AddXY#2!#2!!!%
                            }%
                     838
                     839 \else
                            \BIC@AfterFi{%
                     840
                     841 &
                              \BIC@@Shl#1#2!!%
                     842 $
                              \BIC@AddXY#1#2!#1#2!!!%
                            }%
                     843
                     844 \BIC@Fi
                     845 }
```

```
\BigIntCalcShl
                 846 \def\BigIntCalcShl#1!{%
                 847 \romannumeral0%
                 848 & \BIC@@Shl#1!!%
                 849 $ \BIC@AddXY#1!#1!!!%
                 850 }
     \BIC@@Shl
                 851 & \def\BIC@@Shl#1#2!{%
                 852 & \ifx\\#2\\%
                        \BIC@AfterFi{%
                 853 &
                           \BIC@@@Shl0!#1%
                 854 &
                          }%
                 855 &
                       \else
                 856 &
                 857 &
                         \BIC@AfterFi{%
                 858 &
                            \BIC@@Sh1#2!#1%
                 859 &
                          }%
                       \BIC@Fi
                 860 &
                 861 & }%
   \BIC@@@Shl #1: carry
                #2: result
                #3#4: reverted number
                 862 & \def\BIC@@@Shl#1#2!#3#4!{%
                       \ifx\\#4\\%
                 863 &
                 864 &
                          \BIC@AfterFi{%
                 865 &
                            \expandafter\BIC@Space
                            \the\numexpr#3*2+#1\relax#2%
                 866 &
                 867 &
                          }%
                 868 &
                        \else
                 869 &
                          \BIC@AfterFi{%
                 870 &
                            \expandafter\BIC@@@@Shl\the\numexpr#3*2+#1!#2!#4!%
                          }%
                 871 &
                       \BIC@Fi
                 872 &
                 873 & }%
  \BIC@@@@Shl
                 874 & \def\BIC@@@@Shl#1!{%
                       \ifnum#1<10 %
                 875 &
                          \BIC@AfterFi{%
                 876 &
                            \BIC@@@Sh10#1%
                 877 &
                          }%
                 878 &
                 879 &
                        \else
                          \BIC@AfterFi{%
                 880 &
                            \BIC@@@Shl#1%
                 881 &
                 882 &
                          }%
                 883 &
                       \BIC@Fi
                 884 & }%
\bigintcalcShr
                 885 \def\bigintcalcShr#1{%
                     \romannumeral0%
                 887
                      \expandafter\expandafter\BIC@Shr
                 888
                      \bigintcalcNum{#1}!%
                 889 }
      \BIC@Shr
                 890 \def\BIC@Shr#1#2!{%
                     \ifx#1-%
                 891
                        \expandafter-\romannumeral0%
                 892
                        \BIC@AfterFi{%
                 893
```

894

\BIC@@Shr#2!%

```
}%
                  895
                        \else
                  896
                          \BIC@AfterFi{%
                  897
                            \BIC@@Shr#1#2!%
                  898
                  899
                          }%
                  900
                        \BIC@Fi
                  901 }
\BigIntCalcShr
                  902 \def\BigIntCalcShr#1!{%
                       \romannumeral0%
                        \BIC@@Shr#1!%
                  905 }
     \BIC@@Shr
                  906 \def\BIC@@Shr#1#2!{%
                  907
                        \ifcase#1 %
                  908
                          \BIC@AfterFi{ 0}%
                  909
                        \or
                  910
                          \ifx\\#2\\%
                  911
                            \BIC@AfterFiFi{ 0}%
                  912
                          \else
                            \BIC@AfterFiFi{%
                  913
                              \BIC@@@Shr#1#2!!%
                  914
                            }%
                  915
                          \fi
                  916
                  917
                        \else
                          \BIC@AfterFi{%
                  918
                  919
                            \BIC@@@Shr0#1#2!!%
                  920
                          }%
                  921
                        \BIC@Fi
                  922 }
    \BIC@@@Shr
                 #1: carry
                 #2#3: number
                 #4: result
                  923 \def\BIC@@@Shr#1#2#3!#4!{%
                       \ifx\\#3\\%
                  924
                          \ifodd#1#2 %
                  925
                            \BIC@AfterFiFi{%
                  926
                  927 &
                              \verb|\expandafter| BICOShrResult| the \\| numexpr(#1#2-1)/2 \\| relax| 
                              \expandafter\expandafter\expandafter\BIC@ShrResult
                  928 $
                  929 $
                              \csname BIC@ShrDigit#1#2\endcsname
                              #4!%
                  930
                  931
                            }%
                  932
                          \else
                  933
                            \BIC@AfterFiFi{%
                  934 &
                              \verb|\expandafter\BIC@ShrResult\the\numexpr#1#2/2\relax|
                  935 $
                              \expandafter\expandafter\expandafter\BIC@ShrResult
                              \csname BIC@ShrDigit#1#2\endcsname
                  936 $
                              #4!%
                  937
                            }%
                  938
                          \fi
                  939
                        \else
                  940
                  941
                          \ifodd#1#2 %
                  942
                            \BIC@AfterFiFi{%
                              \expandafter\BIC@@@Shr\the\numexpr(#1#2-1)/2\relax1%
                  943 &
                  944 $
                              \expandafter\expandafter\expandafter\BIC@@@Shr
                  945 $
                              \csname BIC@ShrDigit#1#2\endcsname
                              #3!#4!%
                  946
                            }%
                  947
                          \else
                  948
```

```
949
                               \BIC@AfterFiFi{%
                                 \expandafter\BIC@@@Shr\the\numexpr#1#2/2\relax0%
                      950 &
                      951 $
                                 \expandafter\expandafter\BIC@@@@Shr
                      952 $
                                 \csname BIC@ShrDigit#1#2\endcsname
                      953
                                 #3!#4!%
                      954
                               }%
                      955
                             \fi
                           \BIC@Fi
                      956
                      957 }
      \BIC@ShrResult
                      958 & \def\BIC@ShrResult#1#2!{ #2#1}%
                      959 $ \def\BIC@ShrResult#1#2#3!{ #3#1}%
         \BIC@@@@Shr
                     #1: new digit
                     #2: carry
                     #3: remaining number
                     #4: result
                      960 \def\BIC@@@Shr#1#2#3!#4!{%
                      961 \BIC@@@Shr#2#3!#4#1!%
                      962 }
\BIC@ShrDigit[00-19]
                      963 $ \def\BIC@Temp#1#2#3#4{%
                             965 $ }%
                      966 $ \BIC@Temp 0000%
                      967 $ \BIC@Temp 0101%
                      968 $ \BIC@Temp 0210%
                      969 $ \BIC@Temp 0311%
                      970 $ \BIC@Temp 0420%
                      971 $ \BIC@Temp 0521%
                      972 $ \BIC@Temp 0630%
                      973 $ \BIC@Temp 0731%
                      974 $ \BIC@Temp 0840%
                      975 $ \BIC@Temp 0941%
                      976 $ \BIC@Temp 1050%
                      977 $ \BIC@Temp 1151%
                      978 $ \BIC@Temp 1260%
                      979 $ \BIC@Temp 1361%
                      980 $ \BIC@Temp 1470%
                      981 $ \BIC@Temp 1571%
                      982 $ \BIC@Temp 1680%
                      983 $ \BIC@Temp 1781%
                      984 $ \BIC@Temp 1890%
                      985 $ \BIC@Temp 1991%
                     2.14 \setminus BIC@Tim
            \BIC@Tim Macro \BIC@Tim implements "Number times digit".
                      #1: plain number without sign
                     #2: digit
           \BIC@@Tim #1#2: number
                     #3: reverted number
                      986 \def\BIC@@Tim#1#2!{%
                      987
                           \ifx\\#2\\%
                             \BIC@AfterFi{%
                      988
                               \BIC@ProcessTim0!#1%
                      989
                             }%
                      990
                      991 \else
```

```
\BIC@AfterFi{%
                    992
                             \BIC@@Tim#2!#1%
                    993
                    994
                    995
                         \BIC@Fi
                    996 }
 \BIC@ProcessTim
                  #1: carry
                   #2: result
                   #3#4: reverted number
                   #5: digit
                    997 \def\BIC@ProcessTim#1#2!#3#4!#5{%
                         \ifx\\#4\\%
                    998
                           \BIC@AfterFi{%
                    999
                             \expandafter\BIC@Space
                   1000
                   1001 &
                             \theta^3*#5+#1\
                   1002 $
                             \romannumeral0\BIC@TimDigit#3#5#1%
                   1003
                             #2%
                   1004
                           }%
                   1005
                         \else
                           \BIC@AfterFi{%
                   1006
                             \expandafter\BIC@@ProcessTim
                   1007
                             \theta^3*45+41
                   1008 &
                             \romannumeral0\BIC@TimDigit#3#5#1%
                   1009 $
                             !#2!#4!#5%
                   1010
                           }%
                   1011
                   1012
                         \BIC@Fi
                   1013 }
\BIC@@ProcessTim
                  #1#2: carry?, new digit
                   #3: new number
                   #4: old number
                   #5: digit
                   1014 \def\BIC@@ProcessTim#1#2!{%
                   1015
                         \ifx\\#2\\%
                   1016
                           \BIC@AfterFi{%
                   1017
                             \BIC@ProcessTimO#1%
                           }%
                   1018
                         \else
                   1019
                           \BIC@AfterFi{%
                   1020
                             \BIC@ProcessTim#1#2%
                   1021
                           }%
                   1022
                         \BIC@Fi
                   1023
                   1024 }
   \BIC@TimDigit
                  #1: digit 0–9
                   #2: digit 3-9
                   #3: carry 0–9
                   1025 $ \def\BIC@TimDigit#1#2#3{\%
                   1026 $
                           \ifcase#1 % 0
                             \BIC@AfterFi{ #3}%
                   1027 $
                           \or % 1
                   1028 $
                             \BIC@AfterFi{%
                   1029 $
                               \expandafter\BIC@Space
                   1030 $
                                \number\csname BIC@AddCarry#2\endcsname#3 %
                   1031 $
                   1032 $
                             }%
                   1033 $
                           \else
                   1034 $
                             \ifcase#3 %
                   1035 $
                               \BIC@AfterFiFi{%
                                  \expandafter\BIC@Space
                   1036 $
                                  \number\csname BIC@MulDigit#2\endcsname#1 %
                   1037 $
                               }%
                   1038 $
                   1039 $
                             \else
```

```
\BIC@AfterFiFi{%
                   1040 $
                                 \expandafter\BIC@Space
                   1041 $
                                 \romannumeral0%
                   1042 $
                   1043 $
                                 \expandafter\BIC@AddXY
                   1044 $
                                 \number\csname BIC@MulDigit#2\endcsname#1!%
                   1045 $
                                 #3!!!%
                   1046 $
                               }%
                             \fi
                   1047 $
                         \BIC@Fi
                   1048 $
                   1049 $ }%
\BIC@MulDigit[3-9]
                   1050 $ \def\BIC@Temp#1#2{%
                           \expandafter\def\csname BIC@MulDigit#1\endcsname##1{%
                   1052 $
                             \ifcase##1 0%
                             \or ##1%
                   1053 $
                             \or #2%
                   1054 $
                             \else\BigIntCalcError:ThisCannotHappen%
                   1055 $?
                   1056 $
                             \fi
                   1057 $
                           }%
                   1058 $ }%
                   1059 $ \BIC@Temp 3{6\or9\or12\or15\or18\or21\or24\or27}%
                   1060 $ \BIC@Temp 4{8\or12\or16\or20\or24\or28\or32\or36}%
                   1061 $ \BIC@Temp 5{10\or15\or20\or25\or30\or35\or40\or45}%
                   1062 \$ \BIC@Temp 6{12\or18\or24\or30\or36\or42\or48\or54}\%
                   1063 \$ \BIC@Temp 7{14\or21\or28\or35\or42\or49\or56\or63}\%
                   1065 $ \BIC@Temp 9{18\or27\or36\or45\or54\or63\or72\or81}\%
```

2.15 Mul

\bigintcalcMul

```
1066 \def\bigintcalcMul#1#2{%
1067 \romannumeral0%
1068 \expandafter\expandafter\EIC@Mul
1069 \bigintcalcNum{#1}!{#2}%
1070 }
```

\BIC@Mul

```
1071 \def\BIC@Mul#1!#2{%
1072 \expandafter\expandafter\expandafter\BIC@MulSwitch
1073 \bigintcalcNum{#2}!#1!%
1074 }
```

\BIC@MulSwitch Decision table for \BIC@MulSwitch.

x = 0				0
x > 0	y = 0			0
	y > 0	x > y	+	Mul(x, y)
		else		Mul(y, x)
	y < 0	x > -y	_	Mul(x, -y)
		else		Mul(-y,x)
x < 0	y = 0			0
	y > 0	-x > y	_	Mul(-x,y)
		else		Mul(y, -x)
	y < 0	-x > -y	+	Mul(-x, -y)
		else		Mul(-y, -x)

```
1075 \def\BIC@MulSwitch#1#2!#3#4!{%
1076 \ifcase\BIC@Sgn#1#2! % x = 0
1077 \BIC@AfterFi{ 0}%
```

```
\ightharpoonup \ \ifcase\BIC@Sgn#3#4! % y = 0
                                              1079
                                                                        \BIC@AfterFiFi{ 0}%
                                              1080
                                              1081
                                                                    1082
                                                                        1083
                                                                              \BIC@AfterFiFiFi{%
                                              1084
                                                                                    \BIC@ProcessMul0!#1#2!#3#4!%
                                                                              }%
                                              1085
                                                                         \else % x <= y
                                              1086
                                                                              \BIC@AfterFiFiFi{%
                                              1087
                                                                                    \BIC@ProcessMul0!#3#4!#1#2!%
                                              1088
                                                                              }%
                                              1089
                                                                        \fi
                                              1090
                                              1091
                                                                    \else % y < 0
                                              1092
                                                                         \expandafter-\romannumeral0%
                                              1093
                                                                         1094
                                                                              \BIC@AfterFiFiFi{%
                                                                                    \BIC@ProcessMul0!#1#2!#4!%
                                              1095
                                                                              }%
                                              1096
                                              1097
                                                                         \else % x <= -y
                                                                              \BIC@AfterFiFiFi{%
                                              1098
                                                                                    \BIC@ProcessMul0!#4!#1#2!%
                                              1099
                                                                              }%
                                              1100
                                                                        \fi
                                              1101
                                              1102
                                                                    \fi
                                                              \else % x < 0
                                              1103
                                                                   \ightharpoonup \ \if case\BIC@Sgn#3#4! % y = 0
                                              1104
                                                                        \BIC@AfterFiFi{ 0}%
                                              1105
                                              1106
                                                                    \expandafter-\romannumeral0%
                                              1107
                                                                         \infnum\BIC@PosCmp#2!#3#4!=1 % -x > y
                                              1108
                                              1109
                                                                              \BIC@AfterFiFiFi{%
                                              1110
                                                                                    \BIC@ProcessMul0!#2!#3#4!%
                                                                              }%
                                              1111
                                              1112
                                                                         \else % -x <= y
                                              1113
                                                                              \BIC@AfterFiFiFi{%
                                              1114
                                                                                    \BIC@ProcessMul0!#3#4!#2!%
                                                                              }%
                                              1115
                                              1116
                                                                        \fi
                                                                   \else % y < 0
                                              1117
                                                                        \mbox{\colored} \mbox{\color
                                              1118
                                                                              \BIC@AfterFiFiFi{%
                                              1119
                                              1120
                                                                                   \BIC@ProcessMul0!#2!#4!%
                                              1121
                                                                              }%
                                                                         \else % -x <= -y
                                              1122
                                              1123
                                                                              \BIC@AfterFiFiFi{%
                                              1124
                                                                                   \BIC@ProcessMul0!#4!#2!%
                                              1125
                                                                              }%
                                              1126
                                                                        \fi
                                              1127
                                                                    \fi
                                                              \BIC@Fi
                                              1128
                                              1129 }
   \BigIntCalcMul
                                              1130 \def\BigIntCalcMul#1!#2!{%
                                              1131 \romannumeral0%
                                                              \BIC@ProcessMul0!#1!#2!%
                                              1132
                                              1133 }
\BIC@ProcessMul #1: result
                                              #2: number x
                                              #3#4: number y
```

1078

```
1134 \def\BIC@ProcessMul#1!#2!#3#4!{%
                      \ifx\\#4\\%
                1135
                        \BIC@AfterFi{%
                1136
                1137
                          \expandafter\expandafter\BIC@Space
                1138
                          \bigintcalcAdd{\BIC@Tim#2!#3}{#10}%
                1139
                        }%
                1140
                      \else
                        \verb|\BIC@AfterFi|{%}|
                1141
                          \verb|\expandafter| expandafter | BIC@ProcessMul| \\
                1142
                          \bigintcalcAdd{\BIC@Tim#2!#3}{#10}!#2!#4!%
                1143
                        }%
                1144
                      \BIC@Fi
                1145
                1146 }
                2.16 Sqr
\bigintcalcSqr
                1147 \def\bigintcalcSqr#1{%
                1148 \romannumeral0%
                1149
                      \expandafter\expandafter\expandafter\BIC@Sqr
                1150
                      \bigintcalcNum{#1}!%
                1151 }
      \BIC@Sqr
                1152 \def\BIC@Sqr#1{%
                1153
                       \ifx#1-%
                         \expandafter\BIC@@Sqr
                1154
                1155
                       \else
                1156
                         \expandafter\BIC@@Sqr\expandafter#1%
                1157
                       \fi
                1158 }
     \BIC@@Sqr
                1159 \def\BIC@@Sqr#1!{%
                1160 \BIC@ProcessMul0!#1!#1!%
                1161 }
                2.17 Fac
\bigintcalcFac
                1162 \def\bigintcalcFac#1{%
                1163
                      \romannumeral0%
                1164
                      \expandafter\expandafter\BIC@Fac
                1165
                      \bigintcalcNum{#1}!%
                1166 }
      \BIC@Fac
                1167 \def\BIC@Fac#1#2!{%
                1168
                     \ifx#1-%
                        \BIC@AfterFi{ 0\BigIntCalcError:FacNegative}%
                1169
                      \else
                1170
                        \ifnum\BIC@PosCmp#1#2!13!<0 %
                1171
                          \ifcase#1#2 %
                1172
                             \BIC@AfterFiFiFi{ 1}% 0!
                1173
                          \or\BIC@AfterFiFiFi{ 1}% 1!
                1174
                1175
                          \or\BIC@AfterFiFiFi{ 2}% 2!
                1176
                          \or\BIC@AfterFiFiFi{ 6}% 3!
                1177
                          \or\BIC@AfterFiFiFi{ 24}% 4!
                1178
                          \or\BIC@AfterFiFiFi{ 120}% 5!
                          \or\BIC@AfterFiFiFi{ 720}% 6!
                1179
                          \or\BIC@AfterFiFiFi{ 5040}% 7!
                1180
```

```
\or\BIC@AfterFiFiFi{ 40320}% 8!
                  1181
                             \or\BIC@AfterFiFiFi{ 362880}% 9!
                  1182
                             \or\BIC@AfterFiFiFi{ 3628800}% 10!
                  1183
                             \or\BIC@AfterFiFiFi{ 39916800}% 11!
                  1184
                  1185
                             \or\BIC@AfterFiFiFi{ 479001600}% 12!
                  1186 ?
                             \else\BigIntCalcError:ThisCannotHappen%
                            \fi
                  1187
                  1188
                           \else
                             \BIC@AfterFiFi{%
                  1189
                               \BIC@ProcessFac#1#2!479001600!%
                  1190
                            }%
                  1191
                  1192
                          \fi
                        \BIC@Fi
                  1193
                  1194 }
\BIC@ProcessFac #1: n
                  #2: result
                  1195 \def\BIC@ProcessFac#1!#2!{%
                  1196 \ifnum\BIC@PosCmp#1!12!=0 %
                          \BIC@AfterFi{ #2}%
                  1197
                       \else
                  1198
                          \BIC@AfterFi{%
                  1199
                             \expandafter\BIC@@ProcessFac
                  1200
                             \romannumeral0\BIC@ProcessMul0!#2!#1!%
                  1201
                  1202
                             !#1!%
                  1203
                          }%
                  1204
                       \BIC@Fi
                  1205 }
\BIC@@ProcessFac #1: result
                  #2: n
                  1206 \def\BIC@@ProcessFac#1!#2!{%
                  1207 \expandafter\BIC@ProcessFac
                  1208 \romannumeral0\BIC@Dec#2!{}%
                  1209
                       !#1!%
                  1210 }
                  2.18 Pow
  \bigintcalcPow #1: basis
                  #2: power
                  1211 \def\bigintcalcPow#1{%
                  1212 \romannumeral0%
                  1213 \verb| \expandafter\expandafter\expandafter\expandafter\\ |
                  1214 \bigintcalcNum{#1}!%
                  1215 }
        \BIC@Pow #1: basis
                  #2: power
                  1216 \def\BIC@Pow#1!#2{%
                  1217 \expandafter\expandafter\expandafter\BIC@PowSwitch
                  1218 \bigintcalcNum{#2}!#1!%
                  1219 }
  \BIC@PowSwitch #1#2: power y
                  #3#4: basis x
                  Decision table for \BIC@PowSwitch.
```

y = 0			1
y=1			x
y=2	x < 0		Mul(-x, -x)
	else		Mul(x,x)
y < 0	x = 0		DivisionByZero
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	else $(x > 1)$		0
y > 2	x = 0		0
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	$x < -1 \ (x < 0)$	ifodd(y)	$-\operatorname{Pow}(-x,y)$
		else	Pow(-x,y)
	else $(x > 1)$		Pow(x, y)

```
1220 \def\BIC@PowSwitch#1#2!#3#4!{%
     \left( \frac{1}{2}\right) 
1221
               \int x#100 \% y = 0
1222
               \left| y = 1 \right|
1223
1224
               \left| y \right| = 2
1225
               \else4 % y > 2
1226
               \fi\fi\fi
1227
             \else
               \int 1-3 \% y < 0
1228
               \left( y > 2 \right)
1229
               \fi
1230
             \fi
1231
        \BIC@AfterFi{ 1}% y = 0
1232
1233
      \ \ y = 1
        \BIC@AfterFi{ #3#4}%
1234
1235
      \ \ y = 2
        1236
          \BIC@AfterFiFi{%
1237
            \BIC@ProcessMul0!#4!#4!%
1238
          }%
1239
        \leq % x >= 0
1240
          \BIC@AfterFiFi{%
1241
            \BIC@ProcessMul0!#3#4!#3#4!%
1242
1243
          }%
1244
        \fi
1245
      \or % y < 0
1246
        \left( \frac{1}{x}\right) 
1247
                 \int x#300 % x = 0
1248
                 \left( x = 1 \right)
1249
                 \else3 % x > 1
1250
                 \fi\fi
               \else
1251
                 \ifcase\BIC@MinusOne#3#4! %
1252
1253
                   3 \% |x| > 1
1254
                 \or
                   2 \% x = -1
1255
1256 ?
                 \else\BigIntCalcError:ThisCannotHappen%
1257
                 \fi
1258
               \fi
1259
          \BIC@AfterFiFi{ 0\BigIntCalcError:DivisionByZero}% x = 0
        1260
          \BIC@AfterFiFi{ 1}% x = 1
1261
        1262
1263
          \ifcase\BIC@ModTwo#2! % even(y)
```

```
\BIC@AfterFiFiFi{ 1}%
1264
          \or % odd(y)
1265
            \BIC@AfterFiFiFi{ -1}%
1266
          \else\BigIntCalcError:ThisCannotHappen%
1267 ?
1268
1269
        1270
          \BIC@AfterFiFi{ 0}%
1271 ?
        \else\BigIntCalcError:ThisCannotHappen%
1272
        \fi
      1273
        \left( \frac{1}{2} \right)^{4}
1274
                  \int ifx#300 \% x = 0
1275
                  \left( x = 1 \right)
1276
                  \left( x > 1 \right)
1277
                  \fi\fi
1278
1279
                \else
                  \ifx#3-%
1280
                    \ifcase\BIC@MinusOne#3#4! %
1281
                      3 \% x < -1
1282
1283
                    \else
                      2 \% x = -1
1284
                    \fi
1285
                  \else
1286
                    4 \% x > 1
1287
1288
1289
                \fi
          BIC@AfterFiFi{ 0}% x = 0
1290
1291
        \operatorname{\ \ \ \ \ \ \ \ \ \ } 1
1292
          BIC@AfterFiFi{ 1}% x = 1
1293
        \or \% x = -1
          \ifcase\BIC@ModTwo#1#2! % even(y)
1294
1295
            \BIC@AfterFiFiFi{ 1}%
          \or % odd(y)
1296
             \BIC@AfterFiFiFi{ -1}%
1297
          \else\BigIntCalcError:ThisCannotHappen%
1298 ?
1299
          \fi
1300
        \ifcase\BIC@ModTwo#1#2! % even(y)
1301
1302
            \BIC@AfterFiFiFi{%
               \BIC@PowRec#4!#1#2!1!%
1303
            }%
1304
          \or % odd(y)
1305
             \expandafter-\romannumeral0%
1306
1307
             \BIC@AfterFiFiFi{%
1308
               \BIC@PowRec#4!#1#2!1!%
1309
1310 ?
          \else\BigIntCalcError:ThisCannotHappen%
1311
          \fi
1312
        \or % x > 1
1313
          \BIC@AfterFiFi{%
             \BIC@PowRec#3#4!#1#2!1!%
1314
          }%
1315
        \else\BigIntCalcError:ThisCannotHappen%
1316?
1317
1318 ? \else\BigIntCalcError:ThisCannotHappen%
      \BIC@Fi
1320 }
```

2.18.1 Help macros

\BIC@ModTwo \BIC@ModTwo expects a number without sign and returns digit 1 or 0 if the number is odd or even.

```
1321 \def\BIC@ModTwo#1#2!{%
               1322 \ifx\\#2\\%
               1323
                       \ifodd#1 %
               1324
                         \BIC@AfterFiFi1%
               1325
                       \else
               1326
                         \BIC@AfterFiFi0%
               1327
                       \fi
               1328
                    \else
                       \BIC@AfterFi{%
               1329
                         \BIC@ModTwo#2!%
               1330
                       }%
               1331
                     \BIC@Fi
               1332
               1333 }
               Macro \BIC@MinusOne expects a number and returns digit 1 if the number equals
\BIC@MinusOne
               minus one and returns 0 otherwise.
               1334 \def\BIC@MinusOne#1#2!{%
               1335 \ifx#1-%
                       \BIC@@MinusOne#2!%
               1336
                     \else
               1337
               1338
                       0%
               1339 \fi
               1340 }
\BIC@@MinusOne
               1341 \def\BIC@@MinusOne#1#2!{%
               1342 \ifx#11%
                       \ifx\\#2\\%
               1343
               1344
                         1%
               1345
                       \else
                         0%
               1346
               1347
                       \fi
               1348
                    \else
               1349
                       0%
               1350 \fi
               1351 }
               2.18.2 Recursive calculation
  \BIC@PowRec
                    Pow(x, y) {
                      PowRec(x, y, 1)
                    PowRec(x, y, r) {
                       if y == 1 then
                         return r
                       else
                         ifodd y then
                          return PowRec(x*x, y div 2, r*x) % y div 2 = (y-1)/2
                          return PowRec(x*x, y div 2, r)
                        fi
                    }
                   #1: x (basis)
               #2#3: y (power)
               #4: r (result)
               1352 \def\BIC@PowRec#1!#2#3!#4!{%
                     \frac{1}{x}21\left(\frac{3}{0}\right) = 1
               1353
                       1354
                         \BIC@AfterFiFi{%
               1355
                           \BIC@ProcessMul0!#1!#4!%
               1356
               1357
                         }%
```

```
1358
                         \else
                           \BIC@AfterFiFi{%
                 1359
                             \BIC@ProcessMul0!#4!#1!%
                 1360
                 1361
                 1362
                         \fi
                 1363
                       \or
                 1364
                         \ifcase\BIC@ModTwo#2#3! % even(y)
                 1365
                           \BIC@AfterFiFi{%
                             \expandafter\BIC@@PowRec\romannumeral0%
                 1366
                             \BIC@@Shr#2#3!%
                 1367
                             !#1!#4!%
                 1368
                           }%
                 1369
                         \or % odd(y)
                 1370
                           \infnum\BIC@PosCmp#1!#4!=1 % x > r
                 1371
                 1372
                             \BIC@AfterFiFiFi{%
                                \expandafter\BIC@@PowRec\romannumeral0%
                 1373
                                \BIC@ProcessMul0!#1!#4!%
                 1374
                                !#1!#2#3!%
                 1375
                             }%
                 1376
                 1377
                           \else
                             \BIC@AfterFiFiFi{%
                 1378
                                \expandafter\BIC@@PowRec\romannumeral0%
                 1379
                                \BIC@ProcessMul0!#1!#4!%
                 1380
                                !#1!#2#3!%
                 1381
                             }%
                 1382
                 1383
                           \fi
                         \else\BigIntCalcError:ThisCannotHappen%
                 1384 ?
                 1385
                 1386~?~ \verb|\else| BigIntCalcError: ThisCannot Happen \%
                       \BIC@Fi
                 1387
                 1388 }
  \BIC@@PowRec
                #1: y/2
                 #2: x
                 #3: new r (r \text{ or } r * x)
                 1389 \def\BIC@@PowRec#1!#2!#3!{%
                      \expandafter\BIC@PowRec\romannumeral0%
                 1390
                       \BIC@ProcessMul0!#2!#2!%
                 1391
                 1392
                       !#1!#3!%
                 1393 }
 \BIC@@@PowRec #1: r*x #2: x #3: y
                 1394 \def\BIC@@@PowRec#1!#2!#3!{%
                       \expandafter\BIC@@PowRec\romannumeral0%
                 1395
                 1396
                       \BIC@@Shr#3!%
                 1397
                       !#2!#1!%
                 1398 }
                 2.19
                        Div
\bigintcalcDiv #1: x
                 #2: y (divisor)
                 1399 \def\bigintcalcDiv#1{%
                      \romannumeral0%
                       \expandafter\expandafter\BIC@Div
                 1401
                      \bigintcalcNum{#1}!%
                 1402
                 1403 }
      \BIC@Div #1: x
                 1404 \def\BIC@Div#1!#2{%
```

 $\verb|\BIC@DivSwitchSign| Decision table for \verb|\BIC@DivSwitchSign|.$

y = 0		DivisionByZero
y > 0	x = 0	0
	x > 0	DivSwitch(+, x, y)
	x < 0	DivSwitch(-, -x, y)
y < 0	x = 0	0
	x > 0	DivSwitch(-, x, -y)
	x < 0	DivSwitch(+, -x, -y)

```
#1: y (divisor)
#2: x
1412 \def\BIC@DivSwitchSign#1#2!#3#4!{%
1413 \ifcase\BIC@Sgn#1#2! % y = 0
       \BIC@AfterFi{ 0\BigIntCalcError:DivisionByZero}%
1414
      1415
        \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % x = 0
1416
          \BIC@AfterFiFi{ 0}%
1417
        1418
1419
          \BIC@AfterFiFi{%
1420
            \BIC@DivSwitch{}#3#4!#1#2!%
         }%
1421
      \else % x < 0
1422
         \BIC@AfterFiFi{%
1423
            \BIC@DivSwitch-#4!#1#2!%
1424
         }%
1425
        \fi
1426
1427
     \else % y < 0
1428
        \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % x = 0
1429
         \BIC@AfterFiFi{ 0}%
1430
        1431
          \BIC@AfterFiFi{%
            \BIC@DivSwitch-#3#4!#2!%
1432
          }%
1433
        \else % x < 0
1434
          \BIC@AfterFiFi{%
1435
            \BIC@DivSwitch{}#4!#2!%
1436
1437
          }%
        \fi
1438
      \BIC@Fi
1439
1440 }
```

\BIC@DivSwitch Decision table for \BIC@DivSwitch.

y = x		sign 1
y > x		0
y < x	y = 1	sign x
	y=2	sign Shr(x)
	y=4	sign Shr(Shr(x))
	else	sign ProcessDiv (x, y)

```
#1: sign
                                                  #2: x
                                                  #3#4: y (y \neq 0)
                                                  1441 \def\BIC@DivSwitch#1#2!#3#4!{%
                                                  1442 \ifcase\BIC@PosCmp#3#4!#2!% y = x
                                                                        \BIC@AfterFi{ #11}%
                                                  1443
                                                  1444
                                                                 \or % y > x
                                                                        \BIC@AfterFi{ 0}%
                                                  1445
                                                  1446
                                                                 \else % y < x
                                                  1447
                                                                        \ifx\\#1\\%
                                                  1448
                                                  1449
                                                                              \expandafter-\romannumeral0%
                                                  1450
                                                                        \fi
                                                                        \ifcase\ifx\\#4\\%
                                                  1451
                                                                                                 \int x#310 \% y = 1
                                                  1452
                                                                                                  \left( x \right) = 10^{10}
                                                  1453
                                                                                                  \left(\frac{342 \% y = 4}{6}\right)
                                                  1454
                                                                                                  \ensuremath{\mbox{\ensuremath{\mbox{\sc v}}}\xspace \ensuremath{\mbox{\sc v}}\xspace \ensuremath{
                                                  1455
                                                  1456
                                                                                                  fi\fi\fi
                                                  1457
                                                                                            \else
                                                  1458
                                                                                                  3 \% y > 2
                                                  1459
                                                                                             \fi
                                                                              \BIC@AfterFiFi{ #2}% y = 1
                                                  1460
                                                                        \ \ y = 2
                                                  1461
                                                                              \BIC@AfterFiFi{%
                                                  1462
                                                                                   \BIC@@Shr#2!%
                                                  1463
                                                                              }%
                                                  1464
                                                  1465
                                                                        1466
                                                                              \BIC@AfterFiFi{%
                                                                                    \expandafter\BIC@@Shr\romannumeral0%
                                                  1467
                                                  1468
                                                                                          \BIC@@Shr#2!!%
                                                  1469
                                                                              }%
                                                  1470
                                                                         1471
                                                                              \BIC@AfterFiFi{%
                                                  1472
                                                                                    \BIC@DivStartX#2!#3#4!!!%
                                                                              }%
                                                  1473
                                                  1474 ?
                                                                         \else\BigIntCalcError:ThisCannotHappen%
                                                  1475
                                                                        \fi
                                                                  \BIC@Fi
                                                  1476
                                                  1477 }
  \BIC@ProcessDiv #1#2: x
                                                  #3#4: y
                                                  #5: collect first digits of x
                                                  #6: corresponding digits of y
                                                  1478 \def\BIC@DivStartX#1#2!#3#4!#5!#6!{%
                                                                  \ifx\\#4\\%
                                                  1480
                                                                         \BIC@AfterFi{%
                                                  1481
                                                                               \BIC@DivStartYii#6#3#4!{#5#1}#2=!%
                                                  1482
                                                                        }%
                                                  1483
                                                                 \else
                                                  1484
                                                                        \BIC@AfterFi{%
                                                                              \BIC@DivStartX#2!#4!#5#1!#6#3!%
                                                  1485
                                                                        }%
                                                  1486
                                                                  \BIC@Fi
                                                  1487
                                                  1488 }
\BIC@DivStartYii #1: y
                                                  #2: x, =
                                                  1489 \def\BIC@DivStartYii#1!{%
                                                                  \expandafter\BIC@DivStartYiv\romannumeral0%
                                                  1490
                                                  1491
                                                                   \BIC@Shl#1!%
```

```
1492 !#1!%
                    1493 }
  \BIC@DivStartYiv #1: 2y
                    #2: y
                    #3: x, =
                    1494 \def\BIC@DivStartYiv#1!{%
                    1495 \expandafter\BIC@DivStartYvi\romannumeral0%
                    1496 \BIC@Shl#1!%
                    1497 !#1!%
                    1498 }
  \BIC@DivStartYvi #1: 4y
                    #2: 2y
                    #3: y
                    #4: x, =
                    1499 \def\BIC@DivStartYvi#1!#2!{%
                    1500 \expandafter\BIC@DivStartYviii\romannumeral0%
                    1501 \BIC@AddXY#1!#2!!!%
                    1502 !#1!#2!%
                    1503 }
\BIC@DivStartYviii #1: 6y
                    #2: 4y
                    #3: 2y
                    #4: y
                    #5: x, =
                    1504 \def\BIC@DivStartYviii#1!#2!{%
                    1505 \expandafter\BIC@DivStart\romannumeral0%
                    1506 \BIC@Shl#2!%
                    1507 !#1!#2!%
                    1508 }
     \BIC@DivStart #1: 8y
                    #2: 6y
                    #3: 4y
                    #4: 2y
                    #5: y
                    #6: x, =
                    1509 \def\BIC@DivStart#1!#2!#3!#4!#5!#6!{%
                    1510 \BIC@ProcessDiv#6!!#5!#4!#3!#2!#1!=%
                    1511 }
   \BIC@ProcessDiv #1#2#3: x, =
                    #4: result
                    #5: y
                    #6: 2y
                    #7: 4y
                    #8: 6y
                    #9: 8y
                    1512 \def\BIC@ProcessDiv#1#2#3!#4!#5!{%
                    1513 \ifcase\BIC@PosCmp#5!#1!% y = #1
                    1514
                            \ifx#2=%
                              \BIC@AfterFiFi{\BIC@DivCleanup{#41}}%
                    1515
                    1516
                            \else
                              \BIC@AfterFiFi{%
                    1517
                                \BIC@ProcessDiv#2#3!#41!#5!%
                    1518
                              ጉ%
                    1519
                            \fi
                    1520
                    1521 \or % y > #1
```

```
1522
                           \ifx#2=%
                             \BIC@AfterFiFi{\BIC@DivCleanup{#40}}%
                   1523
                   1524
                           \else
                   1525
                              \ifx\\#4\\%
                   1526
                                \BIC@AfterFiFiFi{%
                   1527
                                  \BIC@ProcessDiv{#1#2}#3!!#5!%
                               }%
                   1528
                   1529
                              \else
                                \BIC@AfterFiFiFi{%
                   1530
                                  \BIC@ProcessDiv{#1#2}#3!#40!#5!%
                   1531
                                }%
                   1532
                   1533
                             \fi
                           \fi
                   1534
                         \else % y < #1
                   1535
                            \BIC@AfterFi{%
                   1536
                   1537
                              \BIC@@ProcessDiv{#1}#2#3!#4!#5!%
                   1538
                           }%
                         \BIC@Fi
                   1539
                   1540 }
 \BIC@DivCleanup
                  #1: result
                   #2: garbage
                   1541 \def\BIC@DivCleanup#1#2={ #1}%
\BIC@@ProcessDiv
                   1542 \def\BIC@@ProcessDiv#1#2#3!#4!#5!#6!#7!{%
                   1543 \ifcase\BIC@PosCmp#7!#1!% 4y = #1
                           \ifx#2=%
                   1544
                             \BIC@AfterFiFi{\BIC@DivCleanup{#44}}%
                   1545
                   1546
                           \else
                   1547
                            \BIC@AfterFiFi{%
                               \BIC@ProcessDiv#2#3!#44!#5!#6!#7!%
                   1548
                   1549
                            }%
                   1550
                           \fi
                   1551
                         1552
                           \label{eq:local_posemp} if case\BIC@PosCmp#6!#1!% 2y = #1
                   1553
                              \ifx#2=%
                                \BIC@AfterFiFiFi{\BIC@DivCleanup{#42}}%
                   1554
                   1555
                              \else
                                \BIC@AfterFiFiFi{%
                   1556
                                  \BIC@ProcessDiv#2#3!#42!#5!#6!#7!%
                   1557
                                }%
                   1558
                   1559
                              \fi
                   1560
                           \or % 2y > #1
                   1561
                              \ifx#2=%
                                \verb|\BIC@AfterFiFiFi{\BIC@DivCleanup{#41}}||
                   1562
                   1563
                                \BIC@AfterFiFiFi{%
                   1564
                                  \BIC@DivSub#1!#5!#2#3!#41!#5!#6!#7!%
                   1565
                                }%
                   1566
                             \fi
                   1567
                           \else % 2v < #1
                   1568
                              \BIC@AfterFiFi{%
                   1569
                   1570
                                \expandafter\BIC@ProcessDivII\romannumeral0%
                   1571
                                \BIC@SubXY#1!#6!!!%
                   1572
                                !#2#3!#4!#5!23%
                   1573
                                #6!#7!%
                             }%
                   1574
                           \fi
                   1575
                         \else % 4y < #1
                   1576
                           \BIC@AfterFi{%
                   1577
                              \BIC@@@ProcessDiv{#1}#2#3!#4!#5!#6!#7!%
                   1578
```

```
1579
                            }%
                          \BIC@Fi
                    1580
                    1581 }
                  Next token group: #1-#2 and next digit #3.
      \BIC@DivSub
                    1582 \def\BIC@DivSub#1!#2!#3{%
                    1583
                         \expandafter\BIC@ProcessDiv\expandafter{%
                    1584
                            \romannumeral0%
                    1585
                            \BIC@SubXY#1!#2!!!%
                    1586
                    1587
                         }%
                    1588 }
\BIC@ProcessDivII #1: x'-2y
                    #2#3: remaining x, =
                    #4: result
                    #5: y
                    #6: first possible result digit
                    #7: second possible result digit
                    1589 \def\BIC@ProcessDivII#1!#2#3!#4!#5!#6#7{%
                          \ightharpoonup #5!#1!% y = #1
                    1590
                            \ifx#2=%
                    1591
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                    1592
                    1593
                            \else
                              \BIC@AfterFiFi{%
                    1594
                                \BIC@ProcessDiv#2#3!#4#7!#5!%
                    1595
                    1596
                              }%
                    1597
                            \fi
                    1598
                          1599
                            \ifx#2=%
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#6}}%
                    1600
                    1601
                            \else
                              \BIC@AfterFiFi{%
                    1602
                                \BIC@ProcessDiv{#1#2}#3!#4#6!#5!%
                    1603
                    1604
                              }%
                    1605
                            \fi
                    1606
                          \else % y < #1
                    1607
                            \ifx#2=%
                    1608
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                    1609
                            \else
                              \BIC@AfterFiFi{%
                    1610
                                \BIC@DivSub#1!#5!#2#3!#4#7!#5!%
                    1611
                              }%
                    1612
                            \fi
                    1613
                          \BIC@Fi
                    1614
                    1615 }
\BIC@ProcessDivIV #1#2#3: x, =, x > 4y
                    #4: result
                    #5: y
                    #6: 2y
                    #7: 4y
                    #8: 6y
                    #9: 8y
                    1616 \def\BIC@@@ProcessDiv#1#2#3!#4!#5!#6!#7!#8!#9!{%
                         \ifcase\BIC@PosCmp#8!#1!% 6y = #1
                    1617
                            \int ifx#2=%
                    1618
                    1619
                              \BIC@AfterFiFi{\BIC@DivCleanup{#46}}%
                    1620
                            \else
                              \BIC@AfterFiFi{%
                    1621
                                \BIC@ProcessDiv#2#3!#46!#5!#6!#7!#8!#9!%
                    1622
```

```
}%
                     1623
                             \fi
                     1624
                           \or \% 6y > #1
                     1625
                             \BIC@AfterFi{%
                     1626
                     1627
                               \expandafter\BIC@ProcessDivII\romannumeral0%
                     1628
                               \BIC@SubXY#1!#7!!!%
                     1629
                               !#2#3!#4!#5!45%
                     1630
                               #6!#7!#8!#9!%
                             }%
                     1631
                           \else % 6y < #1
                     1632
                             \ifcase\BIC@PosCmp#9!#1!% 8y = #1
                     1633
                               \ifx#2=%
                     1634
                                 \BIC@AfterFiFiFi{\BIC@DivCleanup{#48}}%
                     1635
                     1636
                               \else
                     1637
                                 \BIC@AfterFiFiFi{%
                     1638
                                   \BIC@ProcessDiv#2#3!#48!#5!#6!#7!#8!#9!%
                                 }%
                     1639
                               \fi
                     1640
                             \or % 8y > #1
                     1641
                     1642
                               \BIC@AfterFiFi{%
                                 \expandafter\BIC@ProcessDivII\romannumeral0%
                     1643
                                 \BIC@SubXY#1!#8!!!%
                     1644
                                 !#2#3!#4!#5!67%
                     1645
                                 #6!#7!#8!#9!%
                     1646
                     1647
                             \else % 8y < #1
                     1648
                     1649
                               \BIC@AfterFiFi{%
                                 \expandafter\BIC@ProcessDivII\romannumeral0%
                     1650
                     1651
                                 \BIC@SubXY#1!#9!!!%
                                 !#2#3!#4!#5!89%
                     1652
                     1653
                                 #6!#7!#8!#9!%
                     1654
                               }%
                     1655
                             \fi
                           \BIC@Fi
                     1656
                     1657 }
                     2.20
                             Mod
    \bigintcalcMod #1: x
                     #2: y
                     1658 \def\bigintcalcMod#1{%
                     1659
                           \romannumeral0%
                           \expandafter\expandafter\BIC@Mod
                     1660
                           \bigintcalcNum{#1}!%
                     1661
                     1662 }
          \BIC@Mod #1: x
                     #2: y
                     1663 \def\BIC@Mod#1!#2{%
                           \expandafter\expandafter\expandafter\BIC@ModSwitchSign
                     1665
                           \bigintcalcNum{#2}!#1!%
                     1666 }
    \BigIntCalcMod
                     1667 \def\BigIntCalcMod#1!#2!{%
                     1668
                           \romannumeral0%
                     1669
                           \BIC@ModSwitchSign#2!#1!%
                     1670 }
\BIC@ModSwitchSign Decision table for \BIC@ModSwitchSign.
```

y = 0		DivisionByZero
y > 0	x = 0	0
	else	ModSwitch(+, x, y)
y < 0		ModSwitch(-, -x, -y)

```
#1#2: y
#3#4: x
1671 \def\BIC@ModSwitchSign#1#2!#3#4!{%
1672 \ifcase\ifx\\#2\\%
               \int x#100 \% y = 0
1673
                \else1 % y > 0
1674
                \fi
1675
               \else
1676
                 \int 1^2 x^4 - 2  % y < 0
1677
1678
                 \else1 % y > 0
1679
1680
              \fi
        \BIC@AfterFi{ 0\BigIntCalcError:DivisionByZero}%
1681
1682
      \or \% y > 0
        \left(\frac{4}{\pi}300 \right) = 1 \left(\frac{\pi}{\pi} x = 0\right)
1683
          \BIC@AfterFiFi{ 0}%
1684
        \else
1685
          \BIC@AfterFiFi{%
1686
            \BIC@ModSwitch{}#3#4!#1#2!%
1687
          }%
1688
        \fi
1689
      \else % y < 0
1690
        \int \frac{1}{2} 
1691
1692
                  \int x#300 % x = 0
1693
                  \left( x > 0 \right)
1694
                  \fi
1695
                \else
                  1696
                  \else1 % x > 0
1697
1698
                  \fi
                \fi
1699
          \BIC@AfterFiFi{ 0}%
1700
1701
        1702
          \BIC@AfterFiFi{%
1703
            \BIC@ModSwitch--#3#4!#2!%
          }%
1704
        \else % x < 0
1705
          \BIC@AfterFiFi{%
1706
1707
            \BIC@ModSwitch-#4!#2!%
1708
          }%
1709
        \fi
      \BIC@Fi
1710
1711 }
```

\BIC@ModSwitch Decision table for \BIC@ModSwitch.

y=1		0
y=2	ifodd(x)	sign 1
	else	0
y > 2	x < 0	$z \leftarrow x - (x/y) * y; (z < 0) ? z + y : z$
	x > 0	x - (x/y) * y

```
#1: sign

#2#3: x

#4#5: y

1712 \def\BIC@ModSwitch#1#2#3!#4#5!{%

1713 \ifcase\ifx\\#5\\%
```

```
\int x#410 \% y = 1
                1714
                                \left( x + 421 \% y = 2 \right)
                1715
                                \ensuremath{\mbox{lelse2}} % y > 2
                1716
                                \fi\fi
                1717
                1718
                             \else2 % y > 2
                1719
                             \fi
                1720
                        \BIC@AfterFi{ 0}% y = 1
                1721
                      \ifcase\BIC@ModTwo#2#3! % even(x)
                1722
                          \BIC@AfterFiFi{ 0}%
                1723
                        \or % odd(x)
                1724
                          \BIC@AfterFiFi{ #11}%
                1725
                        \else\BigIntCalcError:ThisCannotHappen%
                1726 ?
                1727
                      \or % y > 2
                1728
                1729
                        \ifx\\#1\\%
                1730
                        \else
                          \expandafter\BIC@Space\romannumeral0%
                1731
                          \expandafter\BIC@ModMinus\romannumeral0%
                1732
                        \fi
                1733
                        \int x^2-x^2 x < 0
                1734
                          \BIC@AfterFiFi{%
                1735
                            \expandafter\expandafter\expandafter\BIC@ModX
                1736
                            \bigintcalcSub{#2#3}{%
                1737
                               \bigintcalcMul{#4#5}{\bigintcalcDiv{#2#3}{#4#5}}%
                1738
                1739
                            }!#4#5!%
                          }%
                1740
                        \leq x > 0
                1741
                1742
                          \BIC@AfterFiFi{%
                            \expandafter\expandafter\BIC@Space
                1743
                            \verb|\bigintcalcSub{#2#3}{%}|
                1744
                1745
                               \bigintcalcMul{#4#5}{\bigintcalcDiv{#2#3}{#4#5}}%
                1746
                            }%
                          }%
                1747
                        \fi
                1748
                1749 ? \else\BigIntCalcError:ThisCannotHappen%
                      \BIC@Fi
                1751 }
\BIC@ModMinus
                1752 \def\BIC@ModMinus#1{%
                1753 \ifx#10%
                        \BIC@AfterFi{ 0}%
                1754
                1755
                      \else
                1756
                        \BIC@AfterFi{ -#1}%
                1757
                      \BIC@Fi
                1758 }
    \BIC@ModX #1#2: z
                #3: x
                1759 \def\BIC@ModX#1#2!#3!{%
                1760 \ifx#1-% z < 0
                1761
                        \BIC@AfterFi{%
                          \expandafter\BIC@Space\romannumeral0%
                1762
                          \BIC@SubXY#3!#2!!!%
                1763
                1764
                        }%
                1765
                      \leq  z >= 0
                1766
                        \BIC@AfterFi{ #1#2}%
                1767
                      \BIC@Fi
                1768 }
                1769 \BIC@AtEnd%
                1770 (/package)
```

3 Test

3.1 Catcode checks for loading

```
1771 (*test1)
1772 \catcode \{=1 \%
1773 \catcode`\}=2 %
1774 \catcode \#=6 %
1775 \catcode \@=11 %
1776 \expandafter\ifx\csname count@\endcsname\relax
1777
                     \countdef\count@=255 %
1778 \fi
1779 \expandafter\ifx\csname @gobble\endcsname\relax
1781 \fi
1782 \verb|\expandafter\ifx\csname @firstofone\endcsname\relax|
1783 \long\def\@firstofone#1{#1}%
1784 \fi
1785 \verb|\expandafter\ifx\csname loop\endcsname\relax|
1786 \expandafter\@firstofone
1787 \ensuremath{\setminus} else
1788 \expandafter\@gobble
1789 \fi
1790 {%
1791
                           \def\loop #1 repeat {\%}
1792
                                  \def\body{#1}%
1793
                                  \iterate
                      }%
1794
                        \def\iterate{%
1795
1796
                                  \body
                                           \let\next\iterate
1797
1798
                                    \else
                                            \let\next\relax
1799
1800
                                    \fi
1801
                                   \next
                        }%
1802
                          1803
1804 }%
1806 \count@=0 %
1807 \loop
                        \edef\RestoreCatcodes{%
1808
                                  \RestoreCatcodes
1809
                                   \catcode\the\count@=\the\catcode\count@\relax
1810
1811
                    }%
1812 \ifnum\count@<255 %
1813 \advance\count@ 1 %
1814 \repeat
1815
1816 \def\RangeCatcodeInvalid#1#2{%
1817
                         \c \ensuremath{$\c \ensuremath{\c \ensuremath{\c}
1818
                        \loop
                                  \catcode\count@=15 %
1819
                         \ifnum\count@<#2\relax
1820
1821
                                  \advance\count@ 1 %
1822
                          \repeat
1823 }
1824 \ensuremath{\mbox{\mbox{$1$}}} 1824 \ensuremath{\mbox{\mbox{$4$}}} \ensuremath{\mbox{$4$}} \ens
                         \count@=#1\relax
1825
                          \loop
1826
                                   \ifnum#3=\catcode\count@
1827
                                   \else
1828
```

```
\errmessage{%
1829
            Character \the\count@\space
1830
            with wrong catcode \the\catcode\count@\space
1831
            instead of \number#3%
1832
1833
          }%
1834
        \fi
1835
      \ifnum\count@<#2\relax
1836
        \advance\count@ 1 %
1837
      \repeat
1838 }
1839 \def\space{ }
1840 \expandafter\ifx\csname LoadCommand\endcsname\relax
      \def\LoadCommand{\input bigintcalc.sty\relax}%
1842 \fi
1843 \def\Test{\%}
      \RangeCatcodeInvalid{0}{47}%
1844
1845
      \RangeCatcodeInvalid{58}{64}%
      \RangeCatcodeInvalid{91}{96}%
1846
      \RangeCatcodeInvalid{123}{255}%
1847
      \catcode`\@=12 %
1848
      \catcode`\\=0 %
1849
      \catcode`\%=14 %
1850
      \LoadCommand
1851
      \RangeCatcodeCheck{0}{36}{15}%
1852
      \RangeCatcodeCheck{37}{37}{14}%
1853
      \RangeCatcodeCheck{38}{47}{15}%
1854
1855
      \RangeCatcodeCheck{48}{57}{12}%
1856
      \RangeCatcodeCheck{58}{63}{15}%
1857
      \RangeCatcodeCheck{64}{64}{12}%
1858
      \RangeCatcodeCheck{65}{90}{11}%
      \RangeCatcodeCheck{91}{91}{15}%
1859
1860
      \RangeCatcodeCheck{92}{92}{0}%
1861
      \RangeCatcodeCheck{93}{96}{15}%
      \RangeCatcodeCheck{97}{122}{11}%
1862
      \RangeCatcodeCheck{123}{255}{15}%
1863
1864
      \RestoreCatcodes
1865 }
1866 \Test
1867 \csname @@end\endcsname
1868 \end
1869 (/test1)
```

3.2 Macro tests

3.2.1 Preamble with test macro definitions

```
1870 (*test2)
1871 \NeedsTeXFormat{LaTeX2e}
1872 \nofiles
1873 \documentclass{article}
1874 (noetex) \let\SavedNumexpr\numexpr
1875 \langle noetex \rangle \ let \ numexpr \ UNDEFINED
1876 \makeatletter
1877 \chardef\BIC@TestMode=1 %
1878 \makeatother
1879 \usepackage{bigintcalc}[2012/04/08]
1880 \langle noetex \rangle \setminus let \setminus numexpr \setminus Saved Numexpr
1881 \usepackage{qstest}
1882 \IncludeTests{*}
1883 \LogTests{log}{*}{*}
1884 \newcommand*{\TestSpaceAtEnd}[1]{%
1885 (noetex) \let\SavedNumexpr\numexpr
1886 (noetex) \let\numexpr\UNDEFINED
```

```
\edef\resultA{#1}%
1887
     \edef\resultB{#1 }%
1888
1889 (noetex) \let\numexpr\SavedNumexpr
     \Expect*{\resultA\space}*{\resultB}%
1891 }
1892 \newcommand*{\TestResult}[2]{%
1893 (noetex) \let\SavedNumexpr\numexpr
1895 \edef\result{#1}%
\Expect*{\result}{#2}%
1897
1898 }
1899 \newcommand*{\TestResultTwoExpansions}[2]{%
1900 (*noetex)
     \begingroup
        1902
1903
        \expandafter\expandafter\expandafter
     \endgroup
1904
1905 (/noetex)
     \expandafter\expandafter\Expect
     \expandafter\expandafter\expandafter{#1}{#2}%
1907
1908 }
1909 \newcount\TestCount
1910 \(\rangle\) \(\newcommand*{\TestArg}[1]{\numexpr#1\relax}\)
1911 (noetex) \newcommand*{\TestArg}[1]{#1}
1912 \newcommand*{\TestTeXDivide}[2]{%
    \TestCount=\TestArg{#1}\relax
1913
      \divide\TestCount by \TestArg{#2}\relax
1914
1915
     \Expect*{\bigintcalcDiv{#1}{#2}}*{\the\TestCount}%
1916 }
1917 \newcommand*{\Test}[2]{%
1918
     \TestResult{#1}{#2}%
1919
     \TestResultTwoExpansions{#1}{#2}%
1920
     \TestSpaceAtEnd{#1}%
1922 \mbox{newcommand}*{\mbox{TestExch}[2]{\mbox{#2}{#1}}}
1923 \newcommand*{\TestInv}[2]{%
1924
     \Test{\bigintcalcInv{#1}}{#2}%
1925 }
1926 \newcommand*{\TestAbs}[2]{%
1927
    \Test{\bigintcalcAbs{#1}}{#2}%
1928 }
1929 \newcommand*{\TestSgn}[2]{%
1930
     \Test{\bigintcalcSgn{#1}}{#2}%
1931 }
1932 \newcommand*{\TestMin}[3]{%
     \Test{\bigintcalcMin{#1}{#2}}{#3}%
1934 }
1935 \newcommand*{\TestMax}[3]{%
1936
     Test{\big(\del{1}{#3}\)}
1937 F
1938 \newcommand*{\TestCmp}[3]{%
     \Test{\bigintcalcCmp{#1}{#2}}{#3}%
1939
1940 }
1941 \newcommand*{\TestOdd}[2]{%
     \Test{\bigintcalcOdd{#1}}{#2}%
1943
     \left( x_{x}\right) 
1944
        \noexpand\Test{%}
1945
         \noexpand\BigIntCalcOdd
         \bigintcalcAbs{#1}!%
1946
       }{#2}%
1947
1948
     }%
```

```
1949
      \x
1950 }
1951 \newcommand*{\TestInc}[2]{%
      \Test{\bigintcalcInc{#1}}{#2}%
1953
      \ifnum\bigintcalcSgn{#1}>-1 %
1954
         \left( x_{x}\right) 
1955
           \noexpand\Test{%
1956
             \noexpand\BigIntCalcInc\bigintcalcNum{#1}!%
1957
           }{#2}%
        }%
1958
        \x
1959
1960
      \fi
1961 }
1962 \newcommand*{\TestDec}[2]{%
      \Test{\bigintcalcDec{#1}}{#2}%
1963
1964
      \ifnum\bigintcalcSgn{#1}>0 %
1965
         \left( x_{x}\right) 
           \noexpand\Test{%}
1966
             \noexpand\BigIntCalcDec\bigintcalcNum{#1}!%
1967
1968
           }{#2}%
1969
        }%
1970
         ١x
      \fi
1971
1972 }
1973 \newcommand*{\TestAdd}[3]{%
1974
      \Test{\bigintcalcAdd{#1}{#2}}{#3}%
1975
      \ifnum\bigintcalcSgn{#1}>0 %
1976
         \ifnum\bigintcalcSgn{#2}> 0 %
           1977
             \left( x_{x}\right) 
1978
               \verb|\noexpand\Test{%}|
1979
1980
                  \noexpand\BigIntCalcAdd
1981
                 \bigintcalcNum{#1}!\bigintcalcNum{#2}!%
               }{#3}%
1982
             }%
1983
1984
             /x
1985
           \else
1986
             \left( x_{x}\right) 
1987
               \noexpand\Test{%
                 \noexpand\BigIntCalcAdd
1988
                 \bigintcalcNum{#2}!\bigintcalcNum{#1}!%
1989
               }{#3}%
1990
             }%
1991
1992
             \x
1993
           \fi
1994
         \fi
1995
      \fi
1996 }
1997 \newcommand*{\TestSub}[3]{%
1998
      \Test{\bigintcalcSub{#1}{#2}}{#3}%
      \ifnum\bigintcalcSgn{#1}>0 %
1999
2000
         \ifnum\bigintcalcSgn{#2}> 0 %
2001
           \ifnum\bigintcalcCmp{#1}{#2}>0 %
2002
             \left( x_{x}\right) 
               \noexpand\Test{%}
2003
2004
                  \noexpand\BigIntCalcSub
2005
                 \bigintcalcNum{#1}!\bigintcalcNum{#2}!%
2006
               }{#3}%
2007
             }%
2008
             /x
           \fi
2009
2010
         \fi
```

```
2011
      \fi
2012 }
2013 \mbox{newcommand}*{\mbox{TestShl}[2]{%}}
      \Test{\bigintcalcShl{#1}}{#2}%
2015
      \left( x_{x}\right) 
2016
        \noexpand\Test{%
2017
          \noexpand\BigIntCalcShl\bigintcalcAbs{#1}!%
2018
        }{\bigintcalcAbs{#2}}%
      }%
2019
2020
      ١x
2021 }
2022 \newcommand*{\TestShr}[2]{%
      \Test{\bigintcalcShr{#1}}{#2}%
2023
      \left( x_{x}\right) 
2024
        \noexpand\Test{%
2025
2026
           \noexpand\BigIntCalcShr\bigintcalcAbs{#1}!%
2027
        }{\bigintcalcAbs{#2}}%
      }%
2028
2029
      \x
2030 }
2031 \newcommand*{\TestMul}[3]{%
      Test{\bigintcalcMul{#1}{#2}}{#3}%
2032
2033
      \left( x_{x}\right) 
2034
        \noexpand\Test{%
           \noexpand\BigIntCalcMul
2035
2036
          \bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
2037
        }{\bigintcalcAbs{#3}}%
2038
      }%
2039
      /x
2040 }
2041 \newcommand*{\TestSqr}[2]{%
2042
      \Test{\bigintcalcSqr{#1}}{#2}%
2043 }
2044 \newcommand*{\TestFac}[2]{%
      \expandafter\TestExch\expandafter{%
2045
2046
        \the\numexpr#2%
2047
      }{\bigintcalcFac{#1}}%
2048 }
2049 \newcommand*{\TestFacBig}[2]{%
2050
      \Test{\bigintcalcFac{#1}}{#2}%
2051 }
2052 \newcommand*{\TestPow}[3]{\%}
      2053
2054 }
2055 \newcommand*{\TestDiv}[3]{%
      \Test{\bigintcalcDiv{#1}{#2}}{#3}%
      \TestTeXDivide{#1}{#2}%
2058 }
2059 \newcommand*{\TestDivBig}[3]{%
2060
      \Test{\bigintcalcDiv{#1}{#2}}{#3}%
      \left( x_{x}\right) 
2061
        \noexpand\Test{%}
2062
          \label{localcabs} $$ \noexpand\BigIntCalcDiv \bigintcalcAbs {#1}!\bigintcalcAbs {#2}!\% $$
2063
2064
        }{\bigintcalcAbs{#3}}%
2065
      }%
2066 }
2067 \newcommand*{\TestMod}[3]{%
2068
      2069
      \ifcase\ifcase\bigintcalcSgn{#1} 0%
2070
                \ifcase\bigintcalcSgn{#2} 1%
2071
2072
                \or 0%
```

```
2073
                                        \else 1%
2074
                                  \else
2075
                                        \ifcase\bigintcalcSgn{#2} 1%
2076
2077
                                        \or 1%
                                        \else 0\%
2078
2079
                                        \fi
                                  fi\relax
2080
                     \left( x_{x}\right) 
2081
                          \noexpand\Test{%
2082
                                \noexpand\BigIntCalcMod
2083
2084
                                \bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
                          }{\bigintcalcAbs{#3}}%
2085
2086
                     }%
2087
                     \x
2088
               \fi
2089 }
3.2.2
                  Time
2090 \verb|\begingroup\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandaft
2091 \expandafter\ifx\csname pdfresettimer\endcsname\relax
2092 \else
2093
                \makeatletter
                \newcount\SummaryTime
2094
                \newcount\TestTime
2095
2096
                \SummaryTime=\z@
                \newcommand*{\PrintTime}[2]{%
2097
2098
                     \typeout{%
2099
                           [Time #1: \strip@pt\dimexpr\number#2sp\relax\space s]%
                    }%
2100
               }%
2101
2102
                \newcommand*{\StartTime}[1]{%
2103
                     \renewcommand*{\TimeDescription}{#1}%
                     \pdfresettimer
2104
2105
2106
                \newcommand*{\TimeDescription}{}%
2107
                \newcommand*{\StopTime}{%
2108
                     \TestTime=\pdfelapsedtime
2109
                     \global\advance\SummaryTime\TestTime
                     \PrintTime\TimeDescription\TestTime
2110
2111
               }%
                \let\saved@qstest\qstest
2112
                \let\saved@endqstest\endqstest
2113
2114
                \def\qstest#1#2{%
2115
                     \saved@qstest{#1}{#2}%
2116
                     \StartTime{#1}%
2117
               }%
2118
                \def\endqstest{%
2119
                     \StopTime
2120
                     \saved@endqstest
               }%
2121
               \AtEndDocument{%
2122
2123
                     \PrintTime{summary}\SummaryTime
2124
               }%
                \makeatother
2125
2126 \fi
3.2.3 Test sets
2127 \makeatletter
2128
2129 \begin{qstest}{inv}{inv}%
               \texttt{\TestInv}\{0\}\{0\}\%
2130
               \texttt{\TestInv}\{1\}\{-1\}\%
2131
```

```
2132
      TestInv{-1}{1}%
      \TestInv{10}{-10}%
2133
2134
      \TestInv{-10}{10}%
      \TestInv{2147483647}{-2147483647}%
      \TestInv{-2147483647}{2147483647}%
2136
2137
      \TestInv{12345678901234567890}{-12345678901234567890}%
2138
      \TestInv{-12345678901234567890}{12345678901234567890}%
2139
      \TestInv{ 0 }{0}%
      \TestInv{ 1 }{-1}%
2140
      \TestInv{--1}{-1}%
2141
      \TestInv{\number\z0}{0}%
2142
      \TestInv{\ifx\relax\relax1\fi}{-1}%
2143
2144
      \TestInv{\ifx\relax\relax-\fi\ifx234\else1\fi}{1}\%
2145 \end{qstest}
2146
2147 \begin{qstest}{abs}{abs}%
2148
     \TestAbs{0}{0}%
      \TestAbs{1}{1}%
2149
     \TestAbs{-1}{1}%
2150
2151
      \TestAbs{10}{10}%
2152
      \TestAbs{-10}{10}%
      \TestAbs{2147483647}{2147483647}%
2153
2154
      \TestAbs{-2147483647}{2147483647}%
2155
      \TestAbs{12345678901234567890}{12345678901234567890}%
      \TestAbs{-12345678901234567890}{12345678901234567890}%
2156
2157
      \TestAbs{ 0 }{0}%
2158
      \TestAbs{ 1 }{1}%
2159
      \TestAbs{--1}{1}%
2160
      \TestAbs{-+-+1}{1}%
      \TestAbs{00000000000}{0}%
2161
      \TestAbs{00000001000}{1000}%
2162
2163
     \TestAbs{\ifx\relax\relax 0\else 1\fi}{0}%
2164 \end{qstest}
2165
2166 \begin{qstest}{sign}{sign}%
2167
     \TestSgn{0}{0}%
2168
     \text{TestSgn}{1}{1}%
2169
     TestSgn{-1}{-1}%
2170
     TestSgn{10}{1}%
2171
      TestSgn{-10}{-1}%
      \texttt{\TestSgn}\{2147483647\}\{1\}\%
2172
      TestSgn{-2147483647}{-1}%
2173
      \TestSgn{12345678901234567890}{1}%
2174
2175
      \TestSgn{-12345678901234567890}{-1}%
2176
      \TestSgn{ 0 }{0}%
2177
      \TestSgn{ 2 }{1}%
2178
      TestSgn{ -2 }{-1}%
2179
      \TestSgn{--2}{1}%
2180
      \TestSgn{
\quad \  \  }{0}%
2181
      \TestSgn{
\underline{0ne}_{1}%
      \TestSgn{\number\m@ne}{-1}\%
2182
      \TestSgn{%
2183
        -+-+\number\z@\number\z@
2184
2185
        \iftrue1\fi\iftrue2\fi\iftrue3\fi
2186
     }{1}%
2187 \end{qstest}
2188
2189 \begin{qstest}{min}{min}%
2190 \TestMin{0}{1}{0}%
2191
     \TestMin{1}{0}{0}%
2192 \quad \texttt{\TestMin} \{-10\} \{-20\} \{-20\} \%
2193 \TestMin{ 1 }{ 2 }{1}%
```

```
2194
      \TestMin{ 2 }{ 1 }{1}%
      \TestMin{1}{1}{1}%
2195
      2196
      \TestMin{\number\@ne}{\number\m@ne}{-1}%
2198 \end{qstest}
2199
2200 \begin{qstest}{max}{max}%
2201
      \text{TestMax}\{0\}\{1\}\{1\}\%
      \text{TestMax}\{1\}\{0\}\{1\}\%
2202
      \TestMax{-10}{-20}{-10}%
2203
      \TestMax{ 1 }{ 2 }{2}%
2204
      \TestMax{ 2 }{ 1 }{2}%
2205
2206
     \text{TestMax}\{1\}\{1\}\{1\}
      \TestMax{\number\@ne}{\number\m@ne}{1}%
2208
2209 \end{qstest}
2210
2211 \begin{qstest}{cmp}{cmp}%
2212
     \TestCmp{0}{0}{0}}
      \texttt{\TestCmp}\{-21\}\{17\}\{-1\}\%
2213
      \texttt{\TestCmp}\{3\}\{4\}\{-1\}\%
2214
      TestCmp{-10}{-10}{0}%
2215
2216
      TestCmp{-10}{-11}{1}%
      \TestCmp{100}{5}{1}%
2217
      TestCmp{9}{10}{-1}%
2218
2219
      \TestCmp{10}{9}{1}%
2220
      TestCmp{ 3 }{ 3 }{ 0}
2221
      TestCmp{-9}{-10}{1}%
2222
      TestCmp{-10}{-9}{-1}%
      TestCmp{-3}{-3}{0}%
2223
      TestCmp{0}{-2}{1}%
2224
      \texttt{\TestCmp}\{0\}\{2\}\{-1\}\%
2225
2226
      TestCmp{2}{0}{1}%
2227
      TestCmp{-2}{0}{-1}%
      \TestCmp{12}{11}{1}%
2228
2229
      \TestCmp{11}{12}{-1}%
2230
      \TestCmp{2147483647}{-2147483647}{1}%
2231
      \TestCmp{-2147483647}{2147483647}{-1}%
2232
      \TestCmp{2147483647}{2147483647}{0}%
2233
      \TestCmp{\number\z@}{\number\@ne}{-1}%
      2234
      TestCmp{ 4 }{ 5 }{-1}%
2235
2236
      TestCmp{ -3 }{ -7 }{1}%
2237 \end{qstest}
2238
2239 \begin{qstest}{odd}{odd}
2240 \tracingmacros=1
2241
     \TestOdd{0}{0}%
2242
      \TestOdd{1}{1}%
2243
      \Test0dd{2}{0}%
      \texttt{\TestOdd}{3}{1}{\%}
2244
2245
      \TestOdd{14}{0}%
      \Test0dd{15}{1}%
2246
2247
      \TestOdd{12345678901234567896}{0}%
      \TestOdd{12345678901234567897}{1}%
2248
2249 \end{qstest}
2250
2251 \begin{qstest}{inc}{inc}%
2252 \TestInc{0}{1}%
2253
     TestInc{1}{2}%
2254 \TestInc{-1}{0}%
2255 \TestInc{10}{11}%
```

```
TestInc{-10}{-9}%
2256
2257
     TestInc{3}{4}%
2258
     \TestInc{999}{1000}%
     \TestInc{-1000}{-999}%
2260
     \TestInc{129}{130}%
2261
     \TestInc{2147483646}{2147483647}%
2262
     \TestInc{-2147483647}{-2147483646}%
2263
     \TestInc{12345678901234567890}{12345678901234567891}%
     2264
     \TestInc{-12345678901234567891}{-12345678901234567890}%
2265
     2266
2267 \end{qstest}
2268
2269 \begin{qstest}{dec}{dec}%
     \TestDec{0}{-1}%
2270
2271
     \TestDec{1}{0}%
2272
     \TestDec{-1}{-2}%
2273
     \TestDec{10}{9}%
     \TestDec{-10}{-11}%
2274
2275
     \TestDec{1000}{999}%
     TestDec{-999}{-1000}%
2276
     \TestDec{130}{129}%
2277
2278
     \TestDec{2147483647}{2147483646}%
     \TestDec{-2147483646}{-2147483647}%
2279
     \TestDec{12345678901234567891}{12345678901234567890}%
     2281
2282
     \TestDec{-12345678901234567890}{-12345678901234567891}%
2283
     2284 \end{qstest}
2285
2286 \begin{qstest}{add}{add}%
2287
     \TestAdd{0}{0}{0}%
2288
     \TestAdd{1}{0}{1}%
2289
     \TestAdd{0}{1}{1}%
2290
     \TestAdd{1}{2}{3}%
2291
     TestAdd{-1}{-1}{-2}%
2292
     \TestAdd{2147483646}{1}{2147483647}%
2293
     \TestAdd{-2147483647}{2147483647}{0}%
2294
     \TestAdd{20}{-5}{15}%
2295
     TestAdd{-4}{-1}{-5}%
     TestAdd{-1}{-4}{-5}%
2296
2297
     TestAdd{-4}{1}{-3}%
     TestAdd{-1}{4}{3}%
2298
2299
     TestAdd{4}{-1}{3}%
2300
     TestAdd{1}{-4}{-3}%
2301
     TestAdd{-4}{-1}{-5}%
2302
     TestAdd{-1}{-4}{-5}%
2303
     TestAdd{ -4 }{ -1 }{-5}%
2304
     TestAdd{ -1 }{ -4 }{-5}%
     TestAdd{ -4 }{ 1 }{-3}%
2305
     TestAdd{ -1 }{ 4 }{3}%
2306
     TestAdd{ 4 }{ -1 }{3}%
2307
     TestAdd{1}{-4}{-3}%
2308
2309
     TestAdd{ -4 }{ -1 }{-5}%
     TestAdd{ -1 }{ -4 }{-5}%
2310
     \TestAdd{876543210}{111111111}{987654321}%
2312
     \TestAdd{999999999}{2}{1000000001}%
2313 \end{qstest}
2314
2315 \begin{qstest}{sub}{sub}
     \TestSub{0}{0}{0}%
2316
     \TestSub{1}{0}{1}%
2317
```

```
TestSub{1}{2}{-1}%
2318
      \TestSub{-1}{-1}{0}%
2319
      \TestSub{2147483646}{-1}{2147483647}%
2320
      \TestSub{-2147483647}{-2147483647}{0}%
2321
2322
      TestSub{-4}{-1}{-3}%
2323
      TestSub{-1}{-4}{3}%
2324
      TestSub{-4}{1}{-5}%
      TestSub{-1}{4}{-5}%
2325
2326
      TestSub{4}{-1}{5}%
      TestSub{1}{-4}{5}%
2327
      TestSub{-4}{-1}{-3}%
2328
      TestSub{-1}{-4}{3}%
2329
      TestSub{ -4 }{ -1 }{-3}%
2330
      TestSub{ -1 }{ -4 }{3}%
2331
      TestSub{ -4 }{ 1 }{-5}%
2332
2333
      TestSub{ -1 }{ 4 }{-5}%
2334
      TestSub{ 4 }{ -1 }{5}%
      TestSub{ 1 }{ -4 }{5}%
2335
      TestSub{ -4 }{ -1 }{-3}%
2336
2337
      TestSub{ -1 }{ -4 }{3}%
      \TestSub{1000000000}{2}{999999998}%
2338
      \TestSub{987654321}{111111111}{876543210}%
2339
2340 \end{qstest}
2341
2342 \left[ shl \right] 
2343
      \TestSh1{0}{0}%
2344
      TestShl{1}{2}%
2345
      TestShl{2}{4}%
2346
      \TestSh1{5621}{11242}%
      \TestSh1{1073741823}{2147483646}%
2347
2348 \end{qstest}
2349
2350 \begin{qstest}{shr}{shr}
      TestShr{0}{0}%
2351
      \TestShr{1}{0}%
2352
2353
      TestShr{2}{1}%
2354
      TestShr{3}{1}%
2355
      TestShr{4}{2}%
2356
      TestShr{5}{2}%
      TestShr{6}{3}%
2357
2358
      \TestShr{7}{3}%
      \TestShr{8}{4}%
2359
2360
      \TestShr{9}{4}%
2361
      \TestShr{10}{5}%
2362
      \TestShr{11}{5}%
2363
      \TestShr{12}{6}%
2364
      \TestShr{13}{6}%
2365
      \TestShr{14}{7}%
2366
      \TestShr{15}{7}%
2367
      \TestShr{16}{8}%
      \TestShr{17}{8}%
2368
2369
      \TestShr{18}{9}%
      \TestShr{19}{9}%
2370
2371
      \TestShr{20}{10}%
2372
      \TestShr{21}{10}%
2373
      \TestShr{22}{11}%
2374
      \TestShr{11241}{5620}%
2375
      TestShr{73054202}{36527101}%
2376
      \TestShr{2147483646}{1073741823}%
2377 \end{qstest}
2378
2379 \left[ qstest \right] \{ mul \} \{ mul \}
```

```
\TestMul{0}{0}{0}%
2380
      \TestMul{1}{0}{0}%
2381
      \TestMul{0}{1}{0}%
2382
      \TestMul{1}{1}{1}%
2384
      \TestMul{3}{1}{3}%
2385
      TestMul{1}{-3}{-3}%
2386
      TestMul{-4}{-5}{20}%
2387
      \TestMul{3}{7}{21}%
      \TestMul{7}{3}{21}%
2388
      TestMul{3}{-7}{-21}%
2389
      \TestMul{7}{-3}{-21}%
2390
      TestMul{-3}{7}{-21}%
2391
      TestMul{-7}{3}{-21}%
2392
      TestMul{-3}{-7}{21}%
2393
      \TestMul{-7}{-3}{21}%
2394
2395
      \TestMul{12}{11}{132}%
2396
      \TestMul{999}{333}{332667}%
      \TestMul{1000}{4321}{4321000}%
2397
      \TestMul{12345}{173955}{2147474475}%
2398
2399
      \TestMul{1073741823}{2}{2147483646}%
      \TestMul{2}{1073741823}{2147483646}%
2400
2401
      \TestMul{-1073741823}{2}{-2147483646}%
2402
      \TestMul{2}{-1073741823}{-2147483646}%
      \TestMul{6706022400}{13}{87178291200}%
2403
2404 \end{qstest}
2405
2407
      \TestSqr{0}{0}%
2408
      \TestSqr{1}{1}%
      \TestSqr{2}{4}%
2409
      \texttt{\TestSqr}{3}{9}\%
2410
2411
      \TestSqr{4}{16}%
2412
      \TestSqr{9}{81}%
2413
      \TestSqr{10}{100}%
      \TestSqr{46340}{2147395600}%
2414
2415
      \TestSqr{-1}{1}%
2416
      \TestSqr{-2}{4}%
      \TestSqr{-46340}{2147395600}%
2417
2418 \end{qstest}
2419
2420 \left[ \frac{qstest}{fac} \right]
      \TestFac{0}{1}%
2421
2422
      \TestFac{1}{1}%
2423
      \TestFac{2}{2}%
2424
      TestFac{3}{2*3}%
2425
      \TestFac{4}{2*3*4}%
2426
      TestFac{5}{2*3*4*5}%
2427
      \TestFac{6}{2*3*4*5*6}%
2428
      \TestFac{7}{2*3*4*5*6*7}%
2429
      \TestFac{8}{2*3*4*5*6*7*8}%
      \TestFac{9}{2*3*4*5*6*7*8*9}%
2430
      \TestFac{10}{2*3*4*5*6*7*8*9*10}%
2431
      \TestFac{11}{2*3*4*5*6*7*8*9*10*11}%
2432
2433
      \TestFac{12}{2*3*4*5*6*7*8*9*10*11*12}%
2434
      \TestFacBig{13}{6227020800}%
2435
      \TestFacBig{14}{87178291200}%
2436
      \TestFacBig{15}{1307674368000}%
2437
      \TestFacBig{16}{20922789888000}%
2438
      \TestFacBig{17}{355687428096000}%
      \TestFacBig{18}{6402373705728000}%
2439
      \TestFacBig{19}{121645100408832000}%
2440
2441
      \TestFacBig{20}{2432902008176640000}%
```

```
\TestFacBig{21}{51090942171709440000}%
2442
      \TestFacBig{22}{1124000727777607680000}%
2443
2444 \end{qstest}
2446 \begin{qstest}{pow}{pow}
2447
      \TestPow{-2}{0}{1}%
2448
      TestPow{-1}{0}{1}%
2449
      TestPow{0}{0}{1}%
2450
      \TestPow{1}{0}{1}%
      TestPow{2}{0}{1}%
2451
      \TestPow{3}{0}{1}%
2452
      TestPow{-2}{1}{-2}%
2453
2454
      TestPow{-1}{1}{-1}%
      \TestPow{1}{1}{1}%
2455
      \TestPow{2}{1}{2}%
2456
2457
      \TestPow{3}{1}{3}%
2458
      \TestPow{-2}{2}{4}%
      TestPow{-1}{2}{1}%
2459
      TestPow{0}{2}{0}%
2460
      \TestPow{1}{2}{1}%
2461
2462
      \TestPow{2}{2}{4}%
      \TestPow{3}{2}{9}%
2463
2464
      \TestPow{0}{1}{0}%
2465
      \TestPow{1}{-2}{1}%
      \TestPow{1}{-1}{1}%
2467
      TestPow{-1}{-2}{1}%
2468
      \TestPow{-1}{-1}{-1}%
2469
      TestPow{-1}{3}{-1}%
2470
      TestPow{-1}{4}{1}%
2471
      TestPow{-2}{-1}{0}%
      TestPow{-2}{-2}{0}%
2472
2473
      \TestPow{2}{3}{8}%
2474
      \TestPow{2}{4}{16}%
2475
      \TestPow{2}{5}{32}%
      \TestPow{2}{6}{64}%
2476
2477
      \TestPow{2}{7}{128}%
2478
      \TestPow{2}{8}{256}%
2479
      \TestPow{2}{9}{512}%
2480
      \TestPow{2}{10}{1024}%
2481
      TestPow{-2}{3}{-8}%
      TestPow{-2}{4}{16}%
2482
      TestPow{-2}{5}{-32}%
2483
2484
      TestPow{-2}{6}{64}%
2485
      TestPow{-2}{7}{-128}%
2486
      \TestPow{-2}{8}{256}%
      \TestPow{-2}{9}{-512}%
2488
      TestPow{-2}{10}{1024}%
2489
      \TestPow{3}{3}{27}%
2490
      \TestPow{3}{4}{81}%
2491
      \TestPow{3}{5}{243}%
      TestPow{-3}{3}{-27}%
2492
2493
      TestPow{-3}{4}{81}%
      TestPow{-3}{5}{-243}%
2494
2495
      \TestPow{2}{30}{1073741824}%
2496
      \TestPow{-3}{19}{-1162261467}%
      \TestPow{5}{13}{1220703125}%
2498
      \TestPow{-7}{11}{-1977326743}%
2499 \end{qstest}
2500
2501 \begin{qstest}{div}{div}
2502
      \TestDiv{1}{1}{1}%
     \TestDiv{2}{1}{2}%
2503
```

```
TestDiv{-2}{1}{-2}%
2504
```

- \TestDiv{2}{-1}{-2}% 2505
- \TestDiv{-2}{-1}{2}% 2506
- \TestDiv{15}{2}{7}% 2507
- 2508 \TestDiv{-16}{2}{-8}%
- 2509 \TestDiv{1}{2}{0}%
- 2510 \TestDiv{1}{3}{0}%
- 2511 \TestDiv{2}{3}{0}%
- $TestDiv{-2}{3}{0}%$ 2512
- $TestDiv{2}{-3}{0}%$ 2513
- $TestDiv{-2}{-3}{0}%$ 2514
- $TestDiv{13}{3}{4}$ % 2515
- 2516 $TestDiv{-13}{-3}{4}%$
- $TestDiv{-13}{3}{-4}%$ 2517
- $TestDiv{-6}{5}{-1}%$ 2518
- 2519 $TestDiv{-5}{5}{-1}%$
- 2520 $TestDiv{-4}{5}{0}$ %
- $\text{TestDiv}{-3}{5}{0}%$ 2521
- $TestDiv{-2}{5}{0}%$ 2522
- 2523 \TestDiv{-1}{5}{0}%
- 2524 \TestDiv{0}{5}{0}% \TestDiv{1}{5}{0}% 2525
- 2526 \TestDiv{2}{5}{0}%
- \TestDiv{3}{5}{0}% 2527
- 2528
- \TestDiv{4}{5}{0}%
- 2529 \TestDiv{5}{5}{1}% 2530
- \TestDiv{6}{5}{1}% 2531
- $\text{TestDiv}{-5}{4}{-1}%$
- 2532 $TestDiv{-4}{4}{-1}%$
- 2533 $TestDiv{-3}{4}{0}%$
- $TestDiv{-2}{4}{0}%$ 2534
- 2535 $TestDiv{-1}{4}{0}%$ 2536 $TestDiv{0}{4}{0}%$
- 2537 \TestDiv{1}{4}{0}%
- \TestDiv{2}{4}{0}% 2538
- 2539 \TestDiv{3}{4}{0}%
- 2540 \TestDiv{4}{4}{1}%
- 2541 $TestDiv{5}{4}{1}%$
- 2542 \TestDiv{12345}{678}{18}%
- 2543 \TestDiv{32372}{5952}{5}%
- \TestDiv{284271294}{18162}{15651}% 2544
- 2545 \TestDiv{217652429}{12561}{17327}%
- \TestDiv{462028434}{5439}{84947}% 2546
- 2547\TestDiv{2147483647}{1000}{2147483}% 2548 \TestDiv{2147483647}{-1000}{-2147483}%
- 2549
- \TestDiv{-2147483647}{1000}{-2147483}% 2550 TestDiv{-2147483647}{-1000}{2147483}%
- 2551 \TestDiv{0}{3}{0}%
- 2552 \TestDiv{1}{3}{0}%
- 2553 \TestDiv{2}{3}{0}%
- \TestDiv{3}{3}{1}% 2554
- 2555 $TestDiv{4}{3}{1}%$
- \TestDiv{5}{3}{1}% 2556
- \TestDiv{6}{3}{2}% 2557
- 2558 $TestDiv{7}{3}{2}%$
- 2559 \TestDiv{8}{3}{2}%
- 2560 \TestDiv{9}{3}{3}%
- 2561 \TestDiv{10}{3}{3}%
- 2562 \TestDiv{11}{3}{3}% 2563 \TestDiv{12}{3}{4}%
- \TestDiv{13}{3}{4}% 2564
- 2565 \TestDiv{14}{3}{4}%

```
\TestDiv{15}{3}{5}%
2566
      \TestDiv{16}{3}{5}%
2567
      \TestDiv{17}{3}{5}%
2568
      \TestDiv{18}{3}{6}%
2569
2570
      \TestDiv{19}{3}{6}%
2571
      \TestDiv{20}{3}{6}%
2572
      \TestDiv{21}{3}{7}%
2573
      \TestDiv{22}{3}{7}%
2574
      \TestDiv{23}{3}{7}%
      \TestDiv{24}{3}{8}%
2575
      \TestDiv{25}{3}{8}%
2576
      \TestDiv{26}{3}{8}%
2577
2578
      \TestDiv{27}{3}{9}%
      \TestDiv{28}{3}{9}%
2579
      \TestDiv{29}{3}{9}%
2580
2581
      \TestDiv{30}{3}{10}%
2582
      \TestDiv{31}{3}{10}%
      \TestDivBig{17363436332507}{24702}{702916214}%
2583
2584 \end{qstest}
2585
2586 \left\{ mod \right\} \pmod{mod}
      TestMod{-6}{5}{4}%
2587
      \TestMod{-5}{5}{0}%
2588
      TestMod{-4}{5}{1}%
2589
      TestMod{-3}{5}{2}%
2590
2591
      TestMod{-2}{5}{3}%
2592
      TestMod{-1}{5}{4}%
2593
      \TestMod{0}{5}{0}%
2594
      \TestMod{1}{5}{1}%
2595
      TestMod{2}{5}{2}%
      \TestMod{3}{5}{3}%
2596
2597
      TestMod{4}{5}{4}%
2598
      \TestMod{5}{5}{0}%
2599
      \TestMod{6}{5}{1}%
      TestMod{-5}{4}{3}%
2600
2601
      TestMod{-4}{4}{0}%
2602
      TestMod{-3}{4}{1}%
2603
      TestMod{-2}{4}{2}%
      TestMod{-1}{4}{3}%
2604
2605
      \TestMod{0}{4}{0}%
      \TestMod{1}{4}{1}%
2606
2607
      TestMod{2}{4}{2}%
      \TestMod{3}{4}{3}%
2608
2609
      TestMod{4}{4}{0}%
2610
      TestMod{5}{4}{1}%
2611
      TestMod{-6}{-5}{-1}%
2612
      TestMod{-5}{-5}{0}%
2613
      TestMod{-4}{-5}{-4}%
2614
      TestMod{-3}{-5}{-3}%
2615
      TestMod{-2}{-5}{-2}%
      TestMod{-1}{-5}{-1}%
2616
2617
      TestMod{0}{-5}{0}%
      TestMod{1}{-5}{-4}%
2618
2619
      TestMod{2}{-5}{-3}%
2620
      TestMod{3}{-5}{-2}%
2621
      TestMod{4}{-5}{-1}%
2622
      TestMod{5}{-5}{0}%
2623
      TestMod{6}{-5}{-4}%
2624
      TestMod{-5}{-4}{-1}%
2625
      TestMod{-4}{-4}{0}%
      TestMod{-3}{-4}{-3}%
2626
2627
      TestMod{-2}{-4}{-2}%
```

```
TestMod{-1}{-4}{-1}%
2628
      TestMod{0}{-4}{0}%
2629
2630
      TestMod{1}{-4}{-3}%
      TestMod{2}{-4}{-2}%
2631
      TestMod{3}{-4}{-1}%
2632
2633
      TestMod{4}{-4}{0}%
2634
      TestMod{5}{-4}{-3}%
2635
      \TestMod{2147483647}{1000}{647}%
      TestMod{2147483647}{-1000}{-353}%
2636
     \TestMod{-2147483647}{1000}{353}%
2637
     TestMod{-2147483647}{-1000}{-647}%
2638
     \TestMod{ 0 }{ 4 }{0}%
2639
2640
     \TestMod{ 1 }{ 4 }{1}%
     TestMod{ -1 }{ 4 }{3}%
2641
     TestMod{ 0 }{ -4 }{0}%
2642
     TestMod{ 1 }{ -4 }{-3}%
2643
2644
     TestMod{ -1 }{ -4 }{-1}%
2645
     \TestMod{18362}{25}{12}%
2646 \end{qstest}
2647
2648 \newcommand*{\TestError}[2]{%
2649
      \begingroup
        \expandafter\def\csname BigIntCalcError:#1\endcsname{}%
2650
2651
        \Expect*{#2}{0}%
        \expandafter\def\csname BigIntCalcError:#1\endcsname{ERROR}%
2652
2653
        \Expect*{#2}{OERROR}%
2654
      \endgroup
2655 }
2656 \begin{qstest}{error}{error}
      \TestError{FacNegative}{\bigintcalcFac{-1}}%
2657
      \TestError{FacNegative}{\bigintcalcFac{-2147483647}}%
2658
2659
     2660
     \TestError{DivisionByZero}{\bigintcalcDiv{1}{0}}%
2661
      \TestError{DivisionByZero}{\bigintcalcMod{1}{0}}%
2662 \end{qstest}
2663
2664 \begin{document}
2665 \end{document}
2666 (/test2)
```

4 Installation

4.1 Download

Package. This package is available on CTAN¹:

CTAN:macros/latex/contrib/oberdiek/bigintcalc.dtx The source file.

CTAN: macros/latex/contrib/oberdiek/bigintcalc.pdf Documentation.

Bundle. All the packages of the bundle 'oberdiek' are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

CTAN:install/macros/latex/contrib/oberdiek.tds.zip

TDS refers to the standard "A Directory Structure for TEX Files" (CTAN:tds/tds.pdf). Directories with texmf in their name are usually organized this way.

¹ftp://ftp.ctan.org/tex-archive/

4.2 Bundle installation

Unpacking. Unpack the oberdiek.tds.zip in the TDS tree (also known as texmf tree) of your choice. Example (linux):

```
unzip oberdiek.tds.zip -d ~/texmf
```

Script installation. Check the directory TDS:scripts/oberdiek/ for scripts that need further installation steps. Package attachfile2 comes with the Perl script pdfatfi.pl that should be installed in such a way that it can be called as pdfatfi. Example (linux):

```
chmod +x scripts/oberdiek/pdfatfi.pl
cp scripts/oberdiek/pdfatfi.pl /usr/local/bin/
```

4.3 Package installation

Unpacking. The .dtx file is a self-extracting docstrip archive. The files are extracted by running the .dtx through plain TFX:

```
tex bigintcalc.dtx
```

TDS. Now the different files must be moved into the different directories in your installation TDS tree (also known as texmf tree):

```
\label{localcond} \begin{array}{ll} \mbox{bigintcalc.sty} & \rightarrow \mbox{tex/generic/oberdiek/bigintcalc.sty} \\ \mbox{bigintcalc.pdf} & \rightarrow \mbox{doc/latex/oberdiek/bigintcalc.pdf} \\ \mbox{test/bigintcalc-test1.tex} & \rightarrow \mbox{doc/latex/oberdiek/test/bigintcalc-test1.tex} \\ \mbox{test/bigintcalc-test2.tex} & \rightarrow \mbox{doc/latex/oberdiek/test/bigintcalc-test3.tex} \\ \mbox{bigintcalc-test3.tex} & \rightarrow \mbox{source/latex/oberdiek/bigintcalc.dtx} \end{array}
```

If you have a docstrip.cfg that configures and enables docstrip's TDS installing feature, then some files can already be in the right place, see the documentation of docstrip.

4.4 Refresh file name databases

If your TEX distribution (teTEX, mikTEX, ...) relies on file name databases, you must refresh these. For example, teTEX users run texhash or mktexlsr.

4.5 Some details for the interested

Attached source. The PDF documentation on CTAN also includes the .dtx source file. It can be extracted by AcrobatReader 6 or higher. Another option is pdftk, e.g. unpack the file into the current directory:

```
pdftk bigintcalc.pdf unpack_files output .
```

Unpacking with IATEX. The .dtx chooses its action depending on the format:

plain TEX: Run docstrip and extract the files.

LATEX: Generate the documentation.

If you insist on using LATEX for docstrip (really, docstrip does not need LATEX), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{bigintcalc.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the .dtx or the .drv to generate the documentation. The process can be configured by the configuration file ltxdoc.cfg. For instance, put this line into this file, if you want to have A4 as paper format:

\PassOptionsToClass{a4paper}{article}

An example follows how to generate the documentation with pdfIATEX:

```
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
```

5 Catalogue

The following XML file can be used as source for the TeX Catalogue. The elements caption and description are imported from the original XML file from the Catalogue. The name of the XML file in the Catalogue is bigintcalc.xml.

```
2667 (*catalogue)
2668 <?xml version='1.0' encoding='us-ascii'?>
2669 <! DOCTYPE entry SYSTEM 'catalogue.dtd'>
2670 <entry datestamp='$Date$' modifier='$Author$' id='bigintcalc'>
2671
     <name>bigintcalc</name>
      <caption>Integer calculations on very large numbers.</caption>
2672
2673
      <authorref id='auth:oberdiek'/>
      <copyright owner='Heiko Oberdiek' year='2007,2011,2012'/>
2674
      <license type='lppl1.3'/>
2675
      <version number='1.3'/>
2676
2677
      <description>
       This package provides expandable arithmetic operations
2678
       with big integers that can exceed TeX's number limits.
2679
2680
       The package is part of the xref refid='oberdiek'>oberdiek bundle.
2681
2682 </description>
2683 <documentation details='Package documentation'
         href='ctan:/macros/latex/contrib/oberdiek/bigintcalc.pdf'/>
2684
     <ctan file='true' path='/macros/latex/contrib/oberdiek/bigintcalc.dtx'/>
2685
2686
     <miktex location='oberdiek'/>
2687
     <texlive location='oberdiek'/>
     <install path='/macros/latex/contrib/oberdiek/oberdiek.tds.zip'/>
2688
2689 </entry>
2690 (/catalogue)
```

6 History

```
[2007/09/27 \text{ v}1.0]
```

• First version.

```
[2007/11/11 v1.1]
```

• Use of package pdftexcmds for LuaTeX support.

[2011/01/30 v1.2]

• Already loaded package files are not input in plain TEX.

[2012/04/08 v1.3]

• Fix: pdftexcmds wasn't loaded in case of LATEX.

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