Appendix: Mathematical Derivations for Base Morphic Field (BMF)

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1 Action and Euler-Lagrange Equation

The action is:

$$S[\phi_0, g, M] = \int \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi_0 \partial_\nu \phi_0 - \frac{\lambda}{4} (\phi_0 - \phi_{vac})^4 + f(\phi_0, M) F_{\alpha\beta} F^{\alpha\beta} + \kappa \int M(\phi_0(x'), \partial\phi_0(x')) K(x, x') \, d^4$$

where $K(x, x') = \theta(x^0 - x'^0) \frac{e^{-|x-x'|^2/\sigma^2}}{(2\pi\sigma^2)^{3/2}}$ is the causal kernel. Varying w.r.t. ϕ_0 yields:

$$\Box_g \phi_0 + \lambda(\phi_0 - \phi_{vac})^3 + \kappa \int \left[\frac{\partial M}{\partial \phi_0(x')} + \partial'_\mu \left(\frac{\partial M}{\partial(\partial'_\mu \phi_0(x'))} \right) \right] K(x, x') \, d^4 x' = 0.$$

2 Noether's Theorem and Stress-Energy Tensor

For translation symmetry, the conserved current leads to:

$$T_{\mu\nu} = \partial_\mu \phi_0 \partial_\nu \phi_0 - g_{\mu\nu} \left[\frac{1}{2} \partial^\alpha \phi_0 \partial_\alpha \phi_0 - \frac{\lambda}{4} (\phi_0 - \phi_{vac})^4 + \kappa M \right],$$

with $\nabla^\mu T_{\mu\nu} = 0$ on-shell.

3 Stability via Lyapunov Functional

The Lyapunov functional is:

$$E = \int \left[\frac{1}{2} (\partial_t \delta\phi_0)^2 + \frac{1}{2} |\nabla \delta\phi_0|^2 + \frac{\lambda}{2} (\delta\phi_0)^2 + \frac{\lambda}{4} (\delta\phi_0)^4 + \kappa \int \delta M \, d\delta\phi_0 \right] d^3 x,$$

where $\dot{E} \leq 0$ ensures $\phi_0 \to \phi_{vac}$.

4 Feedback Request

Please review for: - Consistency with QFT (e.g., renormalization of κ term). - Causal kernel validity (e.g., σ scale). - Suggestions for physical ties (e.g., units for ϕ_{vac}).