DTFT:

$$\begin{cases} x[n] = \dfrac{1}{2\pi}\displaystyle\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}\,d\omega \\ X(e^{j\omega}) = \displaystyle\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \end{cases}$$

سوال ۱)

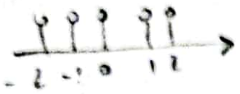
(آ) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (1/5)^{-n} u[-n-1] e^{-j\omega n} = \sum_{n=-\infty}^{-1} (1/5)^{n} e^{-j\omega n} = \sum_{n=1}^{\infty} (1/5)^{n} e^{j\omega n}$

$= \sum_{n=1}^{\infty} (\frac{1}{5}e^{j\omega})^n = \dfrac{\frac{1}{5}e^{j\omega}}{1-\frac{1}{5}e^{j\omega}}$

$(1/5)^n u[n] \leftrightarrow \dfrac{1}{1-\frac{1}{5}e^{-j\omega}} \xRightarrow{-n} (1/5)^{-n} u[-n] \leftrightarrow \dfrac{1}{1-\frac{1}{5}e^{j\omega}} \xRightarrow{-1} (1/5)^{-n} \times (1/5)^{-1} u[-n-1] \leftrightarrow \dfrac{e^{j\omega}}{1-\frac{1}{5}e^{j\omega}}$

(ب) $x(n) = u[n+2] - u[n-3]$

$X(e^{j\omega}) = 1 + e^{j\omega} + e^{-j\omega} + e^{j2\omega} + e^{-j2\omega} = 1 + 2\cos(\omega) + 2\cos(2\omega)$



(ج) $x[n] = 2\delta[n-2]$

$X(e^{j\omega}) = 2e^{-j\omega 2}$

(د) $x[n] = \sin(\frac{5\pi n}{3}) + \cos(\frac{7\pi n}{3}) = \dfrac{e^{j\frac{5\pi n}{3}} - e^{-j\frac{5\pi n}{3}}}{2j} + \dfrac{e^{j\frac{7\pi n}{3}} + e^{-j\frac{7\pi n}{3}}}{2}$

$e^{j\omega_0 n} \leftrightarrow 2\pi\,\delta(\omega-\omega_0)$

$X(e^{j\omega}) = \frac{2\pi}{2j}\left[\delta(\omega - \frac{5\pi}{3}) - \delta(\omega - \frac{5\pi}{3})\right] + \frac{2\pi}{2}\left(\delta(\omega - \frac{7\pi}{3}) + \delta(\omega + \frac{7\pi}{3})\right)$

(ه) $X(e^{j\omega}) = \sum_{n=-N}^{N} \frac{1}{2}\left(1 + \dfrac{e^{j\frac{\pi}{N}n} + e^{-j\frac{\pi}{N}n}}{2}\right)e^{-j\omega n}$

$= \frac{1}{2}\dfrac{\sin(\frac{2N+1}{2}\omega)}{\sin(\frac{1}{2}\omega)} + \frac{1}{2}\dfrac{\sin(\frac{2N+1}{2}(\omega - \frac{\pi}{N}))}{\sin(\frac{1}{2}(\omega - \frac{\pi}{N}))} + \frac{1}{2}\dfrac{\sin(\frac{2N+1}{2}(\omega + \frac{\pi}{N}))}{\sin(\frac{1}{2}(\omega + \frac{\pi}{N}))}$

(و) $x[n] = \underbrace{\dfrac{\sin(\pi/4\, n)}{\pi n}}_{\begin{cases}1 & |\omega|\le \pi/4 \\ 0 & \pi/4 \le |\omega| \le \pi\end{cases}} * \underbrace{\dfrac{\sin \frac{\pi}{4}(n-8)}{\pi(n-8)}}_{e^{-j8\omega} \times \{ \,''}} \rightarrow X(e^{j\omega}) = \begin{cases} e^{-j8\omega} & |\omega| \le \pi/4 \\ 0 & \pi/4 < |\omega| \le \pi \end{cases}$

(آ) $\cos^2 x = \dfrac{1+\cos 2x}{2}, \quad \sin^2 x = \dfrac{1-\cos 2x}{2}$

$X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega = 1 + \frac{1}{2}\underbrace{\cos(2\omega)}_{\frac{e^{j2\omega}+e^{-j2\omega}}{2}} - \frac{1}{2}\underbrace{\cos(6\omega)}_{\frac{e^{j6\omega}+e^{-j6\omega}}{2}}, \quad \cos(\omega) = \dfrac{e^{j\omega}+e^{-j\omega}}{2}$

$\delta[n-n_0] \leftrightarrow e^{-j\omega n_0}$

$\rightarrow x[n] = \delta[n] + \frac{1}{4}\delta(n+2) + \frac{1}{4}\delta(n-2) + \frac{1}{4}\delta(n+6) - \frac{1}{4}\delta(n-6)$

(ادامه سوال ۱)

(ب) $X(e^{j\omega}) = e^{j\omega/2} \quad \underline{-\pi \leq \omega \leq \pi}$

$\mathcal{F}^{-1}\{e^{-j\omega n_0}\} \longleftrightarrow \delta[n-n_0] \quad , n_0 = -1/2 \Big\} \; x[n] = \delta[n-n_0] \; \times$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega/2} \cdot e^{j\omega n} \, d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(1/2+n)\omega} \, d\omega$$

$$= 1/2\pi \left[\frac{1}{j(1/2+n)} e^{j(1/2+n)\omega}\right]_{-\pi}^{\pi} = 1/2\pi \, \frac{1}{j(1/2+n)} \Big(\underbrace{\frac{e^{j(1/2+n)\pi} - e^{-j(1/2+n)\pi}}{2j}}_{\sin}\Big)$$

$$= \frac{\sin((1/2+n)\pi)}{\pi(1/2+n)}$$

(ج) به فرم زیر $X(e^{j\omega}) \begin{cases} 1 & , 0 < |\omega| < \pi/2 \\ 0 & \pi/2 < |\omega| \leq \pi \end{cases} \longrightarrow x[n] = \dfrac{\sin(\pi/2\, n)}{\pi n}$

$X(j\omega) = \begin{cases} 1 & 0 \leq |\omega| \leq W \\ 0 & W \leq |\omega| \leq \pi \end{cases} \longrightarrow x[n] = \dfrac{\sin Wn}{\pi n}$

(د) $X(e^{j\omega}) = \cos(2\omega) + j\sin(2\omega) = \underline{\delta[n+2]}$

$$1/2\pi \int_{-\pi}^{\pi} e^{j\Omega(2+n)} d\Omega$$

(ه) $X(e^{j\omega}) = \cos(\omega) + j\cos(\omega/2) \Rightarrow$

$$x[n] = 1/2\pi \int_{-\pi}^{\pi} \left(\frac{e^{j\Omega} - e^{-j\Omega}}{2j} + \frac{e^{j\Omega/2} + e^{-j\Omega/2}}{2}\right) e^{j\Omega n} \, d\Omega$$

$$= 1/2j\,\delta[n+1] - 1/2j\,\delta[n-1] + \frac{1}{2\pi}\frac{\cos(\pi n)}{n+0.5} - \frac{1}{2\pi}\frac{\cos(\pi n)}{n-0.5}$$

(و) $$x[n] = \frac{1}{2\pi}\int_{0.25\pi}^{0.75\pi} e^{j\Omega(n-4)} d\Omega + \frac{1}{2\pi}\int_{-0.75\pi}^{-0.25\pi} e^{j\Omega(n-4)} d\Omega$$

$$= \frac{\sin(0.75\pi(n-4)) - \sin(0.25\pi(n-4))}{\pi(n-4)}$$

(20)

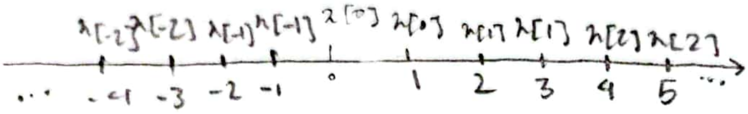
(ت) $X(e^{j\omega}) = \dfrac{2e^{-j\omega}}{-0.25e^{-2j\omega}+1} = \dfrac{A}{1-\frac{1}{2}e^{-j\Omega}} + \dfrac{B}{1+\frac{1}{2}e^{j\Omega}}$ $\qquad a^n u[n] \underset{|a|<1}{\longleftrightarrow} \dfrac{1}{1-ae^{-j\omega}}$

$$\begin{aligned} 2 &= A/2 - B/2 \\ 0 &= A+B \end{aligned} \quad \rightarrow \quad \begin{aligned} A &= 2 \\ B &= 2 \end{aligned} \quad \longrightarrow \quad x[n] = \left[2\left(\tfrac{1}{2}\right)^n - 2\left(-\tfrac{1}{2}\right)^n\right]u[n]$$

(ث) $X(e^{j\omega}) = \dfrac{6-2e^{-j\omega}+0.5e^{-j2\omega}}{(-0.25e^{-j2\omega}+1)(1-0.25e^{-j\omega})} = \dfrac{A}{1+\frac{1}{2}e^{-j\Omega}} + \dfrac{B}{1-\frac{1}{2}e^{-j\Omega}} + \dfrac{C}{1-\frac{1}{4}e^{-j\Omega}}$

$$\begin{aligned} 6 &= A+B+C \\ -2 &= -3/4\,A + 1/4\,B \\ 1/2 &= 1/8\,A - 1/8\,B - 1/4\,C \end{aligned} \quad \rightarrow \quad \begin{aligned} A &= 4 \\ B &= 4 \\ C &= 2 \end{aligned} \quad \longrightarrow \quad x[n]\left[4\left(-\tfrac{1}{2}\right)^n + 4\left(\tfrac{1}{2}\right)^n - 2\left(\tfrac{1}{4}\right)^n\right]u[n]$$

سوال 3)



$$y[n] = x_2[n] + x_2[n-1] \xrightarrow{\mathcal{F}} Y(e^{j\omega}) = X(e^{2j\omega}) + X(e^{2j\omega})e^{-j\omega}$$

سوال 4)

$$h[n] = -h[N-1-n] \Rightarrow H(e^{j\omega}) = -H(e^{-j\omega})e^{-j\omega(N-1)}$$

$$\rightarrow \measuredangle H(e^{j\omega}) = \pi + \underbrace{\measuredangle H(e^{-j\omega})}_{\substack{h[n] \text{ is real} \\ \rightarrow \measuredangle H(e^{-j\omega}) = -\measuredangle H(e^{j\omega})}} + \measuredangle e^{-j\omega(N-1)}$$

$$\rightarrow \measuredangle H(e^{j\omega}) = \pi - \measuredangle H(e^{j\omega}) - \omega(N-1)$$

$$\rightarrow \measuredangle H(e^{j\omega}) = \pi/2 - \frac{\omega(N-1)}{2} \Rightarrow \begin{cases} a = -\omega/2 \\ b = 1/2 \\ c = \pi/2 \end{cases}$$

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$\rightarrow \frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$

$\rightarrow \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 d\omega$

$\rightarrow \quad x[n] * y[n] = X(e^{j\omega}) Y(e^{j\omega})$

$\rightarrow \begin{cases} x[n] = \sum_{k=\langle N \rangle} a_k \exp(jk \frac{2\pi}{N} n) \\ X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - \frac{2\pi k}{N}) \end{cases}$

$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{1 + e^{-j\omega}}$ (T) (5U'a

$x[n] = 2.5 - 0.5 \cos(\pi n)$ $\quad \omega_0 = \pi = \frac{2\pi}{N}$

$N = 2$

$\hookrightarrow m = 1$

(ب) $X(e^{j\omega}) = 1 + \frac{e^{-j\omega}}{(1 - 0.5 e^{-j\omega})(1 + 0.25 e^{-j\omega})} = \frac{1 + 3/4\, e^{-j\omega} - 1/8\, e^{-j2\omega}}{1 - 1/4\, e^{-j\omega} - 1/8\, e^{-j2\omega}}$

$\rightarrow y[n] - 1/4\, y[n-1] - 1/8\, y[n-2] = x[n] + 3/4\, x[n-1] - 1/8\, x[n-2]$

(2) $h[n] = \delta[n] + 2(1/2)^n u[n] + (-1/2)^n u[n] \rightarrow H(e^{j\omega}) = 1 + \frac{2}{1 - \frac{1}{2} e^{-j\omega}} + \frac{1}{1 + 1/2\, e^{-j\omega}}$

سوال 6)

آ) $x[n-n_0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega(n-n_0)}\, d\omega$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{X(e^{j\omega})\, e^{-j\omega n_0}}_{DTFT\{x[n-n_0]\}}\, e^{j\omega n}\, d\omega$$

ب) $x[n] * y[n] \longleftrightarrow C(e^{j\Omega})$

$$C(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty} x[l]\, y[n-l]\, e^{-j\Omega n}$$

$$= \sum_{l=-\infty}^{\infty} x[l]\, e^{-j\Omega l} \underbrace{\sum_{n=-\infty}^{\infty} y[n-l]\, e^{-j\Omega(n-l)}}_{Y(e^{j\Omega})}$$

$$= \sum_{l=-\infty}^{+\infty} Y(e^{j\Omega})\, x[l]\, e^{-j\Omega l}$$

$$= X(e^{j\Omega})\, Y(e^{j\Omega})$$

ج) $x[n]\, y[n] \longleftrightarrow M(e^{j\Omega})$

$$M(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, y[n]\, e^{-j\Omega n}, \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma})\, e^{j\Gamma n}\, d\Gamma$$

$$M(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} y[n]\, \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma})\, e^{j\Gamma n}\, d\Gamma\, e^{-j\Omega n}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma}) \sum_{n=-\infty}^{\infty} y[n]\, e^{j\Gamma n}\, e^{-j\Omega n}\, d\Gamma$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma}) \underbrace{\sum_{n=-\infty}^{\infty} y[n]\, e^{-j(\Omega-\Gamma)n}}_{Y(e^{j(\Omega-\Gamma)})}\, d\Gamma$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma})\, Y(e^{j(\Omega-\Gamma)})\, d\Gamma$$

$$= \frac{1}{2\pi}\, X(e^{j\Omega}) * Y(e^{j\Omega})$$

ادامه سوال 6)

(د) $x_z[n] = 0$, unless n/p is integer $\rightsquigarrow$ مضرب p

$$z[n] = x_z[pn] \rightsquigarrow Z(e^{j\Omega}) = X_z(e^{j\Omega/p})$$

$$X_z(e^{j\Omega/p}) = \sum_{n=-\infty}^{\infty} x_z[n]\, e^{-j\frac{\Omega}{p}n}.$$

$n = pr$ $\rightarrow$ sum over r

$$= \sum_{r=-\infty}^{\infty} x_z[pr]\, e^{-j\Omega r}$$

$$= \sum_{n=-\infty}^{\infty} x_z[pn]\, e^{-j\Omega n}$$

$$= Z(e^{j\Omega})$$

سوال 7)

$1, 3/4, 7/16, 15/64$

(آ) $y[n] - a\,y[n-1] + b\,y[n-2] = x[n]$

$x[n]=0,\ y[n]=0\ ;\ n<0$

$x[0]=1,\ y[0]=1 \rightarrow 1=1$

$x[1]=0,\ y[1]=3/4 \rightarrow 0 = 3/4 - a \rightarrow a = 3/4$

$x[2]=0,\ y[2]=7/16 \rightarrow 0 = 7/16 - a\times 3/4 + b \rightarrow b = 9/16 - 7/16 = -1/16$

(ب) $Y(e^{j\omega})\left(1 - \frac{3}{4}e^{-j\omega} - \frac{1}{16}e^{-j2\omega}\right) = X(e^{j\omega})$

$$\rightarrow H(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}e^{-j\omega} - \frac{1}{16}e^{-j2\omega}} = \frac{A}{1 + d e^{-j\omega}} + \frac{B}{1 + c e^{-j\omega}}$$

$\left.\begin{array}{l} c+d = -3/4 \\ cD = -1/16 \end{array}\right\} \rightarrow c = -3/4 - d$, $cD = -3/4 d - d^2 = -1/16 \rightarrow d^2 + 3/4 d - 1/16 = 0 \rightarrow d \begin{cases} (-3+\sqrt{13})/8 & \rightarrow c = (-3+\sqrt{13})/8 \\ (-3-\sqrt{13})/8 & \rightarrow c = (-3+\sqrt{13})/8 \end{cases}$

← برابر گذاشتن مثبت مال موهومی و حقیقی در بسط آوردن

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$\left.\begin{array}{l} A + B = 1 \rightarrow cA + cB = c \\ cA + dB = 0 \end{array}\right\}$ $(c-d)B = c \rightarrow B = \frac{c}{c-d}$

$A = 1 - B = 1 + \frac{c}{d-c} = \frac{d}{d-c}$

$\rightarrow y[n] = \frac{d}{d-c}[-d]^n u[n] + \frac{c}{c-d}[-c]^n u[n]$ جمع نهایی