Applied Advanced Optimisation iRAT 1

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Question 1

State all the languages you speak.

English, Chinese (Mandarin).

Question 2

Write the solution of $\min_x ||Ax - b||_2^2$**.**

To minimise $(Ax-b)^T(Ax-b) = x^TA^TAx - 2x^TA^Tb + b^Tb$,

we take the derivative with respect to x and set it to zero: $2A^TAx - 2A^Tb = 0$.

Thus $x^* = (A^TA)^{-1}A^Tb$.

Question 3

Explain why in the norm approximation problem, the ℓ_1-norm generates a large number of zero residuals.

Because the ℓ_1-norm adds the absolute values together, when minimising the residues using ℓ_1-norm, we get a very sparse solution.

Question 4

Which penalty function would you pick to reduce sensitivity to outliers?

ℓ_1-norm or the Huber penalty function.

Question 5

Give the solution to the ℓ_2 least-norm problem $\min_x ||x||_2^2 \;\; s.t. \; Ax = b$**.**

$x^* = A^\top(AA^\top)^{-1}b$.

Question 6

Give the solution of the Tikhonov regularisation problem.

We have the general equation of Tikhonov regularisation problem:

$$\min_x ||Ax - b||_2^2 + \gamma ||x||_2^2$$

Objective function:

$$J(x) = ||Ax - b||_2^2 + \gamma ||x||_2^2$$

We take the derivative with respect to x and set it to zero:

$$\nabla_x J(x) = 2A^T Ax - 2A^T b + 2\gamma x = 0$$

$$A^T Ax + \gamma x = A^T b$$

Thus:

$$x = (A^T A + \gamma I)^{-1} A^T b$$

Question 7

Make a comparison between the solution of the nominal least-square, stochastic least-square and worst-case least-square.

The nominal least-square solution achieves best result when $u = 0$, ie. when there are no significant outliers.

The stochastic least-square solution performs better than nominal least-square solution with large u, ie. when the data is noisy.

The worst-Case least-square is least sensitive to large number of outliers, it has a similar performance across all u.

Question 8

Declare a variable x of dimension n in CVXPY.

According to https://www.cvxpy.org/tutorial/intro/index.html, we can declare:

```
import cvxpy as cp
x = cp.Variable(n)
```

Question 9

Declare the constraints $x + y = 1$ and $x - y \geq 1$ in CVXPY.

```
y = cp.Variable()
constraints = [
    x + y == 1,
    x - y >= 1
]
```

Question 10

Which atomic function would you use in CVXPY to compute the ℓ_∞-norm of a variable x?

```
norm_inf_x = cp.norm(x, "inf")
```