Test 4

Dr G. Scarciotti

January 29, 2025

- **Please submit a pdf with your answers on blackboard by Thursday 6th of February at 13:00. Please state name, surname, and CID.**

  Use the handouts to reply to these questions. Do not use generative AI tools. Note that each answer is in the handouts.

  You cannot answer using screenshots. You need to reply by typing or in your own writing. This promotes subconscious retention.

  This is an individual test. You cannot collaborate with other students or any other individual. This must be your work.

  You should be able to complete this in 1 page. Do not write, nor spend too much time on this.

1. (2 marks) Define quasiconvexity and give the modified Jensen's inequality for quasiconvex functions.
2. Define a convex optimisation problem.
3. Write the bisection algorithm to solve a quasiconvex optimisation problem.
4. Consider a general convex optimisation problem and suppose that f_0 is differentiable. State a necessary and sufficient condition for x to be optimal.
5. (2 marks) Consider the convex optimisation problem

$$\begin{array}{ll} \min & f_0(x) \\ s.t. & Ax = b. \end{array}$$

   State necessary and sufficient conditions for x to be optimal.

6. (3 marks) Consider the convex optimisation problem

$$\begin{array}{ll} \min & f_0(x) \\ s.t. & x \succcurlyeq 0. \end{array}$$

   State necessary and sufficient conditions for x to be optimal.

Appendix

The following questions were part of the test in previous years. I removed them to make the test faster to complete. These questions are not marked, but you can use them as extra exercise material.

- Consider the function $f(x) = \dfrac{1}{g(x)}$, where $g(x)$ is positive for all $x \in \mathbb{R}$. You want to use the composition rule to establish whether $f(x)$ is convex, concave or neither. To this end:
  a) Identify the function $h(x)$ (you should consider the function only for $x > 0$).
  b) Find $\tilde{h}$ and show whether this is non-decreasing or non-increasing.
  c) Add a missing assumption on $g(x)$ to apply the composition rule.
- Define log-concavity and give the modified Jensen's inequality for log-concave functions.