《专业外语》期末试题

数学与统计学院 2017级 统计班

2018年11月22日, 26教学楼409室

姓名： here

学号： here

注意事项:

- 遵守考场纪律. 作弊者考试成绩一律以0分记, 并上报学院;
- 考试为开卷, 可以带书、笔记、作业、手机、笔记本电脑和LaTeX文档;自带笔记本电脑完成考试;
- 可以上网查阅相关资料, 但严禁通过手机或电脑相互传递或发布信息;
- 总分100分, 其中95分卷面, 5分为LaTeX编译规范;
- 考试一律用LaTeX完成, 并把Tex及生成的PDF文件(定义为自己的姓名+学号)提交监考老师;
- 英文完成考题.

Question 1. *(30分) Answer the following questions:*

(1). Random variables X and Y are independent. Assume that one of them is continuous. Let $Z = X + Y$. Does Z is continuous? If Z is continuous, give its pdf;

(2). Let X and Y be independent random variables with common standard normal distribution. How about the distribution of $X + Y$ and $X - Y$, and calculate $\mathbf{E}\max\{X, Y\}$;

(3). Assume $(X, Y) \sim N(0, 0; 1, 1; \rho)$ with $|\rho| < 1$. Find the conditional distribution of X given $Y = y$.

Solution.

$\square$

Question 2. *(15分) Let X be a positive continuous random variable with survival function $S(x) = \mathbb{P}(X > x)$ and $\mathbf{E}\, X$ exists. Let $e_X(x) = \mathbf{E}\{(X - x)|X > x\}$ be the mean excess loss function. Show that X follows exponential distribution if and only if $e_X(x) \equiv \theta$, a positive constant, for all $x > 0$.*

Proof. $\square$

Question 3. *(15分) Let the probability mass function of nonnegative integer-valued random variable Y be given by*

$$\mathbb{P}\{Y = k\} = \binom{k+r-1}{k} \left(\frac{1}{1+\beta}\right)^r \left(\frac{\beta}{1+\beta}\right)^k, \quad k = 0, 1, 2, \cdots$$

where parameter $r > 0, \beta = \beta(r) > 0$.

(1). Calculate the characteristic function of Y;

(2). Find the limit distribution of Y under the condition that $r\beta \to \lambda > 0$ as $r \to \infty$.

Solution.

$\square$

Question 4. *(20分) Let $X_1, X_2, \cdots, X_r$ follow the multinomial distribution, i.e.,*

$$\mathbb{P}\Big\{X_1 = k_1, X_2 = k_2, \cdots, X_r = k_r\Big\} = \frac{n!}{(k_1)!(k_2)!\cdots(k_r)!} p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$$

with $\sum_{j=1}^r p_j = 1, \sum_{j=1}^r k_j = n$. Show that

(1). $\mathbf{E}\, X_j = np_j, \mathbf{Var}(X_j) = np_j(1 - p_j)$ for $j = 1, 2, \cdots, r$;

(2). For $i \neq j$, the correlation of X_i and X_j is given by

$$\mathbf{Corr}(X_i, X_j) = -\sqrt{\frac{p_i p_j}{(1-p_i)(1-p_j)}}.$$

Proof. □

Question 5. *(15分) (1). Let X and Y be two continuous random variables with pdfs $f_X(x)$ and $f_Y(y)$, respectively. The conditional density of X given $Y = y$ is denoted by $f_{X|Y=y}(x|y)$. Prove that*

$$f_X(x) = \int_{-\infty}^{\infty} f_{X|Y=y}(x|y) f_Y(y) dy; \tag{1}$$

(2). Assume that $X \sim N(\theta, a)$ given $\Theta = \theta$, and Θ itself follows normal distribution with mean $\mathbf{E}\,\Theta = \mu$ and variance $\mathbf{Var}\,\Theta = b$, where $a > 0, b > 0$. Find the pdf of X by using (1).

Solution.

□