



Repulsive force in point charge-neutral metal system

Abstract

One classic problem in electrostatics is the interaction between a charged particle and a perfectly conducting, neutral metal. Generally speaking, the electric force will be attractive, which make sense intuitively: the opposite charge will be induced closer to the source charge, resulting an attractive force. However, if we consider the configuration of a metallic hemisphere concave up with a point charge moving on the z-axis, some repulsive force appear at a small distance just above the cross section.

Matlab Simulation

1. We consider a two-dimensional analogue of this problem, which means the metallic semicircle is infinitesimally thin (with only surface charge density σ).
2. Solve the Poisson equation for the system, and obtain the electric potential V . We can further obtain the potential energy $U(z)$ and the electric force F_z , which enable us to simulate the motion of the point charge.
3. Adjust the parameters of the simulation to make the particle's motion reasonable.

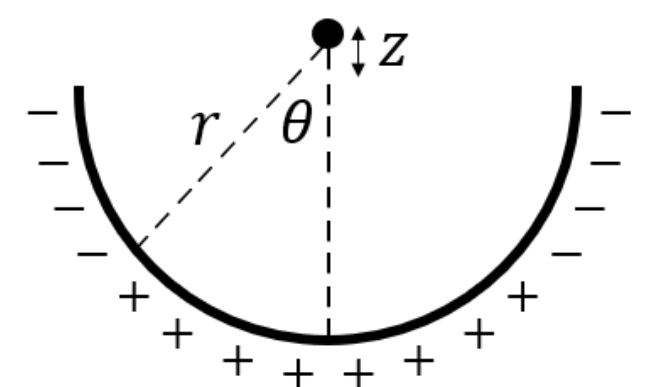


FIG.1 The side view of a point charge reside just next to an infinitesimally thin semicircle with induced charge.

Result

From FIG. 3 and FIG. 4, we find that when the particle is still far from the metallic hemisphere, it will be attracted downwards, as most of us have expected. However, once the position of the particle z is smaller than R (the radius of the semicircle), the force acting on the particle becomes repulsive, and the potential energy will go to 0 as $z \rightarrow 0$. In the end, it will be “pushed” upward, and return to the initial position.

Due to symmetry, we only need to consider the force on the z-axis, which is proportional to $\cos \theta / r^2$. When $z \ll R$, $\cos \theta$ dominates, and the positive charges below, therefore, wins out, resulting in a repulsive force.

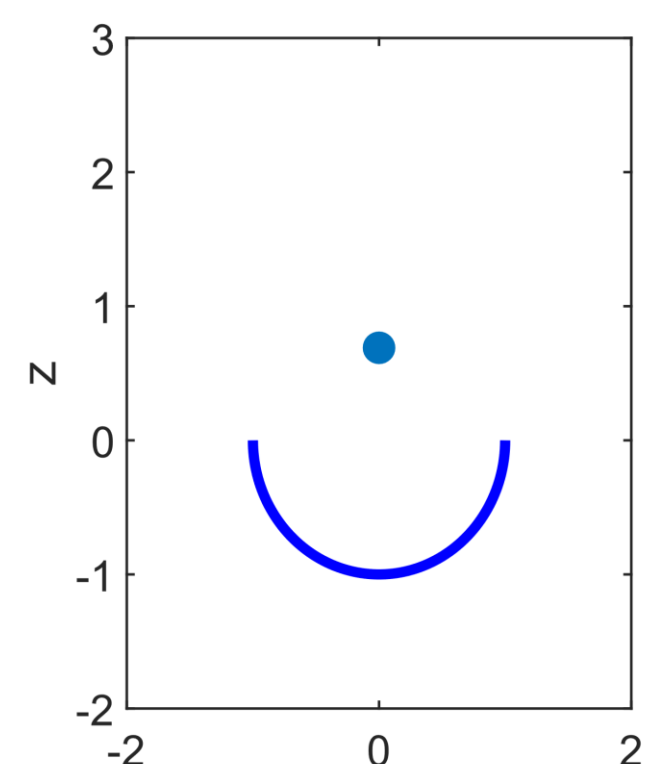


FIG. 2 Matlab simulation

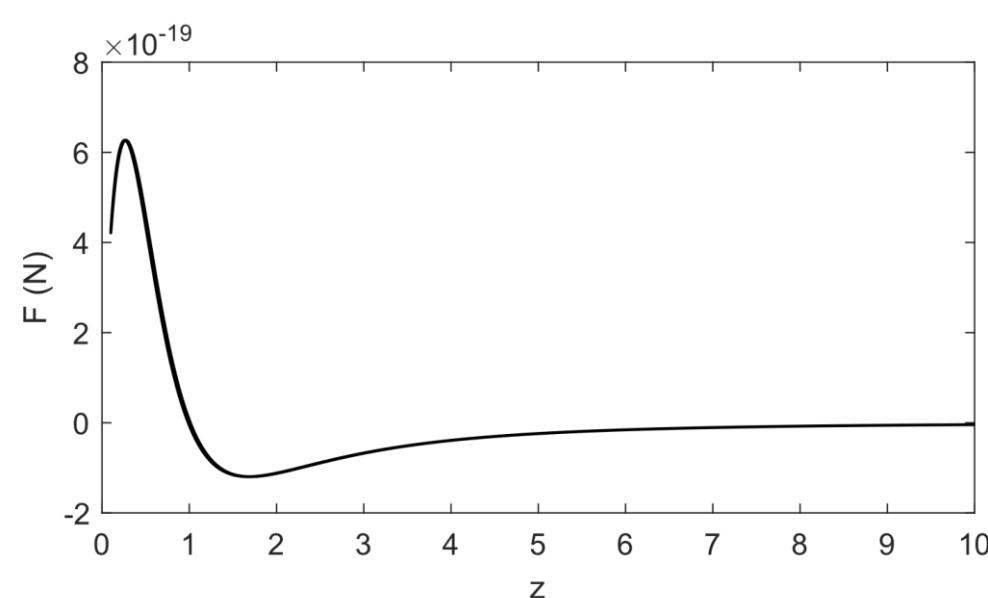
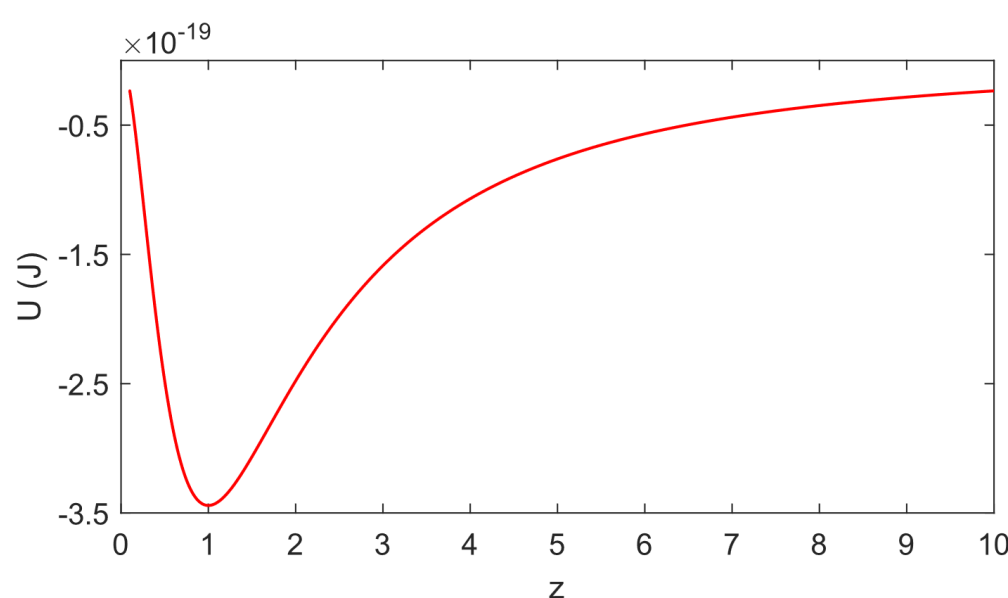


FIG.3 and 4 Potential energy $U(z)$ and electric force $F(z)$ for the 2D system.

References

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2. Griffiths D J Introduction to Electrodynamics 4rd edition
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