QUANTUM TECHNOLOGY (PHY602)

Basics of Quantum Noise and Quantum Error Correction. Description of phase flip error, amplitude error, and explanations of the 3-bit and 5-bit error correcting codes.

Symeonakis Aristotelis & Bampaniotis Fotios
Supervising Professor D. Angelakis

June 2021

Abstract

Throughout the journey of our course we discussed system states and their possibilities. In order for our cold maths to take place we have to isolate our system from the outside world. But why? Cosmic noise prevents us from building chips with massive amounts of qubits because of their delicate nature. Lets try to bring up some of the problems and weapons to overcome this problematic foe.

Preliminary

In order to approach further concepts, some quantum information basics is usual.

The qubit

Since we focus on the informational domain, we define a unit to carry our bits of information, namely, the qubit. In physical implementation it can be a cold ion or a photon or something of the kind, but we wont deal with that here. One qubit can have values of 0 or 1 or anything in between. Keep in mind that a qubit represents one bit of classical information only when stored or measured! Upon processing, a qubit can behave as an infinite number of classical bits because of its property to obtain any value from the space continuum [0, 1]. We will describe the qubit states using Dirac's formalization *bra* and *ket* as shown below:

ket, qubit representation for the 0, 1 basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

bra, qubit representation for the 0, 1 basis:

$$\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}, \langle 1| = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

Two more interesting terms are the *physical* and *logical* qubits. By definition, a physical qubit is something tangible and can be affected by foreign matter and energy. A logical qubit is a theoretical mechanism of portraying information. We choose, for our ease, to not model any noise consequences on our logical qubits when solving algorithms or executing maths but, since real machines contain physical qubits we have to transduce our logical ones to physical ones by extending our operations accordingly.

Operators

As in classical computation, we are in the need of tools capable of physically implementing our mathematical calculations. Instead of classical gates we now have quantum gates or simply put, operators. They are usually represented in the form of a squared matrix with dimensions corresponding to the number of qubits they will act on. Some basic operators are the Pauli sigmas or rotations:

$$\sigma_I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Kraus operators take our state to a post-channel realization.

Trace & Partial trace:

Suppose a matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

the trace of this operator is the sum of its diagonal elements:

$$Tr(a) = a + d.$$

This stands for any squared array regardless its dimensions.

The partial trace comes in handy on multi qubit systems. It's a specialization of the trace for a target qubit in the system. Lets say:

$$\rho_B = Tr_A(\rho) = Tr_A(|\beta_{10}\rangle\langle\beta_{10}|)$$

Channel

A quantum channel is the medium in which a set

of predefined qubit(s) will be transmitted, in the same or a different state, to another peer.
Say in the case we transmit a single qubit, we can model the channel as an operator acting on our qubit:

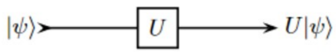


Figure 1: Unitary quantum channel

A channel that induces noise to our qubits is called a *depolarizing channel*. We can represent the channel acting on a density matrix ρ as:

$$\Delta_\lambda(\rho) = \sum_{i=0}^{3} K_i \rho K_i^\dagger,$$

where K_i are the Kraus operators given by

$$K_0 = \sqrt{1 - \frac{3\lambda}{4}} I, K_1 = \sqrt{\frac{\lambda}{4}} X,$$
$$K_2 = \sqrt{\frac{\lambda}{4}} Y, K_3 = \sqrt{\frac{\lambda}{4}} Z.$$

Density matrix

When we are dealing with a multiple number of qubits, called an *ensemble*, having an averaging operator comes in handy. The Density operator does that work by holding all the probabilities of each qubit state to appear.

Density matrix for a single qubit:

$$\rho = \frac{I + \vec{n} \cdot \vec{\sigma}}{2},$$

with n and σ spanned on the Pauli x, y, z.

Density operator for pure states:

$$\rho = |\psi\rangle\langle\psi|.$$

Density operator for mixed states:

$$\rho_a = |a\rangle\langle a|$$

$$\rho_b = |b\rangle\langle b|,$$

in the case our ensemble can be found in either of the states:

$$|a\rangle = \alpha|x\rangle + \beta|y\rangle$$

$$|b\rangle = \gamma|x\rangle + \delta|y\rangle.$$

There are 3 required properties for a density operator:

- The density operator is Hermitian.
- $Tr(\rho) = 1$.
- The density operator is a positive operator.

Commutator – Anticommutator

The *commutator* of two operators A and B is defined as

$$[A, B] = AB - BA.$$

When [A, B] = 0, we say that the operators *A* and *B commute*.

The *anticommutator* of two operators A and B is defined as

$$\{A, B\} = AB + BA.$$

When {A, B} = 0, we say that the operators *A* and *B anticommute*.

Quantum noise

In quantum information there is no need to divide our attention into physical obstacles such as hardware construction, maintenance and isolation but that is not the case here. We define as a *closed system*, that, which has no interaction with the world outside the system's circuit and of course, such a system can exist only in theory. We call the realistic modelization of our quantum systems *open systems*, these can interact with energy outside the circuit, energy that mostly causes problems as we will state shortly.

Noise can produce errors when acting operators or even just by measuring some system's output.

The *bit flip error*, as the name suggests, inverts the state of a qubit. It takes us from a state |ψ⟩ to X|ψ⟩ with probability *p* and does nothing with probability *1-p*. The channel that may flip our sent state |ψ⟩ is called *bit flip channel*. We can model a bit-flip noise as a single qubit having control over our informational qubit:

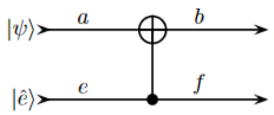
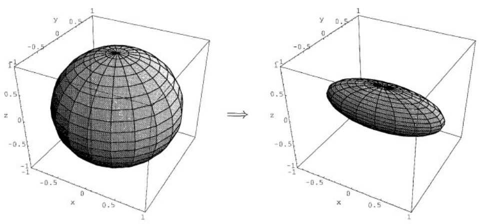


Figure 2: Unitary quantum noise model

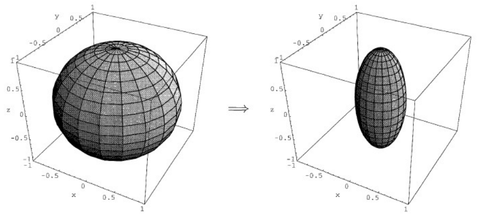
The bit flip error can be visualized on the Bloch sphere as seen below for p = 0.3:



The sphere on the left is the set of all pure states, while the sphere on the right represents the states after going through the channel.

Something that doesn't have a classical analogue is the *phase flip error* which affects the phase of a qubit in such a way that a transmitted state $|\psi\rangle$ will end up $Z|\psi\rangle$ with a probability *p* and stay the same with a probability *1-p*. The channel that flips our state's phase is called a p*hase flip channel*.

The phase flip error can be visualized on the Bloch sphere as seen below for p = 0.3:



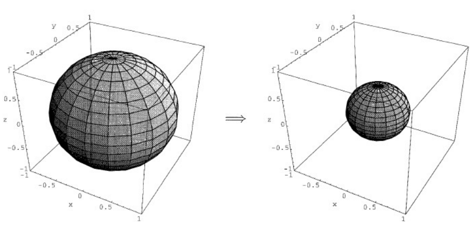
While the x – y plane is uniformly contracted, states on the z axis are left alone.

Pauli Y gate errors are the combination of X and Z, since Y = iXZ.

Our confined qubit space has a certain Hamiltonian energy upon which we base our calculations. When our system interacts with the outside world and there is energy exchange, we cannot always configure what exactly is going on and errors are made.

The depolarizing channel affects a qubit with probability p and makes it polarized, with probability 1 – p the qubit is not affected. A polarized qubit is replaced with mixed state *I/2* and the system state after the effect of the depolarizing channel noise is $\mathcal{E}(\rho) = \frac{pI}{2} + (1-p)\rho$

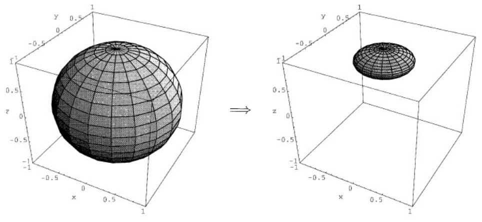
The effect for the depolarizing channel on the Bloch sphere is illustrated below for p = 0.5:



The Bloch sphere contracts uniformly as a function of p.

Another type of quantum noise is *amplitude damping*, which we describe as an operator that degrades an excited state $|1\rangle$ to the ground state $|0\rangle$ while leaving the state $|0\rangle$ unchanged. This effect corresponds to the physical process of losing energy to the environment. The operation of amplitude damping has the general characteristic of a set(s) of states that are left invariant under the operation, in this case the unchanged $|0\rangle$ state, despite of a $|1\rangle$ state losing amplitude.

The effect of amplitude damping is visualized on the Bloch sphere below for p = 0.8, where it can be seen how the sphere shrinks towards the north pole, which is the $|0\rangle$ state.



Finally, *phase damping* is a type of quantum noise which describes the loss of information without the loss of energy.
To understand this type of noise we need to mention that the energy eigenstates of a quantum system accumulate a phase over time, which is proportional to the eigenvalue, while remaining unchanged. When a system evolves for unknown time, information about the relative phases between energy eigenstates is lost.

The effect of phase dumping has already been seen since it is identical to the effect of the phase flip channel.

Quantum error correction (QEC)

Controlling operational error and decoherence is one of the major challenges facing the field of quantum computation. Lets overview some of the basic quantum error correcting tools but first some background theory.

If there are *n* qubits in the quantum system, then error operators will be of *length* n. The *weight* of an error operator is the number of terms not equal to I. For example $X_{10011}Z_{00110}$ has length 5, weight 4.

Let H = {M} be a set of commuting error operators. Since the operators all commute, they can have simultaneous eigenstates. Let C = {$|u\rangle$} be an orthonormal set of simultaneous eigenstates all having eigenvalue +1:

$$M|u\rangle = |u\rangle \forall u \in C, \forall M \in H$$

The set C is a quantum error correcting code and H is its *stabilizer*. The orthonormal states $|u\rangle$ are termed *codewords.* If an operator M satisfies $M|\psi\rangle = |\psi\rangle$ we say that M stabilizes the state $|\psi\rangle$. The stabilizer formalism exploits elements of the Pauli group $\Pi = \{I, X, Y, Z\}$.

Quantum error correction also employs *syndrome qubit* measurements by performing a multi – qubit measurement that does not disturb the quantum information in the encoded state but retrieves information about the error. A syndrome measurement can determine whether a qubit has been corrupted, and if so, which one. What's more, the outcome of this operation tells us not only which physical qubit was affected but also in which of several possible ways it was affected. An all zero syndrome tells you that all parity check equations are satisfied.

Measurement error mitigation can be accomplished with linear algebra. We can take the thumbprint of some noise in an N by N identity matrix and multiply with the state vector of a given system. What we get is a good approach of an erroneous measurement of the above system. To treat the aforementioned problem we take the inverse of the noise thumbprint matrix and multiply it with the noisy state vector to end up with a clean non-error system measurement. Suppose we want to mitigate the measurement error on a bell state circuit. The ideal measurement for 10.000 tries would be:

$$C_{ideal} = \begin{pmatrix} 5000 \\ 0 \\ 0 \\ 5000 \end{pmatrix},$$

were the index of the matrix represents the states $|00\rangle \to |11\rangle$.

We collect the measures of a system from state $|00\rangle$ to $|11\rangle$, represented here in matrix form M

$$M = \begin{pmatrix} 0.9808 & 0.0107 & 0.0095 & 0.0001 \\ 0.0095 & 0.9788 & 0.0001 & 0.0107 \\ 0.0096 & 0.0002 & 0.9814 & 0.0087 \\ 0.0001 & 0.0103 & 0.0090 & 0.9805 \end{pmatrix}.$$

By applying the noise thumbprint M to C_{ideal} we get a replica of the noisy measurement:

$$C_{noisy} = MC_{ideal}$$

$$C_{noisy} = \begin{pmatrix} 1.01978 & -0.01115 & -0.00987 & 0.00011 \\ -0.0099 & 1.02188 & 0.00009 & -0.01115 \\ -0.00997 & 0 & 1.01913 & -0.00904 \\ 0.00009 & -0.01073 & -0.00935 & 1.02009 \end{pmatrix} \begin{pmatrix} 5000 \\ 0 \\ 0 \\ 5000 \end{pmatrix}$$

$$C_{noisy} = \begin{pmatrix} 4904.5 \\ 101 \\ 91.5 \\ 4903 \end{pmatrix}.$$

Linear algebra tells us that

$$C_{ideal} = M^{-1}C_{noisy}$$

So we calculate the inverse of M

$$M^{-1} = \begin{pmatrix} 1.01978 & -0.01115 & -0.00987 & 0.00011 \\ -0.0099 & 1.02188 & 0.00009 & -0.01115 \\ -0.00997 & 0 & 1.01913 & -0.00904 \\ 0.00009 & -0.01073 & -0.00935 & 1.02009 \end{pmatrix}$$

And apply

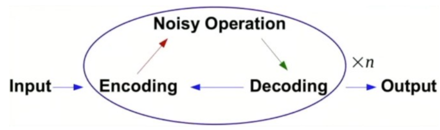
$$C_{ideal} = \begin{pmatrix} 1.01978 & -0.01115 & -0.00987 & 0.00011 \\ -0.0099 & 1.02188 & 0.00009 & -0.01115 \\ -0.00997 & 0 & 1.01913 & -0.00904 \\ 0.00009 & -0.01073 & -0.00935 & 1.02009 \end{pmatrix} \begin{pmatrix} 4904.5 \\ 101 \\ 91.5 \\ 4903 \end{pmatrix}$$

So we end up with the ideal state vector

$$C_{ideal} = \begin{pmatrix} 5000 \\ 0 \\ 0 \\ 5000 \end{pmatrix}.$$

The question now is how to correct errors happening at any time, in any place, at any form.

The simplest way for troubleshooting random noise could be by using repetition codes. The basic concept tells that by transmitting chunks of information in a known repetitive manner, we contradict the random nature of noise which does not encourage acts in the same way twice, let alone thrice. For every repetition the probability of error decreases exponentially.



The 3-qubit, bit flip *Quantum Error Correction Code* (QECC) heals a qubit affected by the equivalent type of noise and having one of the encoded bits flipped. The general algorithm goes as follows:

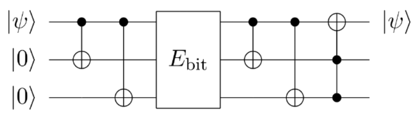


Figure 3: 3-qubit bit flip QECC circuit

Firstly we encode every $|0\rangle$ as $|000\rangle$ and every $|1\rangle$ as $|111\rangle$ so a state like: $|\psi\rangle = \alpha|0\rangle + |1\rangle$ ends up as $|\psi\rangle = \alpha\, |000\rangle + |111\rangle$.
The E_{bit} box represents the bit flip channel explained earlier.
Lets see the bit flip channel in action:

Say we initialize our qubit in the ground state:

$$|\psi_0\rangle = |0\rangle$$

The bit flip noise acts as an X gate:

$$|\psi_1\rangle = X|\psi_0\rangle$$
$$|\psi_1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$|\psi_1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$|\psi_1\rangle = |1\rangle.$$

At the circuit's output, we decode with the CNOT gates and compare the digits in the triplet states with the Toffoli gate to see if there is a culprit with changed value. If that's the case, we flip again the minority bit hoping that only one error occurred.

Because of the measurement at the end of the bit & phase flip circuits, any existing superpositions will collapse, so these are not nifty applications for every circumstance.
An approach, that bit more clever, is to use our information qubits to act on another qubit we don't intend on keeping in a superposition. There is a relative of the above code, using stabilizers.

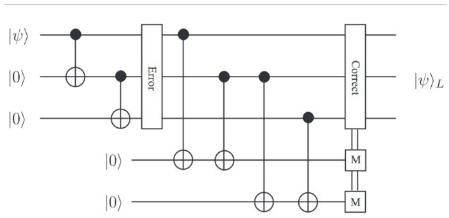


Figure 4: 3-qubit bit flip stabilizer circuit

The encoding part remains but, after passing through the noisy channel, we extract the two syndromes to inspect our state. This is done with a total of four CNOT gates and a Z measurement as shown in Figure 4. If no errors (or in an unfortunate situation, two errors) occur the CNOTs will either do nothing, or swap the syndrome qubit once and twice back to its initial state. In case of a single qubit error in the encoded state, one of the CNOT pairs will act an odd number of times ending up with a different outcome than its even acting sibling. By measuring in the Z plane, we acquire the error syndrome and find the error using Table 1. We can easily get to the conclusion that the stabilizer generators are Z_1Z_2 and Z_2Z_3.

Z_1Z_2	Z_2Z_3	Error type	Action
+1	+1	no error	no action
+1	−1	bit 3 flipped	flip bit 3
−1	+1	bit 1 flipped	flip bit 1
−1	−1	bit 2 flipped	flip bit 2

Table 1: 3-qubit bit flip stabilizer correction pattern

After the discovery of error existence and location, we can simply act the inverse operation to treat the single qubit bit flip error.
What we have achieved by the stabilize version of the code is that it remained encoded during the process and also, no measurement was made on the encoded state thus, maintaining any superposition.

A phase flip QECC works as a remedy on a qubit which has its phase changed by a phase flip type of noise.

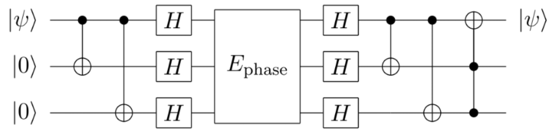


Figure 5: 3-qubit phase flip QECC circuit

Similar to the bit flip QECC, we encode our physical qubit, then we superpose our qubits, so that we can make use of the phase flip and pass them through a phase flip channel E_{phase}.
Lets see the phase flip channel in action:

Suppose we initialize our qubit in the excited state:

$$|\psi_0\rangle = |1\rangle$$

We superpose our qubit so that we can see the phase flip:

$$|\psi_1\rangle = H|\psi_0\rangle$$
$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}$$
$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$$

The phase flip noise acts as a Z gate:

$$|\psi_2\rangle = Z|\psi_1\rangle$$
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}$$
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$$

We then depose our qubit:

$$|\psi_3\rangle = H|\psi_2\rangle$$
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$$
$$|\psi_3\rangle = \frac{1}{2}\begin{pmatrix}2\\ 0\end{pmatrix}$$
$$|\psi_3\rangle = |0\rangle.$$

And end up with a bit flip error.

Now, we decode our state with the use of the CNOT gate and heal the bit flip caused by the phase flip channel with the Toffoli gate.

There is also a stabilizer version of the phase flip QECC. The basic idea is like that of the bit flip stabilizer code but instead of stabilizers Z_1Z_2, Z_1Z_2 we now have X_1X_2, X_2X_3 yielding the corresponding syndromes.

state	probability
$a\|000\rangle + b\|111\rangle$	$(1-p)^3$
$a\|100\rangle + b\|011\rangle$	$p(1-p)^2$
$a\|010\rangle + b\|101\rangle$	$p(1-p)^2$
$a\|001\rangle + b\|110\rangle$	$p(1-p)^2$
$a\|110\rangle + b\|001\rangle$	$p^2(1-p)$
$a\|101\rangle + b\|010\rangle$	$p^2(1-p)$
$a\|011\rangle + b\|100\rangle$	$p^2(1-p)$
$a\|111\rangle + b\|000\rangle$	p^3

Table 2: Encoded output state probability for input $a|0\rangle + b|1\rangle$ with error probability p.

Repetition codes rely on the method of *majority voting* and the probability of success is:

$$(1-p)^3 + 3p(1-p)^2 = 1 - 3p^2 + 2p^3$$

We can correct a qubit affected by any type of noise with a stabilizer code that encodes one qubit onto five. This is the minimum number of encoded qubits for correcting all types of error.

The four stabilizers of this 5-qubit code are:

$$M_1 = X_1\, Z_2\, Z_3\, X_4\, I_5$$
$$M_2 = I_1\, X_2\, Z_3\, Z_4\, X_5$$
$$M_3 = X_1\, I_2\, X_3\, Z_4\, Z_5$$
$$M_4 = Z_1\, X_2\, I_3\, X_4\, Z_5$$

Were I_i, X_i, Y_i, Z_i are the Pauli sigmas acting on the i[th] qubit. $M_i \in S$, were S is the stabilizer state space.

For a single X, Y, Z error we encode the logical qubit on the 5-bit QECC as:

$$|0\rangle_L = \sum_{M \in S} M|00000\rangle$$

$$\begin{aligned}|0\rangle_L = 1/4[&|00000\rangle + M_1|00000\rangle + M_2|00000\rangle \\ &+ M_3|00000\rangle + M_4|00000\rangle + M_1M_2|00000\rangle \\ &+ M_1M_3|00000\rangle + M_1M_4|00000\rangle + M_2M_3|00000\rangle \\ &+ M_2M_4|00000\rangle + M_3M_4|00000\rangle \\ &+ M_1M_2M_3|00000\rangle + M_1M_2M_4|00000\rangle \\ &+ M_1M_3M_4|00000\rangle + M_2M_3M_4|00000\rangle \\ &+ M_1M_2M_3M_4|00000\rangle] = \\ 1/4[&+ |00000\rangle + |10010\rangle + |01001\rangle + |10100\rangle \\ &+ |01010\rangle - |11011\rangle - |00110\rangle - |11000\rangle \\ &- |11101\rangle - |00011\rangle - |11110\rangle - |01111\rangle \\ &- |10001\rangle - |01100\rangle - |10111\rangle + |00101\rangle],\end{aligned}$$

and

$$|1\rangle_L = \mathbf{X}|0\rangle_L.$$

If no error happened, the stabilizers will prompt +1, else, one or more stabilizers will prompt -1. In case an error has occurred, we act the syndrome stabilizer on our state to inverse the corresponding error. Because every possible error syndrome of the 5-qubit QECC is used by the different single qubit errors, we say it's a *perfect* code.

The encoding begins by appending four additional qubits to the information state $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$, yielding $|\psi_1\rangle = |\psi 0000\rangle$. We then prepare a controlled matrix defined by $C_1 = (X_4X_3X_2X_1)^5$ and act on the extended state $C_1|\psi_1\rangle = |\psi_2\rangle$. Next, we prepare the coding matrix occurring from the stabilizers $T_1 = \prod_{i=1}^{4}\{\sum_{j=0}^{1} M_i^j\}$ and act on the controlled state $T_1|\psi_2\rangle = |\psi_C\rangle$ to obtain the encoded state representation $|\psi_C\rangle = \alpha_0|0\rangle_L + \alpha_1|1\rangle_L$.

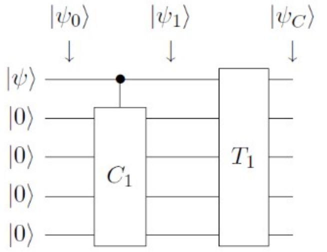


Figure 6: 5-qubit encoding circuit

Now we can pass our state through a noisy channel. Once the encoded state is sent, there is a probability that an error acts on it, so the received state can be modeled as $|\psi_R\rangle = E|\psi_C\rangle$.

The decoding process begins by identifying and correcting any potential error on our received state retrieving the initial quantum information. To identify the errors E we compute and measure the syndromes $S_i = \langle\psi_C|H_i|\psi_C\rangle$. Next we correct the error by applying $E^\dagger|\psi_R\rangle = |\psi'\rangle$ ($|\psi'\rangle = |\psi_R\rangle$ if $E^\dagger E = I$).

Errors pattern E	Syndrome $S_1\ S_2\ S_3\ S_4$
X_1	1 1 1 -1
X_2	-1 1 1 1
X_3	-1-1 1 1
X_4	1-1-1 1
X_5	11- 1 -1
Z_1	-1 1 -1 1
Z_2	1 -11 -1
Z_3	1 1-1 1
Z_4	-1 11 -1
Z_5	1 -1 1 1
Y_1	-1 1-1-1
Y_2	-1 -1 1 1
Y_3	-1-1-1 1
Y_4	-1-1-1-1
Y_5	1-1-1-1

Table 3: Correctable error patterns for the 5-qubit code

Finally to get back to our single qubit, we define a preparation matrix $P = |00000\rangle\langle 00000| + |11111\rangle\langle 11111|$ and act it on $|\psi'\rangle$ so $P|\psi'\rangle = P|\psi_C\rangle = |\psi_2\rangle$. A qubit is appended to our bit-stream resulting to an auxiliary state $|\psi_2\rangle \otimes |0\rangle = |\psi_{20}\rangle$ and we apply a controlled matrix $C_2 = (X_1)^{x_6 x_5 x_4 x_2 x_2}$ to it:

$$C_2|\psi_{20}\rangle = |\psi\rangle$$

and finally

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle.$$

We successfully recovered our initial state from any type of single qubit error!

Conclusion

Even though we tried to present the problems and solutions of quantum noise in a perceivable way, these concepts may still cause a puzzlement to many of us. The key ideas to remember are the Pauli rotations and their importance in error producing and solving, the repetition algorithm properties that discourages random noise patterns and finally the stabilizer method of constructing error correcting codes. Our understanding is that the QEC field has not quite evolved yet and could use some further research.