SOLUTIONS

Physics 471 – Quiz #6

Friday, November 17, 2023

Name:____ (v1)

You do not need to show any calculations for this quiz.

This quiz has questions on both sides of the paper!

1) [3] A particle of mass m is trapped in an infinite square well potential of width L, with boundaries at $x = 0$ and $x = L$. If the initial state vector is $|\psi(t=0)\rangle = |E_2\rangle$ (circle the correct answers for parts b and c):

a) What is $\langle\hat{x}\rangle$ at $t = 0$? $L/2$ by symmetry.

$|E_2\rangle$ is a stationary state.

b) Does $\langle\hat{x}(t)\rangle$ change with time? YES NO

c) Does $\langle\hat{H}\rangle$ (the expectation value of the energy) change with time? YES NO

$\langle\hat{H}\rangle = E_2$ always

See Homework 9, problem 1

2) [3] Consider the same particle in an infinite square well from the previous problem. If the initial state vector is $|\psi(t=0)\rangle = \frac{1}{\sqrt{5}}(|E_1\rangle + 2|E_2\rangle)$ (circle the correct answers for parts a and c):

a) Does $\langle\hat{x}(t)\rangle$ change with time? YES NO

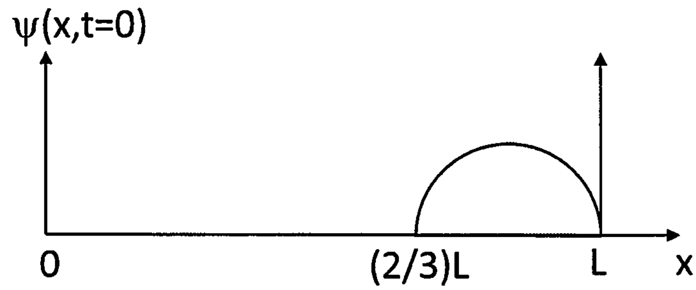
b) What is $\langle\hat{H}\rangle$ at $t = 0$? (Express your answer in terms of E_1 and E_2.)

$$\langle\hat{H}\rangle = \sum_n E_n P(E_n) = \frac{1}{5}E_1 + \frac{4}{5}E_2$$

c) Does $\langle\hat{H}\rangle$ change with time? YES NO

Energy is conserved when $\hat{H}$ doesn't depend on time.

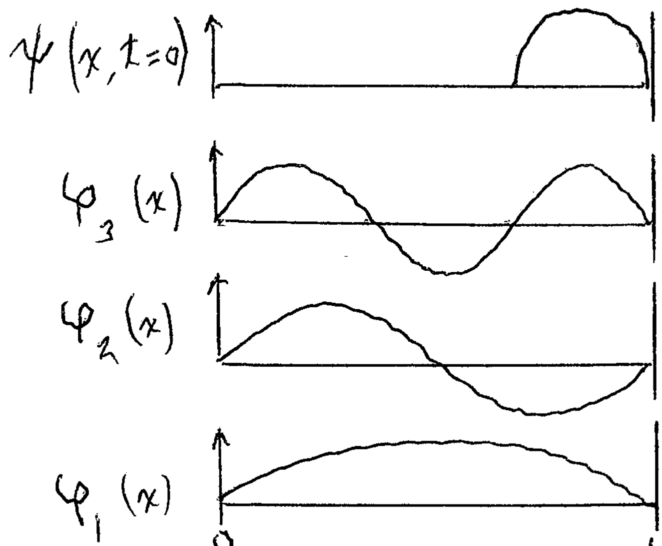
See Homework 9, problem 2



3) [4] The picture to the right shows the initial wave function (at $t = 0$) of a particle in an infinite square well that extends from $x = 0$ to $x = L$. The wave function is a perfect semi-circle; it is nonzero only for $\frac{2}{3}L < x < L$.

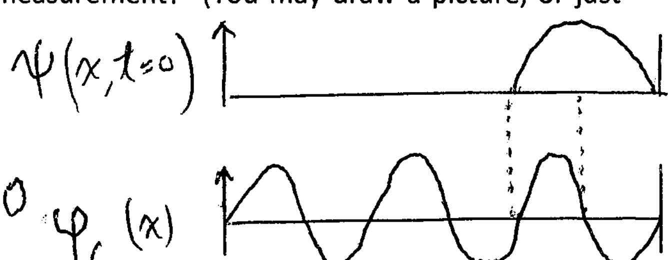
a) [2] If you were to measure the energy of the particle, there are many possible answers you might get: E_1, E_2, E_3, etc. Which of those results do you think is most likely? To answer this question, draw pictures of the lowest 3 energy eigenstate wave functions, $\varphi_n(x)$, for the infinite well. (If you find it helpful, you may want to re-draw $\psi(x, t = 0)$ directly above your other pictures.)

E_3 is most likely, because $\varphi_3(x)$ has the greatest overlap with $\psi(x, t=0)$.



b) [1] Considering all the energy eigenvalues for this infinite square well, which is the smallest one that you will <u>never</u> get from an energy measurement? (You may draw a picture, or just explain your reasoning in words.)

You will <u>never</u> measure E_6, because $\langle E_6 | \psi(t=0) \rangle = 0$



c) [1] If we express $|\psi(t = 0)\rangle$ as a superposition of energy eigenstates: $|\psi(t = 0)\rangle = \sum_n c_n |E_n\rangle$, then we can write the energy measurement probabilities as $P(E_n) = |c_n|^2$. Write down two expressions for c_n: one is a bracket (inner product), the other is an integral. You do <u>not</u> need to write out the explicit forms for the energy eigenstate wave functions, $\varphi_n(x)$, or for the initial wave function $\psi(x, t = 0)$.

$$c_n = \langle E_n | \psi(t=0) \rangle = \int_{-\infty}^{\infty} dx\, \varphi_n^*(x)\, \psi(x, t=0)$$