PHY 481 - Fall 2023
Homework 10

Due Sunday, December 10, 2023

Preface

Homework 10 finishes our discussion of magnetism by focusing on the Ampere's law and the vector potential, which is a useful tool for calculating the magnetic field in some problems. It also introduces the idea of magnetic fields in matter using the H-field.

1 Ampere's Law - Volume Current (5.3)

Consider a thick SLAB of current. The slab is infinite in (both) x and y, but finite in z. So we must think about the volume current density $\mathbf{J}$, rather than $\mathbf{K}$. The slab has thickness $2h$ (It runs from $z = -h$ to $z = +h$) Let's assume that the current is still flowing in the $+x$ direction, and is uniform in the x and y dimensions, but now $\mathbf{J}$ depends on height linearly, i.e. $\mathbf{J} = J_0 \,\mathrm{abs}(z)\, \hat{x}$, inside the slab (but is 0 above or below the slab), where $\mathrm{abs}(z)$ is the absolute value of z.

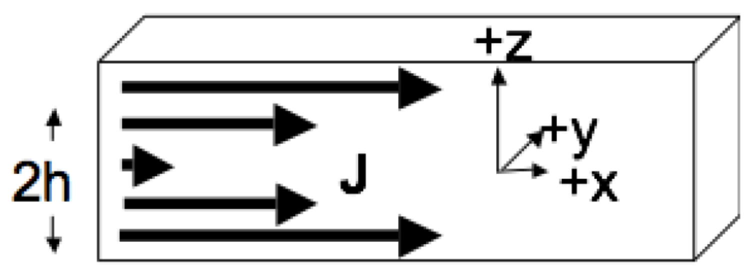

Figure 1: Thick Slab

Find the B field (magnitude and direction) everywhere in space (above, below, and also, most interestingly, inside the slab!)

2 Magnetic Vector Potential (5.4)

1. A long (infinite) wire (cylindrical conductor, radius R, whose axis coincides with the z axis) carries a uniformly distributed current I_0 in the $+z$ direction. Assuming $\nabla \cdot \mathbf{A} = 0$ (the "Coulomb gauge"), and choosing $\mathbf{A} = 0$ at the edge of the wire, show that the vector potential inside the wire could be given by $A = cI_0(1 - s^2/R^2)$. Find the constant c (including units.) Things to explicitly find/discuss: What is the vector direction of $\mathbf{A}$? (Does it "make sense" in any way to you?) Is your answer unique, or is there any remaining "ambiguity" in $\mathbf{A}$? (Note that we're not asking you to derive $\mathbf{A}$ from scratch, just to see that this choice of A "works")

2. What is the vector potential outside that wire? (Make sure that it still satisfies $\nabla \cdot \mathbf{A} = 0$, and make sure that $\mathbf{A}$ is continuous at the edge of the wire, consistent with part 1). Here again, is your answer unique, or is there any remaining "ambiguity" in $\mathbf{A}(outside)$?

3 Semi-classical electron magnetic dipole moment (6.1)

A thin uniform solid torus (a "donut") has total charge Q, mass M, radius R. It rotates around its own central axis at angular frequency ω, as shown.

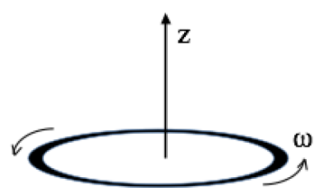

Figure 2: Spinning "Donut"

1. Find the magnetic dipole moment m of this rotating donut. What are the SI units of dipole moment?

2. Compute the ratio m/L, the "magnetic dipole moment" divided by the angular momentum. This is called the "gyromagnetic ratio."

3. What is the gyromagnetic ratio for a uniform spinning sphere? *HINT: This question really doesn't require any additional calculating: picture the sphere as a bunch of rings, and apply the result of part 2.*

4. In quantum mechanics, the angular momentum of a "spinning electron" is $\hbar/2$. Use your results above to deduce the electron's magnetic dipole moment (in SI units.) *This "semi-classical" calculation is low by a factor of almost exactly 2. Dirac developed a relativistic form of quantum mechanics which got the factor of 2 right in the 1930's. In the '40's, Feynman, Schwinger, and Tomonaga calculated tiny extra corrections arising from*

QED (Quantum electrodynamics). See the current best-value for the electron magnetic dipole moment. If you compare theory and measurement, you will be extremely impressed at the agreement (around 15 digits!) It may make you "believe" in quantum physics in a way you might not have before! That's not how it works in practice though- people use this measurement to extract a fundamental constant of nature, and then use that value to predict OTHER experiments.

4 Bound Currents (6.2)

Consider a long magnetic rod, radius a. Imagine that we have set up a permanent azimuthal magnetization $\mathbf{M}(s, \phi, z) = cs\hat{\phi}$, with c= constant, and s is the usual cylindrical radial coordinate. Neglect end effects, assume the cylinder is infinitely long.

1. Calculate the bound currents $\mathbf{K}_b$ and $\mathbf{J}_b$ (on the surface and interior of the rod, respectively).
2. Find the unit of c.
3. Use these bound currents to find the magnetic field $\mathbf{B}$, and also the $\mathbf{H}$-field, inside and outside. (Direction and magnitude)