Physics 471 – Fall 2023

Homework #7 – due Wednesday, October 18 at 11:30am

Point values for each problem are in square brackets

1. [7] **Spin precession and measurement**

At time t = 0, the state of an electron is spin up in the z direction, $|\psi(t=0)\rangle = |+\rangle$

a) [1] If you measure the observable S_y at time t = 0, what are the possible results and the probabilities of those results?

b) [2] *Instead* of performing the measurement of S_y in part a), starting from the initial state $|+\rangle$ the system is allowed to evolve in a uniform magnetic field in the x-direction $\boldsymbol{B} = B_0\hat{\boldsymbol{x}}$. Calculate the state of the system after a time *t* has passed. (You may leave your answer in terms of the energy eigenstates.)

c) [4] In part b), after time *t*, suppose the observable S_y is now measured. What is the probability that you will measure the value $+\hbar/2$? Sketch it as a function of time.
What is the earliest time t when the probability is exactly the same as it was at t=0 (which you computed in part a)?

2. [7] **<u>Atomic beam engineering</u>**

A beam of silver atoms with electron spin ½ travels along the +y-axis. The beam passes through a series of two Stern-Gerlach spin-analyzing magnets, each of which is designed to analyze the spin component along the z-axis. The first Stern-Gerlach analyzer allows only particles with **spin up** (along the z-axis) to pass through. The second Stern-Gerlach analyzer allows only particles with **spin down** (along the z-axis) to pass through. The particles travel at speed *v* between the two analyzers, which are separated by a region of space that has a length *d* in which there is a uniform magnetic field B_0 pointing in the x-direction. (This is *just like* the previous question, the results of which will prove essential here!)

a) [2] Sketch the setup. If B_0=0, how many particles will be transmitted through the 2nd analyzer? Explain your reasoning.

b) [3] If B_0>0, find the smallest value of *d* such that 50% of the particles transmitted by the first z-analyzer are transmitted by the second z-analyzer.

c) [1] Very briefly, explain physically why particles are transmitted in part b (but not part a)

d) [1] Under the same conditions as part b), what is the smallest value of *d* such that 100% of the particles are transmitted by the second analyzer? You should be able to get the answer without doing any additional calculations, just using your results from part b).

3. [6] **Variation on Atomic Beam Engineering**

Two Physics 471 students want to have some fun. They set up an experiment similar to the one in the previous problem: as before, the first Stern-Gerlach analyzer allows only particles with **spin up along the z-axis** to pass through, but this time the second Stern-Gerlach analyzer allows only particles with **spin up along the x-axis** to pass through. The analyzers are separated by a region of space where there is a uniform magnetic field B_0. The students are free to choose the direction of B_0, but they want to arrange it so that 100% of the silver atoms that were transmitted by the first analyzer are also transmitted by the second analyzer.

Student A finished the two previous problems and says that they should orient the magnetic field B_0 along the $+\hat{\boldsymbol{y}}$ direction so she can use her results from those problems.

Student B looked at Figure 3.9(b) in McIntyre and thought it would be cool to orient the field B_0 in the x-z plane at an angle of 45° with respect to the z-axis. (In spherical coordinates, B_0 would be pointing in the $\hat{\boldsymbol{n}}$ direction given by $\theta = \pi/4$ and $\phi = 0$.)

a) [1] If they follow student A's suggestion, what is the minimum time the particles must spend in the magnetic field so that 100% of the particles transmitted by the initial **z-analyzer** are transmitted by the final **x-analyzer**? Define $\omega_0 = \frac{eB_0}{m}$ and express your answer in terms of ω_0. Hint: Although the field is oriented along +y axis rather than the +x axis as in problems 1 and 2, the physics is the same, so the results should be the same.

b) [2] If they follow student B's suggestion, what is the minimum time the particles must spend in the magnetic field so that 100% of the particles transmitted by the initial **z-analyzer** are transmitted by the final **x-analyzer**? Hint: You may be tempted to use Equation (3.48), but that's not necessary. Take seriously the claim that "expectation values in quantum mechanics obey the laws of classical mechanics" and think about how the direction of the spin vector varies in time. McIntyre's Figure 3.9(b) should be helpful.

c) [2] If they follow student A's suggestion and do the calculations correctly, but they accidentally orient the B-field along the $-\hat{\boldsymbol{y}}$ direction instead of the $+\hat{\boldsymbol{y}}$ direction, what fraction of the particles transmitted by the initial **z-analyzer** will be transmitted by the final **x-analyzer**? Could they get that fraction back to 100% by changing the time that the particles spend in the magnetic field?

d) [1] If they follow student B's suggestion but they accidentally orient the B-field in the opposite direction, what fraction of the particles transmitted by the initial **z-analyzer** will be transmitted by the final **x-analyzer**?