Phys 471 HW #6

1.

$H|E_1\rangle = E_1|E_1\rangle$, $H|E_2\rangle = E_2|E_2\rangle$

$|\psi\rangle = c_1(t)|E_1\rangle + c_2(t)|E_2\rangle$ $\qquad H = (|E_1\rangle, |E_2\rangle)$

$(3.8) + \frac{d\, c_1(t)}{dt} = -i\frac{E_1}{\hbar} c_1(t)$ $\qquad \frac{d}{dt}c_2(t) = -i\frac{E_2}{\hbar}c_2(t)$

$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle = c_1(t)H|E_1\rangle + c_2(t)H|E_2\rangle$

$\cancel{i\hbar\frac{d}{dt}c_1(t)|E_1\rangle} = c_1(t)E_1|E_1\rangle + c_2(t)E_2|E_2\rangle$

wait you said in matrix?

$|E_1\rangle = \begin{pmatrix} E_1 \\ 0 \end{pmatrix}$ $\quad |E_2\rangle = \begin{pmatrix} 0 \\ E_2 \end{pmatrix}$ $\quad H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$

$|\psi(t)\rangle = \begin{pmatrix} c_1 E_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 E_2 \end{pmatrix} = \begin{pmatrix} c_1(t)E_1 \\ c_2(t)E_2 \end{pmatrix}$

$i\hbar\frac{d}{dt}\begin{pmatrix} c_1(t)E_1 \\ c_2(t)E_2 \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}\begin{pmatrix} c_1(t)E_1 \\ c_2(t)E_2 \end{pmatrix} = \cancel{c_1(t)E_1^2 + c_2(t)E_2^2}$

$= \begin{pmatrix} c_1 E_1^2 \\ c_2 E_2^2 \end{pmatrix}$ Thus

$\frac{i\hbar\frac{d}{dt}c_1(t)E_1}{i\hbar E_1} = \frac{c_1(t)E_1^2}{i\hbar E_1}$ $\Big|$ $\frac{d}{dt}c_1(t) = \frac{-i\,c(t)E_1}{\hbar}$ ✓

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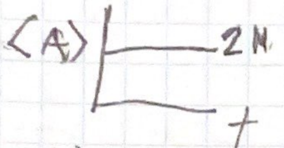
1. 2. a) $\hat{A} = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}$ $\det(\hat{A} - \lambda I) = 0$ $\begin{pmatrix} -\lambda & M \\ M & -\lambda \end{pmatrix}$ $\lambda^2 - M^2 = 0$

$\lambda = M, -M$

b) $|\psi(t)\rangle = e^{-i a_1 t/\hbar} |a_1\rangle = e^{-i\frac{Mt}{\hbar}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|\psi(0)\rangle = \sum_n c_n |E_n\rangle$ $c_n = \delta_{1n}$

c) $\langle A \rangle = \langle \psi | \hat{A} | \psi \rangle = \langle a_1 | e^{i a_1 t/\hbar} \hat{A} e^{-i a_1 t/\hbar} | a_1 \rangle$



$= \langle a_1 | \hat{A} | a_1 \rangle$ The e's cancel and become $e^0 = 1$

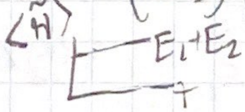
$= (1\ 1) \begin{pmatrix} M \\ M \end{pmatrix} = 2M$

$a_1 = M$ $|a_1\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|a_2\rangle = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

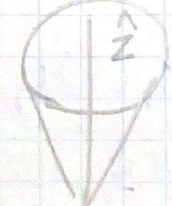
$\langle \hat{H} \rangle = (1\ 1) e^{\frac{iMt}{\hbar}} \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-\frac{iMt}{\hbar}}$

$= E_1 + E_2$



This makes sense since the average energy is independent of time (does not decay or grow)

3. $|+\rangle_n = \cos\frac{\theta}{2} |+\rangle + \sin\frac{\theta}{2} e^{i\phi} |-\rangle$



a) $\theta = \frac{\pi}{2}$ $\phi = -\frac{\pi}{4}$

$= \cos\frac{\pi}{4} |+\rangle + \sin\left(\frac{\pi}{4}\right) e^{i\left(-\frac{\pi}{4}\right)} |-\rangle$

$= \sqrt{2}/2\ |+\rangle + \sqrt{2}/2\ e^{-i\frac{\pi}{4}}$ $= \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ e^{-i\frac{\pi}{4}} \end{pmatrix}$

Since $B = B_0 \hat{z}$

$\hat{H}$ & $\hat{S}_z$ have the same basis states

b) $|\psi(0)\rangle = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ e^{-i\pi/4} \end{pmatrix} = \sum_n c_n |E_n\rangle$

$|\psi(t)\rangle = \begin{pmatrix} 1\, e^{-i\omega t/2} \\ 0\, e^{-i\omega t/2} \end{pmatrix}$

$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$C_1(t) = \frac{\sqrt{2}}{2}$? $C_2(t) = -\frac{\sqrt{2}}{2} e^{-\frac{\pi}{4}}$?

Go to office hours

a) b) $|\psi(t)\rangle = \frac{\sqrt{2}}{2}\begin{pmatrix} e^{-i\omega t/2} \\ e^{i\omega} \end{pmatrix}$

b) $i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$

$H = -\mu B_z \hat{S}_z = \omega_0 \hat{S}_z = \omega_0 \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Eigenstates $\begin{pmatrix} \omega_0 \frac{\hbar}{2} & 0 \\ 0 & -\omega_0 \frac{\hbar}{2} \end{pmatrix}$

$H = \pm \omega_0 \frac{\hbar}{2}$

$|\psi(0)\rangle = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ e^{i\frac{\pi}{4}} \end{pmatrix}$

$|\psi(t)\rangle = c_1(t)\begin{pmatrix} \omega_0 \frac{\hbar}{2} \\ 0 \end{pmatrix} + c_2(t)\begin{pmatrix} 0 \\ -\omega_0 \frac{\hbar}{2} \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}\Bigg|_{t=0}$

c_1

$|\psi(t)\rangle = \frac{\sqrt{2}}{2} e^{-i\omega_0 t/2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{\sqrt{2}}{2} e^{-i\omega_0 \frac{t}{2}} e^{-i\frac{\pi}{4}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|\psi(t)\rangle = \frac{\sqrt{2}}{2}\begin{pmatrix} e^{-i\omega_0 t/2} \\ e^{-i\left(\frac{\pi}{4} + \omega_0 \frac{t}{2}\right)} \end{pmatrix}$

c) $P_{+y} = \left|{}_y\langle + | \psi(t)\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}(1 \;\; -i)\,|\psi(t)\rangle\right|^2$

$= \left|\frac{1}{2} e^{-i\omega_0 t/2} + \frac{1}{2} \underbrace{e^{i\frac{\pi}{2}} e^{-i\left(\frac{\pi}{4} + \omega_0 \frac{t}{2}\right)}}\right|^2 \rightarrow e^{i\left(\frac{\pi}{4} - \omega_0 \frac{t}{2}\right)} = e^{i\frac{\pi}{4}} e^{-i\frac{\omega_0 t}{2}}$

$= \left|\frac{1}{2} e^{-i\omega_0 t/2}\left(1 + e^{i\frac{\pi}{4}}\right)\right|^2 = \frac{1}{2}$

$$P_{+y} = \left|{}_y\langle + | \psi \rangle\right|^2 = \left|\frac{1}{\sqrt{2}}(1\ \ -i)\, e^{-i\omega_0 t/2}\begin{pmatrix}\cos\theta/2 \\ e^{i(\phi+\omega_0 t)}\sin\frac{\theta}{2}\end{pmatrix}\right|^2$$

$$= \frac{1}{2}\left|\cos\left(\frac{\theta}{2}\right) e^{i(\phi+\omega_0 t)} e^{\frac{\pi}{2}} \sin\frac{\theta}{2}\right|^2$$

~~for~~ $\theta = \frac{\pi}{2}$ $\phi = -\frac{\pi}{4}$

$$= \frac{1}{2}\left|\left(\frac{1}{\sqrt{2}} - ie^{i(-\frac{\pi}{4}+\omega_0 t)}\frac{1}{\sqrt{2}}\right)\right|^2$$

$$= \frac{1}{4}\left(1 - ie^{i(-\frac{\pi}{4}+\omega_0 t)}\right)$$

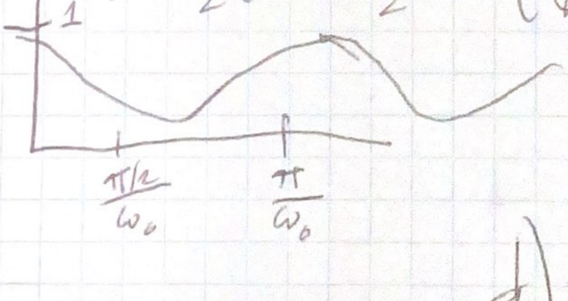
$$= \frac{1}{4}\left(1 - ie^{i(-\frac{\pi}{4}+\omega_0 t)}\right)\left(1 + ie^{-i(-\frac{\pi}{4}+\omega_0 t)}\right)$$

$$= \frac{1}{4}\left(1^2 + \cancel{e^{2(-\frac{\pi}{4}+\omega_0 t)}}\right)$$ cancels?

$$= \frac{1}{4}$$

If $P_{+y} = \frac{1}{2}(1 + \sin\theta\cos(-\phi + \omega_0 t))$

then $P_{+y} = \frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{4} + \omega_0 t\right)\right)$



returns to original state after $\frac{2\pi}{\omega_0}$ seconds

d)

e) $\langle \hat{S}_x \rangle = \langle \Psi(t) | S_x | \Psi(t) \rangle$

$= \hbar ab \cos(\omega_0 t)$

$= \frac{\hbar}{2} e^{-i\frac{\pi}{4}} \cos(\omega_0 t)$

$\langle S_y \rangle = \frac{ab\hbar}{2} \cdot 2i\sin(\omega_0 t)$

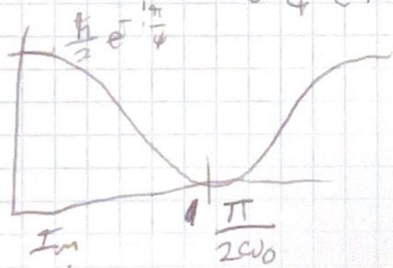
$= \frac{\hbar}{2} e^{-i\frac{\pi}{4}} \cdot i\sin(\omega_0 t)$

$\langle S_z \rangle = \frac{\hbar}{2}(a^2 - b^2) = \frac{\hbar}{2}\left(\frac{1}{2} - \frac{1}{2} e^{-i\frac{\pi}{2}}\right)$

$= \frac{\hbar}{4}\left(1 - e^{-i\frac{\pi}{2}}\right)$

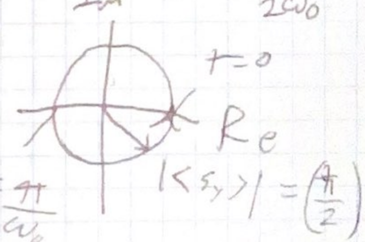
$\langle H \rangle = \omega_0 \langle S_z \rangle = \omega_0 \frac{\hbar}{4}\left(1 - e^{-i\frac{\pi}{2}}\right)$

$\langle S_x \rangle =$



$\langle S_z \rangle =$

$\langle S_y \rangle =$



$\langle S \rangle$ rotates through the imaginary plane with constant magnitude