Physics 471 – Fall 2023

Homework #9 – due Friday, November 10 due to midterms in PHY481 & AST308

Read Sections 5.3, 5.4, and 5.7 in McIntyre before you start!

1. [7] A particle of mass *m* is in an infinite square well potential of width *L*, as in McIntyre's section 5.4. For all parts of this question, suppose we have an initial state vector $|\psi(t=0)\rangle = |E_2\rangle$, the 2nd energy eigenstate. *(This is also called "the 1st excited state", since E_1 is the lowest state or "ground state".)*

a) [1] At $t = 0$, what is the expectation value of the energy? *(Please answer in terms of m, L, and constants of nature such as π and $\hbar$.)* Find the state vector at time *t*. What are the possible outcome(s) of an energy measurement now, with what probability(ies)? What is the expectation value of energy at time *t*? *Hint: You do not need any wavefunctions for this part.*

b) [2] Find the position probability density function at time *t*: $|\psi(x,t)|^2$. *(You may use any results from McIntyre you want, just be sure to reference equation numbers.)* Use this to compute $\langle x(t)\rangle$ for the above state. Is it time dependent? Is it physically reasonable? Why?

c) [1] Compute $\langle p(t)\rangle$. Does your answer make physical sense? Why?

d) [1] Calculate the probability that a measurement of position will find it somewhere between $L/4$ and $3L/4$.

e) [2] Compute Δx and Δp for the above state, and comment on their product $\Delta x \Delta p$. *Feel free to use a computer for any integrals you find nasty.* Recall from earlier this term how we define the standard deviation:

$$\Delta x \equiv \sqrt{\langle\psi|x^2|\psi\rangle - \langle\psi|x|\psi\rangle^2}$$

2. [10] The potential is the same as in the previous problem, but now you have an initial state vector $|\psi(t=0)\rangle = A(|E_1\rangle + 3i|E_2\rangle)$

a) [2] Normalize the state vector (that is, find *A*). *Hint: exploit the orthonormality of energy eigenstates!* At $t = 0$, what are the possible outcomes of a measurement of the energy, and with what probabilities would they occur? What is the expectation value of the energy at $t = 0$? (*Express your answer first in terms of E_1 and E_2, and then in terms of m and L. You do not need any wavefunctions for this part, nor for the next part.)*

b) [1] Find the state vector at time *t*. (You may leave the exponentials in terms of E_1 and E_2.) At this time, what are the possible outcomes of a measurement of the energy, with what probabilities would they occur, and what is the expectation value of the energy?

(continued on back)

c) [3] Find the position probability density at arbitrary time *t*: $|\psi(x,t)|^2$ Express this as a real, sinusoidal function in time. (For convenience, please define the constant $\omega = 3\pi^2\hbar/2mL^2$.) What is the period of oscillation of $|\psi(x,t)|^2$? Use a computer to plot $|\psi|^2$ at 5 times:

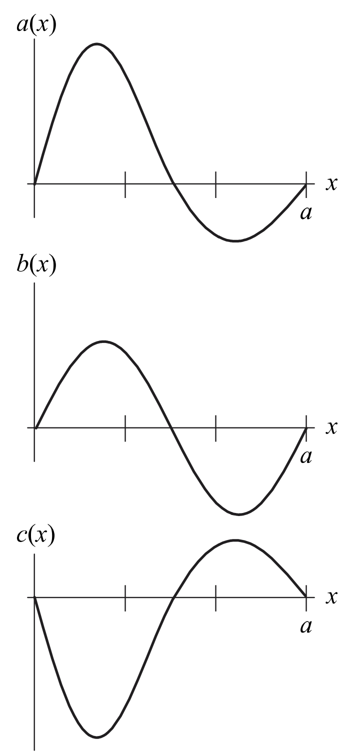
$t_0 = 0$, $t_1 = \frac{\pi\hbar/2}{E_2-E_1} = \frac{\pi}{2\omega}$, $t_2 = \frac{\pi\hbar}{E_2-E_1} = \frac{\pi}{\omega}$, $t_3 = \frac{3\pi\hbar/2}{E_2-E_1} = \frac{3\pi}{2\omega}$, and $t_4 = \frac{2\pi\hbar}{E_2-E_1} = \frac{2\pi}{\omega}$.

Comment briefly on your description/explanation of what is happening here physically.

d) [2] Compute $\langle x(t)\rangle$ for the above state. What is its maximum value? Briefly, discuss how you physically interpret your result. (Contrast it with the similar question about $\langle x(t)\rangle$ in problem #1 above.)

e) [2] Suppose you measure the energy of the particle above and happen to get E_1. What is now the state of the particle? (Write it as a ket, and then write it in the position representation.) Then, calculate the probability that a subsequent measurement of position of this particle will find it somewhere between L/4 and 3L/4. Compare this probability with what you got for the similar question in problem 1d), and make a physical argument for why you could have guessed in advance which answer should be larger. *(Hint, use a sketch!)*

3. [3] Shown at right are the graphs of three normalized wavefunctions. *(Some casual observations: note that all three functions cross the x axis at x/2. Also, I see that b(x) is perfectly anti-symmetric about the midpoint, whereas a(x) = -c(x) is not, it's noticeably asymmetric about the midpoint. The scales may not be exactly right, but all three are normalized.)*



a) [2] Estimate whether each of the following inner products is *positive, negative,* or *zero.* In each case, explain your reasoning. I encourage you to draw whatever graphs you need to aid your explanations.

i) $a(x)$ and $b(x)$ (i.e. what is the sign of $\langle a|b\rangle$)
ii) $a(x)$ and $c(x)$
iii) $b(x)$ and $c(x)$

b) [1] Rank the three inner products by absolute value from greatest to least. Explain how you performed your ranking.