PHY 481 - Fall 2023
Homework 05

Due Friday October 20, 2023

Preface

Homework 05 starts with the conductor problem, which requires new methods to approach finding V or $\vec{E}$ as we are often unable to find ρ a priori. In the presence of an external electric field, charges will shift in a conductor, thus complicating matters of finding ρ. Gauss's law can be used to calculate the E-field in cases of very high symmetry. Once the E-field is known, then the potential can be calculated by performing the appropriate path integral. We will also use this to calculate capacitance of coaxial conductors. In addition, we will solve a problem using image charge method. Finally, we will plot a potential in 3D using Python.

1 Gauss' Law and Cavities

A **metal** sphere of radius R, carrying a charge $+q$, is surrounded by a thick concentric **metal** shell (inner radius a, outer radius b). The shell carries no net charge. Where requested, please explain your reasoning.

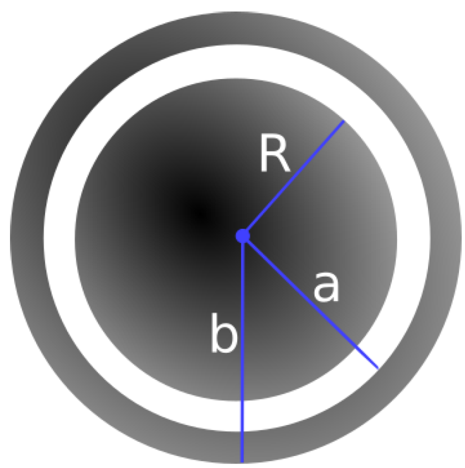

Figure 1: Concentric Metal Shells

1. Sketch the charge distribution everywhere. If the charge is zero anywhere, indicate that explicitly.

2. From part 1, you probably noticed the charge distributes in some way on the metals. Determine the surface charge density σ at R, at a, and at b.

3. Find and sketch the electric field everywhere; explain how you know the field you have drawn is correct (by looking at the discontinuities above and below surfaces). If the field is zero anywhere, indicate that explicitly.

4. Find and sketch the potential everywhere, use $r \to \infty$ as your reference point for $V = 0$. Is the potential continuous as expected?

2 Coax capacitors

Consider a coaxial cable with an inner conducting cylinder has radius a and the outer conducting cylindrical shell has inner radius b. It is physically easy to set up any fixed potential difference ΔV between the inner and outer conductors. In practice, the cable is always electrically neutral.

1. Assuming charge per length $+\lambda$ and $-\lambda$ on the inner and outer cylinders, derive a formula for the voltage difference ΔV between the cylinders.

2. Assuming infinitely long cylinders, find the **energy stored per length** (W/L) inside this capacitor. *Notice we are asking for the energy per unit length, the answer is not infinity!* Let's do it two ways so we can check: **First Method:** find the capacitance per length (C/L) of this system, and then use stored energy $W = \frac{1}{2}C(\Delta V)^2$.

3. **Second Method:** Integrate the energy density stored in the E field in order to obtain the **energy stored per unit length**.

3 The method of images

Griffiths Problem 3.10: A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy-plane.)

1. Find the potential in the region above the plane.

2. Find the charge density σ induced on the conducting plane.

4 Python: Visualizing Solutions to the Laplace Equation

In Example 3.3 of the book, Griffiths solves the problem of a U-shaped infinite slot with the base in the yz-plane, see below.

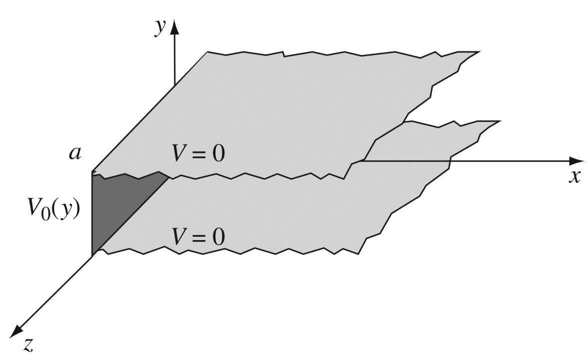

Figure 2: U-shaped Slot

Griffiths goes on to solve the problem for the specific case where the potential $V_0(y)$ at the base ($x = 0$) is just a constant $V_0(y) = V_0 = \mathrm{constant}$. This solution was analytic, but it contained an infinite series:

$$V(x, y) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_0}{\pi n} \sin\left(\frac{n\pi y}{a}\right) \exp\left[-\frac{n\pi x}{a}\right] \tag{1}$$

While perfectly analytic, this solution is hard to visualize. What does that solution look like? Take $V_0 = 10$ V and $a = 1$ m.

1. Plot the approximate solution in 3D space using Python's "mplot3D" for just the first term in the sum (i.e., only for $n = 1$). Download this Jupyter notebook: HW05_3dPotentialPlot.ipynb from D2L, which walks you through how to plot in 3D.

2. Plot the approximate solution in 3D space for the sum of first 5 terms. What do you notice about the boundary where $V = V_0$?

3. Test the plot by summing different number of terms to see how the plot starts to look constant $V = V_0$ at the boundary. Plot the approximate solution for the sum of first 100 terms such that the boundary where $V = V_0$ looks very close to constant.