SOLUTIONS

Physics 471 – Quiz #7

Friday, December 1, 2023

Name:____ (v1)

You do <u>not</u> need to show any calculations for this quiz.

This quiz has questions on both sides of the paper!

1) [5] Two free particles of mass m are represented by Gaussian wave packets. The initial momentum-space wave functions (or "momentum distributions") for the two particles are:

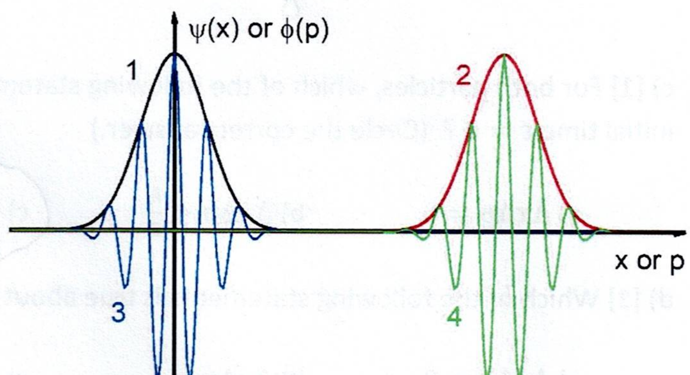
$$\phi_A(p) = \left(\frac{1}{2\pi\beta^2}\right)^{\frac{1}{4}} e^{-p^2/4\beta^2} \qquad\qquad \phi_B(p) = \left(\frac{1}{2\pi\beta^2}\right)^{\frac{1}{4}} e^{-(p-p_0)^2/4\beta^2}$$

where p_0 and β are positive constants with units of momentum. At t = 0, the position-space wave functions of the two particles have the following forms:

$$\psi_A(x,t=0) = \left(\frac{1}{2\pi\alpha^2}\right)^{\frac{1}{4}} e^{-x^2/4\alpha^2} \qquad\qquad \psi_B(x,t=0) = \left(\frac{1}{2\pi\alpha^2}\right)^{\frac{1}{4}} e^{ip_0x/\hbar}e^{-x^2/4\alpha^2}$$

where $\alpha = \frac{\hbar}{2\beta}$.

The plot to the right contains four curves, numbered 1 – 4. The axes cross each other at the origin. Each curve might represent a wave function in real space or in momentum space.



a) [0.5] Which of the curves above could represent $\phi_A(p)$? 1

b) [0.5] Which of the curves above could represent $\phi_B(p)$? 2

c) [1] Which of the curves above could represent the real part of $\psi_A(x,t=0)$? 1

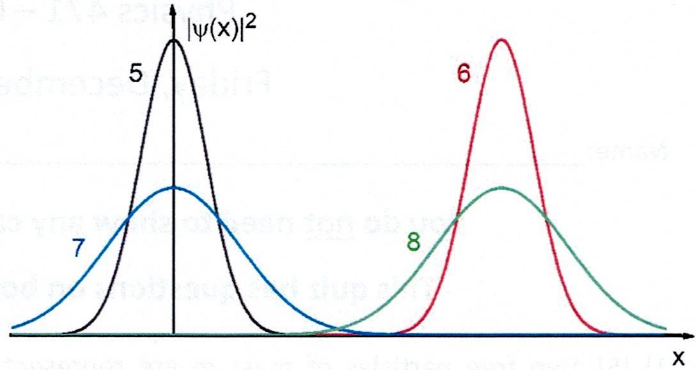
d) [1] Which of the curves above could represent the real part of $\psi_B(x,t=0)$? 3

e) [1] What is $\langle\hat{p}\rangle$ for particle A? 0

f) [1] What is $\langle\hat{p}\rangle$ for particle B? p_0

(v1)

2) [4] As time passes, the wave packets from the previous page evolve. The picture to the right shows a new set of curves, numbered 5 – 8. As before, the axes cross each other at the origin. Each curve might represent the modulus squared of a wave function, i.e. $|\psi|^2$.



a) [1] Assume that curve 5 represents $|\psi_A(x, t=0)|^2$. Which one of the curves might represent $|\psi_A(x,t)|^2$ at the time $t = \sqrt{3}\tau$, where $\tau = \frac{m\hbar}{2\beta^2}$ is the characteristic spreading time for these wave packets? (Note that, on the homework, the same characteristic time was also called Ω^{-1}.)

7

b) [1] Assume that curve 5 also represents $|\psi_B(x, t=0)|^2$. Which one of the curves might represent $|\psi_B(x,t)|^2$ at the time $t = \sqrt{3}\tau$?

8

c) [1] For both particles, which of the following statements is true about the product $\Delta x \Delta p$ at the initial time $t = 0$? (Circle the correct answer.)

a) $\Delta x \Delta p = 0$ b) $\Delta x \Delta p < \frac{\hbar}{2}$ c) $\Delta x \Delta p = \frac{\hbar}{2}$ d) $\Delta x \Delta p > \frac{\hbar}{2}$

d) [1] Which of the following statements is true about the product $\Delta x \Delta p$ at the time $t = \sqrt{3}\tau$?

a) $\Delta x \Delta p = 0$ b) $\Delta x \Delta p < \frac{\hbar}{2}$ c) $\Delta x \Delta p = \frac{\hbar}{2}$ d) $\Delta x \Delta p > \frac{\hbar}{2}$

3) [1] Now consider a completely different situation, again with two free particles of mass m, but now particle C is represented by an initial wave packet $|\psi_C(x, t=0)|^2$ that looks like curve 5 in the picture above, while particle D is represented by an initial wave packet $|\psi_D(x, t=0)|^2$ that looks like curve 7 above. Which wave packet will spread out more quickly with time? (Circle the correct answer.)

a) Wave packet C b) Wave packet D c) They will both spread out at the same rate.

Narrower wave packet $\Leftrightarrow$ wider momentum distribution

SOLUTIONS

Physics 471 – Quiz #7

Friday, December 1, 2023

Name:____ (v2)

You do not need to show any calculations for this quiz.

This quiz has questions on both sides of the paper!

1) [5] Two free particles of mass m are represented by Gaussian wave packets. The initial momentum-space wave functions (or "momentum distributions") for the two particles are:

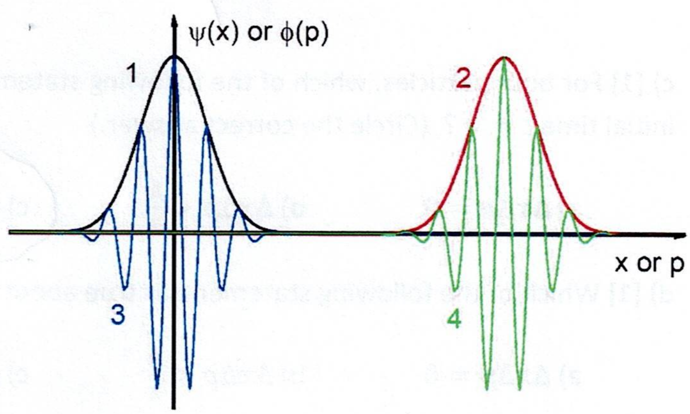
$$\phi_A(p) = \left(\frac{1}{2\pi\beta^2}\right)^{\frac{1}{4}} e^{-(p-p_0)^2/4\beta^2} \qquad \phi_B(p) = \left(\frac{1}{2\pi\beta^2}\right)^{\frac{1}{4}} e^{-p^2/4\beta^2}$$

where p_0 and β are positive constants with units of momentum. At t = 0, the position-space wave functions of the two particles have the following forms:

$$\psi_A(x, t=0) = \left(\frac{1}{2\pi\alpha^2}\right)^{\frac{1}{4}} e^{ip_0x/\hbar} e^{-x^2/4\alpha^2} \qquad \psi_B(x, t=0) = \left(\frac{1}{2\pi\alpha^2}\right)^{\frac{1}{4}} e^{-x^2/4\alpha^2}$$

where $\alpha = \frac{\hbar}{2\beta}$.

The plot to the right contains four curves, numbered 1 – 4. The axes cross each other at the origin. Each curve might represent a wave function in real space or in momentum space.



a) [0.5] Which of the curves above could represent $\phi_A(p)$? 2

b) [0.5] Which of the curves above could represent $\phi_B(p)$? 1

c) [1] Which of the curves above could represent the real part of $\psi_A(x, t = 0)$? 3

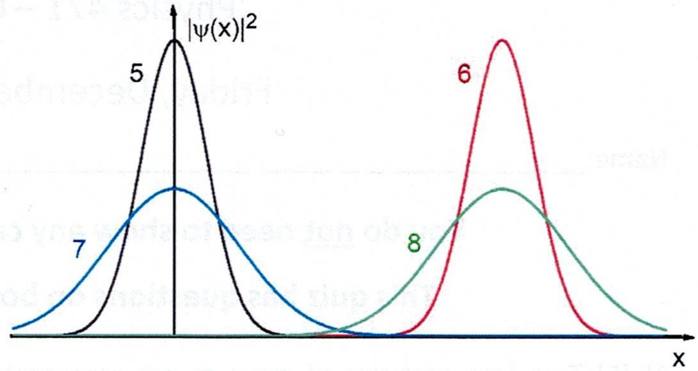
d) [1] Which of the curves above could represent the real part of $\psi_B(x, t = 0)$? 1

e) [1] What is $\langle\hat{p}\rangle$ for particle A? p_0

f) [1] What is $\langle\hat{p}\rangle$ for particle B? 0

(v2)

2) [4] As time passes, the wave packets from the previous page evolve. The picture to the right shows a new set of curves, numbered 5 – 8. As before, the axes cross each other at the origin. Each curve might represent the modulus squared of a wave function, i.e. $|\psi|^2$.



a) [1] Assume that curve 5 represents $|\psi_A(x, t = 0)|^2$. Which one of the curves might represent $|\psi_A(x,t)|^2$ at the time $t = \sqrt{3}\tau$, where $\tau = \frac{m\hbar}{2\beta^2}$ is the characteristic spreading time for these wave packets? (Note that, on the homework, the same characteristic time was also called Ω^{-1}.)

8

b) [1] Assume that curve 5 also represents $|\psi_B(x, t = 0)|^2$. Which one of the curves might represent $|\psi_B(x,t)|^2$ at the time $t = \sqrt{3}\tau$?

7

c) [1] For both particles, which of the following statements is true about the product $\Delta x \Delta p$ at the initial time $t = 0$? (Circle the correct answer.)

a) $\Delta x \Delta p = 0$ b) $\Delta x \Delta p < \frac{\hbar}{2}$ c) $\Delta x \Delta p = \frac{\hbar}{2}$ d) $\Delta x \Delta p > \frac{\hbar}{2}$

d) [1] Which of the following statements is true about the product $\Delta x \Delta p$ at the time $t = \sqrt{3}\tau$?

a) $\Delta x \Delta p = 0$ b) $\Delta x \Delta p < \frac{\hbar}{2}$ c) $\Delta x \Delta p = \frac{\hbar}{2}$ d) $\Delta x \Delta p > \frac{\hbar}{2}$

3) [1] Now consider a completely different situation, again with two free particles of mass m, but now particle C is represented by an initial wave packet $|\psi_C(x, t = 0)|^2$ that looks like curve 5 in the picture above, while particle D is represented by an initial wave packet $|\psi_D(x, t = 0)|^2$ that looks like curve 7 above. Which wave packet will spread out more quickly with time? (Circle the correct answer.)

a) Wave packet C b) Wave packet D c) They will both spread out at the same rate.

Narrower wave packet $\Leftrightarrow$ wider momentum distribution