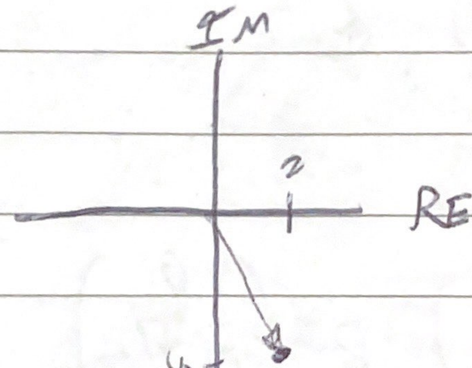
1. $z = 2-4i$

a) $(2-4i)(2+4i) = 20$

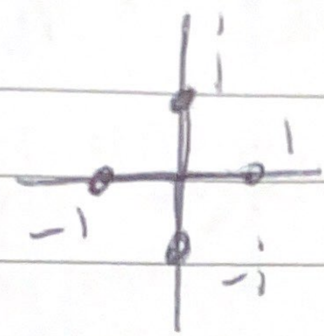
b)



c) $A = \sqrt{20}$

$\theta = \tan^{-1}\left(\frac{-4}{2}\right) = -\tan^{-1}(2) = -1.11$ rad

$\boxed{2)}$ a)



$1 = e^0$

$i = e^{\frac{\pi}{2}}$

$-1 = e^{\pi}$

$-i = e^{\frac{3\pi}{2}}$

$\cancel{e^0 \cdot e^{\frac{\pi}{2}}}$

b) $1 \cdot i = i$

$i \cdot i = -1$

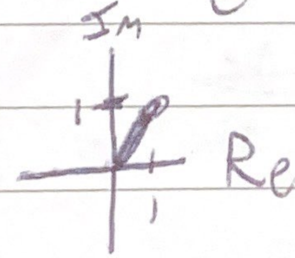
$-1 \cdot i = -i$

$-i \cdot i = 1$

#3. Multiplying by i rotates 90° $\left(\frac{\pi}{2}\right)$ ccw

a) $z = i e^{-i\frac{\pi}{4}} = e^{i\frac{\pi}{4}}$ $A = 1$ $\theta = \frac{\pi}{4}$

b)



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$\boxed{4)}$ a) $A = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & -1/2 \end{pmatrix}$ $C = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

$$AB = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1/2 \end{pmatrix} = \begin{pmatrix} 0 & -i/2 \\ -i & 0 \end{pmatrix}$$

$$AC = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{5} \\ 4/5 \end{pmatrix} = \begin{pmatrix} i4/5 \\ -i3/5 \end{pmatrix}$$

b) $Av = \lambda v$, $[A - \lambda] = \begin{bmatrix} -\lambda & i \\ -i & -\lambda \end{bmatrix} = \lambda^2 + 1 = 0$

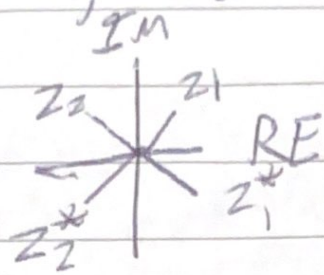
$\lambda = 1$ $\qquad \lambda = \sqrt{-1} = i$

c) ~~No, you would ne~~

No, the ~~[illegible]~~ sizes are not compatable

5) $z_1 = e^{i\frac{\pi}{4}}$, $z_2 = e^{i\frac{3\pi}{4}}$

a)



b) $z_1 z_2 = e^{i\frac{\pi}{4}} e^{i\frac{3\pi}{4}} = e^{i\pi}$

$z_2 / z_1 = e^{i\frac{3\pi}{4}} / e^{i\frac{\pi}{4}} = e^{i\frac{\pi}{2}}$

$z_2 z_1^* = e^{i\frac{3\pi}{4}} e^{-i\frac{\pi}{4}} = e^{i\frac{\pi}{2}}$

c) $e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$

$z_1 = e^{i\frac{\pi}{4}} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

$z_2 = e^{i\frac{3\pi}{4}} = \sim \quad \sim = -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

$\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = -\frac{1}{2} + \frac{i}{2} - \frac{i}{2} - \frac{1}{2} = -1$

6) $Re(z) = \frac{z + z^*}{2}$ $\quad Im(z) = (z - z^*)/x$

$z = 4 + 6i$

a) $\frac{4 + 6i + (4 - 6i)}{2} = \frac{2\,4}{2} = 4$

$\frac{4 + 6i - (4 - 6i)}{x} = \frac{2\,6i}{x}$ $\quad x = 2i$

6)

b) $\cancel{Re(z_1 z_2) = Re(z_1 Re z_2) + Im(z_1) Im(z_2)}$

$z_1 = a_1 + b_1 i \quad z_2 = a_2 + b_2 i$

$z_1 z_2 = (a_1 + b_1 i)(a_2 + b_2 i) = a_1 a_2 + a_1 b_2 i + a_2 b_1 i$

$- b_1 b_2$

$Re(z_1 z_2) = a_1 a_2 - b_1 b_2$

$= Re(z_1) Re(z_2) - Im(z_1) Im(z_2)$

$Im(z_1 z_2) = a_1 b_2 i + a_2 b_1 i$

$= Re(z_1) Im(z_2) + Re(z_2) Im(z_1)$

c) $z^2 = z z = (a + bi)(a + bi) = a^2 + 2abi - b^2$

$\neq a^2 + b^2$

$Re(z^2) = a^2 - b^2 \neq Re(|z|^2) = a^2 + b^2$

explanation: they aren't equal

d) $z = Ae^{i\theta}$

$Ae^{i\theta} = A\cos\theta + Ai\sin\theta$

$Re(z) = A\cos\theta \quad Im(z) = i \cdot A\sin\theta$

$z^* = Ae^{-i\theta} \quad |z| = A$

e) $e^{i\theta} = \cos\theta + i\sin\theta \quad e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

Maria Goeppert Mayer was a German-American physicist who lived from 1906 to 1972. She was born on June 28, 1906, in Kattowitz, Germany (now Katowice, Poland), and later became a naturalized American citizen. Her most important contribution came in 1949 when she proposed the nuclear shell model, which explained the structure of atomic nuclei based on the arrangement of protons and neutrons in discrete energy levels or "shells," similar to the electron shells in atoms. I'm not sure if her research in atomic structure counts as quantum physics, but it doesn't change the fact that I think Mayer is cool. This groundbreaking work provided a fundamental framework for understanding nuclear behavior and earned her the Nobel Prize in Physics in 1963, making her the second woman to receive the Nobel Prize in physics.