Physics 471 – Fall 2023

Homework #8 – due Wednesday, November 1

Point values for each problem are in square brackets

1. [7] **More practice with quantum measurements**

An operator $\widehat{\boldsymbol{A}}$ (representing observable A) has two normalized eigenstates $|\psi_1\rangle$ and $|\psi_2\rangle$, with eigenvalues a_1 and a_2, respectively. Operator $\widehat{\boldsymbol{B}}$ (representing observable B) has two normalized eigenstates $|\varphi_1\rangle$ and $|\varphi_2\rangle$, with eigenvalues b_1 and b_2. Suppose these eigenstates are related by the following:

$$|\psi_1\rangle = \frac{1}{3}|\varphi_1\rangle + \frac{\sqrt{8}}{3}|\varphi_2\rangle, \quad |\psi\rangle_2 = \frac{\sqrt{8}}{3}|\varphi_1\rangle - \frac{1}{3}|\varphi_2\rangle$$

a) [1] Show that, assuming $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are properly normalized, $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthonormal. That means show that they are orthogonal to each other and that each one of them is normalized.

b) [1] Let's start in some unspecified random state, and then observable A is measured. Further, assume that the result of the measurement is the particular value a_1. What is the state of the system immediately after this measurement?

c) [1] Immediately after the measurement of A (which, recall, happened to yield a_1), the observable B is then measured. What are the possible results of the B measurement, and what are their probabilities?

d) [4] Consider three scenarios following up on the "setup" in part C:

- i) Suppose I tell you that, when we measured B, we found b_1. And immediately after that, we measure A once again. What is the probability of getting outcome a_1 again? *Hint: If you think the answer is trivially "1" or trivially "1/2", I suggest you think again!*

- ii) Suppose *instead* that you measure B, but you do not know the outcome! (To be clear: B has been measured, nature knows the result, but YOU do not. So as far as you are concerned, there are two possibilities, b_1 or b_2, with relative probabilities given in part c above.) If you now remeasure A, what is the probability of getting a_1?

- iii) Go back a step—suppose that after our very first measurement of A (when, you will recall, we happened to find a_1), we had completely neglected to measure B at all, and simply measured A again, right away. What are the possible result(s) of the second measurement of A, with what probabilities?

- iv) Do you think the operators $\widehat{\boldsymbol{A}}$ and $\widehat{\boldsymbol{B}}$ commute? Why/why not?

2. [9] **The spin singlet state and the EPR experiment**

Consider the entangled state of two spin-1/2 particles: $|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2)$

a) [1] Show that this 2-spin state $|\psi\rangle$ is properly normalized.

b) [1] Is this state $|\psi\rangle$ an eigenstate of $\widehat{S_{1z}}$ (i.e. the operator associated with measuring the z-component of the spin of particle 1 only)? If so, what is the eigenvalue? If not, why not?

c) [1] Is state $|\psi\rangle$ an eigenstate of the "total z-component of spin operator" $\widehat{S_{1z}} + \widehat{S_{2z}}$? If so, what is the eigenvalue? If not, why not?

d) [3] Show that this entangled state $|\psi\rangle$ is physically equivalent to the more general state: $\frac{1}{\sqrt{2}}(|+\rangle_{1n}|-\rangle_{2n} - |-\rangle_{1n}|+\rangle_{2n})$, where $\hat{n}$ is a unit vector in an arbitrary direction. Hint: It may be easier to start with the more general state and work your way backwards to the initial state.

This result shows why the two detectors in the EPR experiment record perfect anticorrelations in their spin measurements, independent of the orientation of their detectors, when both detectors are aligned along the same direction!

e) [3] Given the state $|\psi\rangle$, show that the probability of observer A to measure particle 1 with spin up is 50% for <u>any</u> orientation of observer A's Stern-Gerlach apparatus. Hint: To find this probability, sum over the joint probabilities for observer A to measure spin up for particle 1 along the chosen direction and observer B to measure any spin orientation for particle 2. To simplify the calculation, choose observer B's Stern-Gerlach apparatus to be oriented along the z direction. You'll have to add the probabilities for the cases where B measures spin up and spin down.

3. [4] **More about entanglement**

a) [1] Is this 2-particle state $|\alpha\rangle = \frac{1}{2}(|+\rangle_1|+\rangle_2 - |+\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2 - |-\rangle_1|-\rangle_2)$ entangled? (Explain why or why not.)

b) [3] Consider the state $|\beta\rangle = \frac{1}{\sqrt{6}}|+\rangle_1|+\rangle_2 + \frac{1}{\sqrt{6}}|+\rangle_1|-\rangle_2 + \frac{2}{\sqrt{6}}|-\rangle_1|+\rangle_2$

(i) What is the probability that observer #1 measures $+\frac{\hbar}{2}$?

(ii) What is the probability that BOTH observers measure $+\frac{\hbar}{2}$?

(iii) Finally, what is the probability that observers 1 and 2 get "opposite" measurements of S_z?

<u>For each of the above, explain your reasoning, don't just write down an answer</u>.