PHY 481 - Fall 2023
Homework 04

Due Friday September 29, 2023

Preface

Homework 04 emphasizes alternative methods to direct integration (Coulomb's Law) for solving the electric field problem including the use of Gauss's Law. In addition, it introduces the concept of the Dirac delta function as a tool for describing distributions of charge. This homework makes use of what you learned in Secs. 1.5, and 2.2 and what you know from 2.1 (i.e. superposition of ($\vec{E}$) will also be important.

1 Describing charge distributions with delta functions

The Dirac delta function is an important theoretical tool for describing distributions of a variety of physical quantities (e.g., mass, charge) where a point object (or system of point objects) is the model we intend to use. In addition, it can be used to describe distributions where these quantities exist in highly constrained spaces (e.g. on a plane or spherical shell). In this class, we will use the Dirac delta function to describe how a charges are distributed. In this problem, you will get familiar with the Dirac delta function for a set point charges on a line.

The linear charge density for a series of charges on the x-axis is given by:

$$\lambda(x) = \sum_{n=0}^{10} q_0 n^2 \delta\left(x - \left(\frac{1}{10}\,\text{meter}\right) n\right) \tag{1}$$

1. Draw a picture of this charge distribution in the xy-plane (in 2D) - be sure to label the individual charges and locations clearly
2. What is the total charge of this distribution? Explicitly write the integral needed and then evaluate it.

2 Spherical charge distributions are special

As you might have picked up by now, spherically symmetric charge distributions are very special. We have a number of theoretical tools we can bring to bear on them and the results we produce are often quite simple in a mathematical sense. In this problem, you will explore these distributions a bit more and gain intuition about these spherically symmetric distributions of charge.

Consider a sphere of radius R, centered on the origin, with a radially symmetric charge distribution $\rho(r)$:

1. What $\rho(r)$ is required for the electric field **in the sphere** to have the power law form $\mathbf{E} = cr^n\hat{\mathbf{r}}$, where c and n are constants? *Hint: Use the differential form of the Gauss' law and divergence of a vector in spherical coordinates (in Griffiths flyleaf)* .

2. You will see that the case of $n = -2$ is special. How so? Calculate $\nabla \cdot \mathbf{E}$ for $n = -2$ to find $\rho(r)$. *Hint:* $\nabla \cdot \frac{\hat{\mathbf{r}}}{r^2} = 4\pi\delta^3(\mathbf{r})$

3. What kind of charge distribution is required for the radial E-field inside the sphere to be of constant magnitude; that is, what $\rho(r)$ produces $E(r) =$ constant (inside only)?

3 Cube with a hole

What happens when you have problems were the symmetries are mixed? How do you tackle a problem with two different geometries? In this problem, you will explore how to deal with situations where they are two "competing" geometries for the problem. Sometimes you will need to bring two (or more!) aspects of your theoretical toolbox to bear on a problem.

Consider a cube (edge length a) with a uniform charge distributed throughout its volume (ρ). We carve a spherical cavity out of it of radius d, such that the cavity is centered at the center of the cube:

1. Does Gauss's Law hold for this problem? Can Gauss's Law be used on this problem? If so, what surface do you use? If not, why?

2. Let the center of the cube (and thus the center of the cavity) be located at the origin $\langle 0, 0, 0\rangle$. **Explain** how you would determine the electric field at point P a distance z from the center of the cube. If there is a direct integral involved, then show how you would set it up - but don't do it! If Gauss's Law can be used, then apply it.

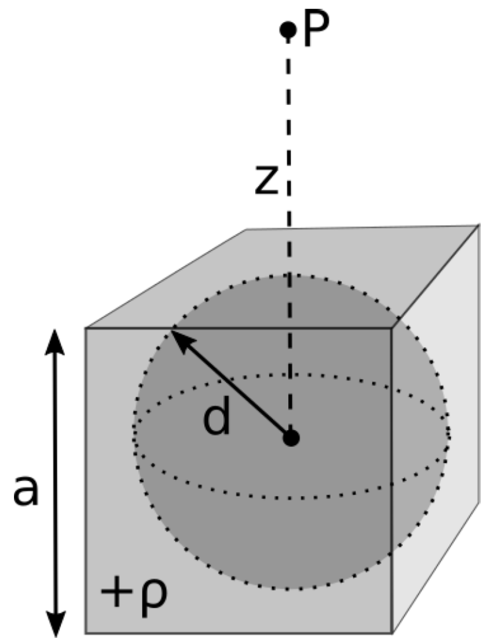

Figure 1: Cube with a hole

4 Python: Electric field of a line charge

As discussed in the book, you can break up a distribution of charge into chunks - each a point charge - and add up the contributions to the electric field of each chunk. This process forms the basis of numerical superposition, which you began to explore in the last homework. In this problem, you will extend that work to a line of charge. You will solve this problem using a Jupyter notebook. You can download it from D2L.

Using numerical superposition, adding up the contributions to the electric field due to each chunk, you will solve the following problems:

1. We want to compute and represent the electric field of the charge at a distance of $0.01\ \mathrm{m}$ from the line charge along the y-axis. Do this.

2. The analytical formula for the electric field of the rod at that location is:

$$E_{\mathrm{rod}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{y\sqrt{L^2/4 + y^2}} \tag{2}$$

   Compare the value of the electric field at that location for different values of "Nchunks," say for 10, 20, 50, and 100 chunks. How close do you get with 100 chunks? How many chunks do you need to get within 1% of the analytical solution?

3. Using what you have built to find the electric field at this location, find the electric field at a variety of points around the the line charge and represent them with arrows. You can choose the locations, but be systematic.