Thermal physics homework #1

1. Simple probability stuff

1. (a) [2] A coin is tossed 5 times. Write down all possible configurations with 3 heads. A useful notation is to represent each configuration by a row of 5 arrows, with heads and tails denoted by up and down arrows, respectively. What is the probability of getting 3 heads when tossing the coin 5 times?

   (b) [3] A coin is tossed 12 times. Draw a histogram showing the number of states as a function of the number of heads. (See Figure 1.7 in the text as an example.) Use the binomial expansion coefficients to calculate the exact multiplicity of each state.

1a)

u = heads, d = tails

uuudd, uudud, uuddu, uduud, udud u, udduu, duuud, duudu, duduu, dduu (10 different ways)

To double check my result, I'll check the multiplicity $g(N, n) = \dfrac{N!}{n!(N-n)!}$

$$g(5,3) = \frac{5!}{3!(2)!} = 10$$

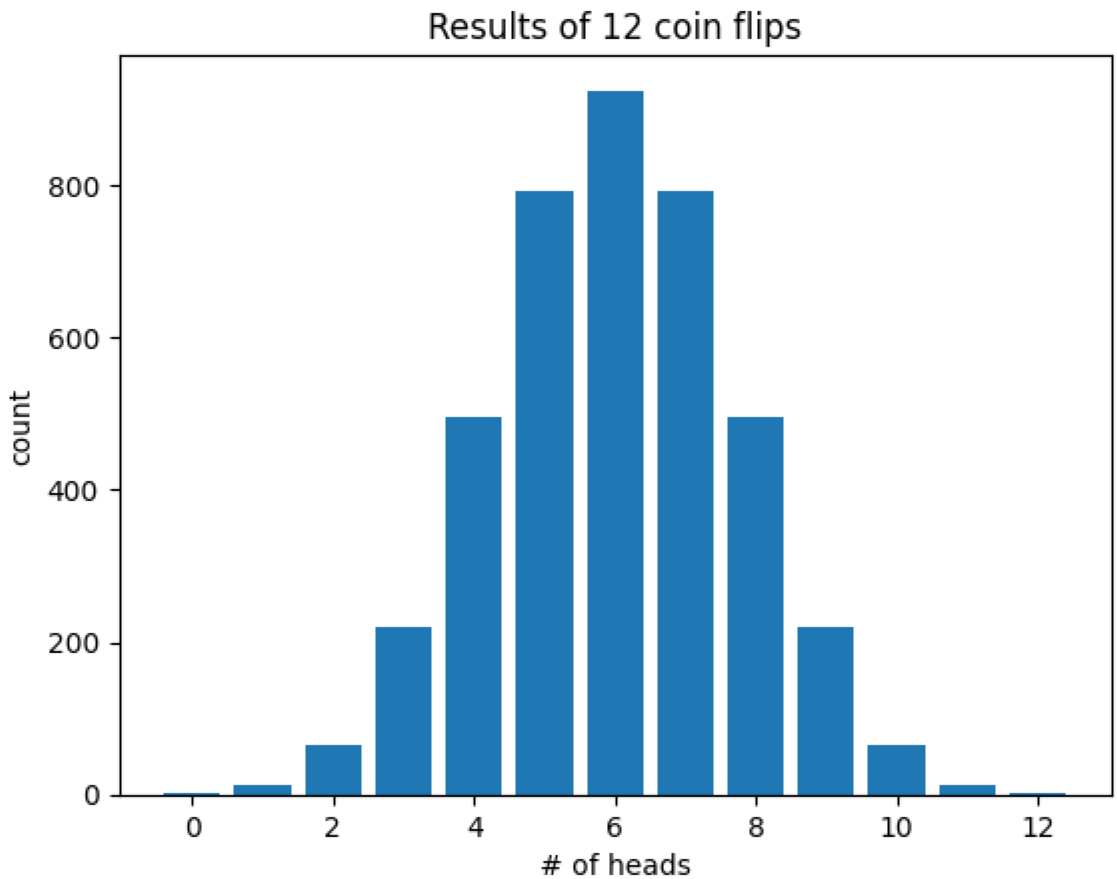
In [ ]:
```
import math

g = math.factorial(5) / (math.factorial(3) * 2)
g
```

Out[ ]: `10.0`

1b)

MS paint drawing? Actually why not have my snake friend draw it

In [ ]:
```
import matplotlib.pyplot as plt

heads = []
for i in range(13):
    multiplicity = math.factorial(12) / (math.factorial(i)*math.factorial(12-i))
    heads.append(multiplicity)

plt.bar(range(len(heads)),heads)
plt.title('Arangements of 12 coin flips')
plt.xlabel('# of heads')
plt.ylabel('count')
plt.show()
```



2. As you can guess, I think I'd rather work with python

(a) [2] To get started, here are some commands you might try:

Random[]
Random[Integer]
Table[Random[Integer], {10}]

Here is a way to toss m coins, find the number of heads, repeat the whole trial N times, and then histogram the result. In the example below I used m=10 and N=200.

NumberOfHeads[m_]:=Sum[Random[Integer], {m}]
ManyTrials[m_,N_]:=Table[NumberOfHeads[m], {N}]
Histogram[ManyTrials[10, 200]]

Notice that when you define a function, the variable name on the left-hand-side of the definition is followed by the underscore, hence m_ instead of m.

2a)

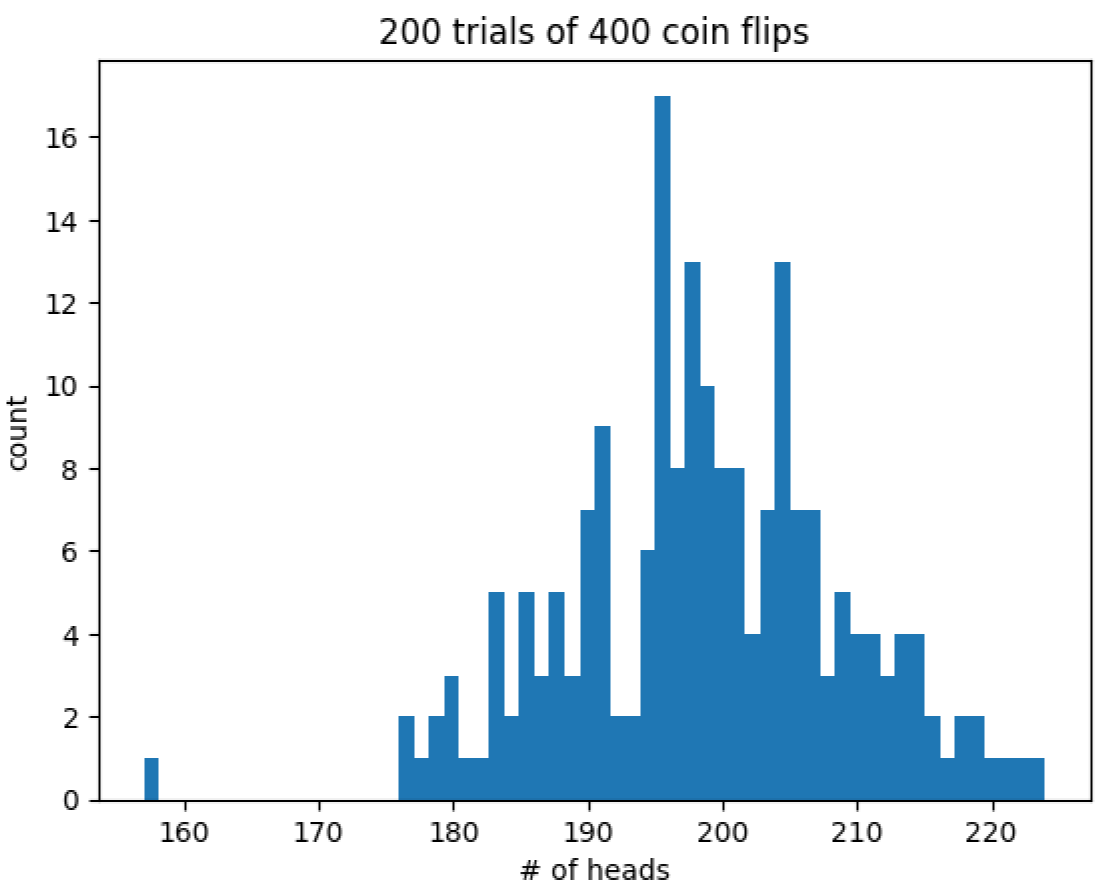
I think I can do these in python

In [ ]:
```
import numpy as np

np.random.random()
np.random.randint(0,2)

coin_flips = []
for i in range(10):
```

```
    coin_flips.append(np.random.randint(0,2))

numheads = sum(coin_flips)

#I've already got our histogram above, just need to use plt.hist() instead
```

2b)

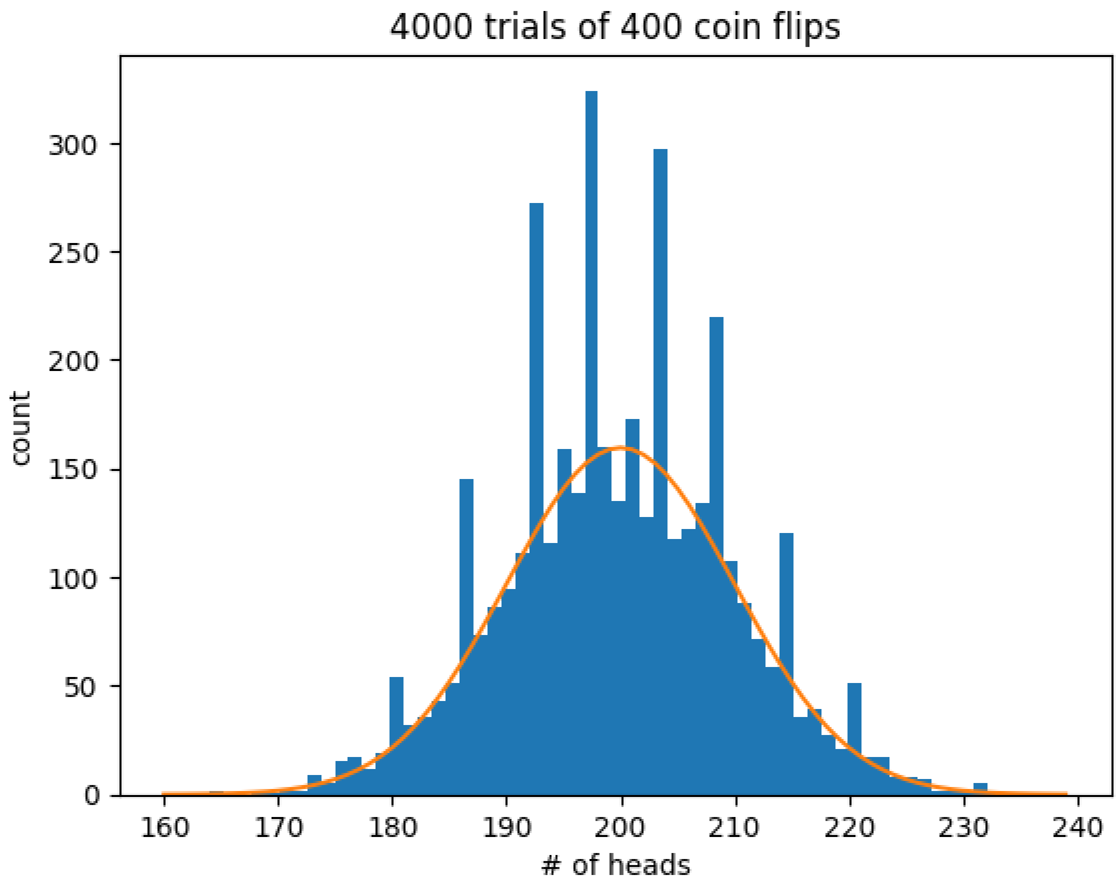
In [ ]:
```
trials = 200
trial_size = 400
bins = 60

histvals = []

for i in range(trials):
    coin_flips = []
    for j in range(trial_size):
        coin_flips.append(np.random.randint(0,2))
    histvals.append(sum(coin_flips))

plt.hist(histvals,bins)
plt.title(f'{trials} trials of {trial_size} coin flips')
plt.xlabel('# of heads')
plt.ylabel('count')
plt.show()
```



2c)

just copy paste the above. I'll also throw 2d into here

In [ ]:
```
trials = 4000
trial_size = 400
bins = 60

histvals = []

for i in range(trials):
    coin_flips = []
    for j in range(trial_size):
        coin_flips.append(np.random.randint(0,2))
    histvals.append(sum(coin_flips))

plt.hist(histvals,bins)
plt.title(f'{trials} trials of {trial_size} coin flips')
plt.xlabel('# of heads')
plt.ylabel('count')
#plt.show()

#print('not sure why we get these peaks but they\'re different with each run')

x = np.arange(160,240,1)
y = 4000*np.sqrt(2/(400*np.pi))*np.exp(-2*(x-200)**2/400)

plt.plot(x,y)
plt.show()
```



2e)

lets find δn

(e) [2] Define n to be the number of heads obtained from the N coin tosses. The mean value of n is called $\overline{n}$ or $<n>$. The standard deviation of n is defined as $\delta n \equiv \sqrt{<(n-\overline{n})^2>}$. What is δn for N=12 and N=400? (You may use results we derived in class.) What is the relative uncertainty in n, defined as $\delta n/\overline{n}$, for the two cases? Is your result for N=400 consistent with the graphs you made in parts (c) and (d)? Discuss how likely (or unlikely) it is to obtain fewer than 30% heads in the two cases.

Standard deviation $\delta n \equiv \sqrt{<(\Delta n)^2>} = \sqrt{Npq}$

$\sqrt{Npq} = \sqrt{N * .25}$

$N = 12$, $\delta n = 1.732$, $\frac{1.732}{6} = 0.289$

$N = 400$, $\delta n = 10$, $\frac{10}{200} = 0.050$

These graphs are mostly as expected. The N=400 graph is very close to the normal curve. According to the relative uncertainty the N=12 trial includes the bottom '60%' within two standard deviations left of the mean, so it is plently likely, however for N=400 the bottom 30% is more than four standard deviations away, and is thus very unlikely to occur.

In [ ]:
```
np.sqrt(400*.25)
1.732/6
```

Out[ ]:
```
0.2886666666666667
```

3a)
considering gas

(a) What is the mean number $\overline{N} = \langle N \rangle$ of molecules located within V? Express your answer in terms of N_0, V_0, and V. (Hint: This problem is just like the biased coin flip problem we discussed in class, where p is the probability to get "heads".)

$V \leq V_0$

There is a $\frac{V}{V_0}$ chance that a single particle is in V, so the mean number is that value multiplied by the total number of particles $\bar{N} = N_0 \frac{V}{V_0}$

(b) Find the relative dispersion $\left\langle \left(N - \overline{N}\right)^2 \right\rangle / \overline{N}^2$ in the number of molecules located within V. (This is the square of the quantity we discussed in class.) Express your answer in terms of $\overline{N}$, V and V_0.

b)
$< (N - \bar{N})^2 >$ is variance $(\delta N)^2 = Npq$.

q is the probability a particle isn't in V, $q = \frac{V_0 - V}{V_0} = 1 - \frac{V}{V_0}$

Substituting in these values we find relative dispersion is $\frac{(N_0(\frac{V}{V_0})(1 - \frac{V}{V_0}))}{N_0 \frac{V}{V_0}}$

In terms of $\bar{N}, V, \text{ and } V_0$, this is

$$\frac{< (N - \bar{N})^2 >}{\bar{N}^2} = \frac{(\bar{N}(1 - \frac{V}{V_0}))}{\bar{N}}$$

(c) What does the answer to part (b) become when $V \ll V_0$? (The answer is not zero.)

c) This approaches $\frac{\bar{N}}{\bar{N}} = 1$

(d) What value should the dispersion $\overline{(N-\bar{N})^2}$ assume when $V \to V_0$? Does the answer in part (b) agree with this?

d) This approaches $\frac{\bar{N}(1-1)}{\bar{N}} = 0$, which makes sense since no gas molecules can be outside the volume

(e) Consider the case $N_0 = 5$, and $V=V_0/4$. At any given time, what is the probability of finding all 5 molecules in V? What is the probability of finding exactly one molecule in V?

This is a binomial with p = 1/4 and N=5, a biased coin flip. The probabilibty of finding all five inside is

$$p^N = \frac{1}{4^5} = .000976$$

And the probability of only finding one inside is

$$pq^4 = \frac{1}{4}(\frac{3}{4})^4 = .0791$$

Yay all done