Problem set #3 - Vincentless

Question 1:

Consider a simple two-period model of intertemporal choice. Suppose that a person receives income \$48,326 in period 1 and additional income \$44,928 in period 2. The market interest rate at which the person can both borrow and save is 4%. Finally, the person's preferences are given by

$$U(c_1, c_2) = \frac{3}{2}(c_1)^{2/3} + \delta \frac{3}{2}(c_2)^{2/3}.$$

(a) Derive the budget constraint that the person faces.

I assume they cannot borrow/save for more than one period, since then they would simply borrow an infinite amount of money in period 1 and never pay it back. The market interest rates mean that person can lose X in period 1 and recieve $X * 1.04$ in period 2, or gain X in period 1 and lose $X * 1.04$ in period 2. As such we get

$$W(X) = (48,326 + X; 44,928 - X * 1.04)$$

I can't quite remember the notation we used in class

(b) Solve for the optimal c_1 and c_2 as a function of δ.

We want to know when U is maximized. This is at the point when the change in U is zero, which means

$$\frac{d}{dc_1}3/2c_1^{2/3} = \frac{d}{dc_2}3/2\delta c_2^{2/3}$$
$$c_1^{-1/3} = \delta c_2^{-1/3}$$
$$c_1 = \sqrt[3]{\delta} c_2$$
$$c_2 = \delta^3 c_1$$

We can double check this with some arbitrary values. Lets say money can be transferred freely between both periods

$$\begin{cases} c_1 + c_2 = 100 \\ \delta = 0.5 \\ c_1(1.125) = 100 \\ c_1 = 88.9,\ c_2 = 11.1 \\ U = 33.61 \end{cases}$$
$$\begin{cases} c_1 = c_2 = 50 \\ U = 30.5 \end{cases}$$
$$\begin{cases} c_1 = 88,\ c_2 = 11 \\ U = 33.4 \end{cases}$$

Sweet

(c) As δ increases, what happens to the optimal c_1 and c_2? Provide some intuition for your answer.

From our formula, it appears they will save more for period 2, c_1 decreases and c_2 increases. This makes sense, their utility function values the future more and so they will consume more in the future

(d) For what values of δ will the person save, and for what values of δ will she borrow?

[Note: Please report your answer to 5 decimal points.]

Okay lets put in our problem values

$$X = c_1 - 48326$$
$$c_1 = X + 48326$$
$$X = \frac{44928 - c_2}{1.04} = \frac{44928 - \delta^3 c_1}{1.04} = \frac{44928 - \delta^3(X + 48326)}{1.04}$$

For what value of δ does $X = 0$?

$$0 = 44928 - \delta^3\,(48326)$$
$$\delta^3 = \frac{44928}{48326}$$
$$\delta = \sqrt[3]{\frac{44928}{48326}} = 0.975990099509$$

If δ is above 0.98, our friend will invest, or below will borrow

(e) For what values of δ is the person's c_1 larger than her c_2? How does this compare to the condition that we discussed in class (for the case of log utility)?

We already have an equation for c_1 and c_2, so

$$c_1 = c_2$$
$$\delta^3 c_1 = c_2$$
$$\delta^3 = 1$$
$$\delta = 1$$

c_1 is greater than c_2 when δ is less than zero

This is obvious if we think logically about the original utility function. I'm not sure if I was in class that day, and I'm too lazy to look through the slides to try to guess the correct answer

(f) Consider the following alternative income streams:

Alternative A: Receive \$55,814 in period 1 and \$37,440 in period 2.

Alternative B: Receive \$44,026 in period 1 and \$49,400 in period 2.

Discuss how the person's optimal consumption path under these alternatives would compare to her optimal consumption path under the initial income stream — that is, discuss whether c_1 is larger, smaller, or the same, and discuss whether c_2 is larger, smaller, or the same. In addition, discuss how the person's period-1 saving under these alternatives would compare to her priod-1 saving under the initial income stream. **[Note: You can and should answer part (f) without re-solving for the person's behavior.]**

Uhh, don't we need δ for this? Nope! Just compare to conditions in intro to part 1

$(55814; 37440)$ converts to $x = 7488,\ (48326; 40538.48)$ with the 1.04 investment scheme. In this case, c_1 and c_2 is lower for the same δ value since the overall wealth of this individual is less when they try to convert to the original income using investment, although their savings are likely larger to balance the two diminishing utilities

The second individual is likely to save more but is otherwise the same for the same reasons