PHY 481 - Fall 2023
Homework 08

Due Friday, November 17, 2023

Preface

Homework 08 investigates polarization that we will use to understand electric fields in matter. Notice that for polarization problems, we can often find the bound charges and solve the problems much like we have done before with free charges. We also focus on developing ideas about the D-field including how to apply Gauss' Law for $\mathbf{D}$ and the relationships between $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{P}$. You might want to spend some time reminding yourself about Gauss's Law (Sec. 2.2.3) for many of these problems as the symmetries you employ here will be reminiscent of prior work.

1 Atomic hydrogen and the polarization model

Griffiths Table 4.1 gives an experimental value for $\alpha/4\pi\varepsilon_0$ for atomic hydrogen. (Read his caption carefully for units!) The "atomic polarizability," α is defined by $\mathbf{p} = \alpha\mathbf{E}$. Study Griffiths' Example 4.1, which tells you how to estimate the atomic polarizability

1. Following the example and using it with this experimental value for $\alpha/4\pi\varepsilon_0$ for atomic hydrogen, estimate the atomic radius of hydrogen. How well did you do, compared, say, with the Bohr radius?

2. Now suppose you have a single hydrogen atom inside a charged parallel-plate capacitor, with plate spacing 1 mm, and voltage 100 V. Determine the "separation distance" d (as defined in that same Example 4.1 problem) of the electron cloud and the proton nucleus. What fraction of the atomic radius of part 2 is this? (You should conclude that 100 V across a 1mm gap capacitor is unlikely to ionize a hydrogen atom, do you agree?)

3. Use your calculations to roughly estimate what voltage (and thus, what E-field) would ionize this single hydrogen atom. (We'd say if you can pull the electron cloud one full atomic radius away, it's breaking down!)

2 Polarized sphere of charge

Consider a dielectric sphere of radius a that has a polarization that is directed radially outward from the center of the sphere, $\mathbf{P} = P_0\mathbf{r}$.

1. Determine the bound charges at the surface, σ_B, and in the volume of the sphere, ρ_B.
2. Find the electric field everywhere and sketch the electric field lines inside and outside the sphere.

3 Bound charges and the D-field I

Consider a long teflon rod, (a dielectric cylinder), radius a. Imagine that we could somehow set up a permanent polarization $\mathbf{P}(s, \phi, z) = k\mathbf{s}(= ks\hat{s})$, where s is the usual cylindrical radial vector from the z-axis, and k is a constant). Neglect end effects, the cylinder is long.

1. Calculate the bound charges σ_{bound} (on the outer surface) and ρ_{bound} (in the interior of the rod). What are the units of your constant k? Show that the units work out in all formulas you have used involving k.
2. Next, use these bound charges (along with Gauss' law, this problem has very high symmetry!) to find the electric field, $\mathbf{E}$ inside and outside the cylinder. (Your answer should include both the direction and magnitude.)
3. Finally, determine the electric displacement field ($\mathbf{D}$) inside and outside the cylinder using the fundamental definition ($\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$) and verify that "Gauss's law for D-fields" works out.

4 Bound charges and the D-field II

1. Now let's hollow out that teflon rod, so it has inner radius b, and outer radius is (still) a. Just to make things a little different here, suppose we now set up a different polarization within the teflon material, namely $\mathbf{P}(s, \phi, z) = k\hat{s}$ for $b < s < a$ and where k is a given constant. We have vacuum for $s < b$ and $s > a$. What does that tell you about $\mathbf{P}$ in those regions? Find the bound charges σ_{bound} (on inner and outer surfaces of the hollow rod) and ρ_{bound} (everywhere else).
2. Use these bound charges, along with Gauss' law, to find the electric field, $\mathbf{E}$, everywhere in space. (Your answer should include the direction and magnitude.)
3. Use Gauss' Law for D-fields to find $\mathbf{D}$ everywhere in space. *This should be quick - use symmetry! Are there any free charges in this problem?*Use this (simple) result for $\mathbf{D}$ along with $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ to find $\mathbf{E}$ everywhere in space. (This should serve as a check for part 5, and shows why sometimes thinking about D-fields is easier and faster!)