v1

Consider the following quantum state: $|\psi\rangle = \frac{\sqrt{3}}{2}|+\rangle + \frac{1}{2}|-\rangle$.

1) [2] Circle the correct answer: $|\psi\rangle$ is an eigenstate of a spin operator pointing:

a) in the x-y plane b) in the x-z plane c) in the y-z plane d) along the x axis

2) [3] Given the state $|\psi\rangle$ above, calculate the expectation value for the z-component of the spin: $\langle \hat{S}_z \rangle$.

$$\langle \hat{S}_z \rangle = \langle \psi | \hat{S}_z | \psi \rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix} = \frac{\hbar}{2}\left(\frac{3}{4} - \frac{1}{4}\right) = \frac{\hbar}{4}$$

3) [3] Given the state $|\psi\rangle$ above, calculate the uncertainty (standard deviation) of the z-component of spin: $\Delta\hat{S}_z = \sqrt{\langle \hat{S}_z^2 \rangle - \langle \hat{S}_z \rangle^2}$.

$$\hat{S}_z^2 = \frac{\hbar^2}{4} \cdot \hat{I} \quad \text{so} \quad \langle S_z^2 \rangle = \frac{\hbar^2}{4}$$

$$\Delta \hat{S}_z = \sqrt{\frac{\hbar^2}{4} - \left(\frac{\hbar}{4}\right)^2} = \hbar \sqrt{\frac{1}{4} - \frac{1}{16}} = \hbar \sqrt{\frac{3}{16}} = \frac{\sqrt{3}\,\hbar}{4}$$

4) [2] The matrix representation of the spin operator along a particular direction $\hat{n}$ is:

$\hat{S}_n = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix}$. What are the eigenvalues of this operator? (You should be able to write down the answer without doing any calculations.)

$$\pm \frac{\hbar}{2}$$

v2

Consider the following quantum state: $|\psi\rangle = \frac{1}{2}|+\rangle + \frac{\sqrt{3}}{2}|-\rangle$.

1) [2] Circle the correct answer: $|\psi\rangle$ is an eigenstate of a spin operator pointing:

a) in the x-y plane b) in the x-z plane c) in the y-z plane d) along the x axis

2) [3] Given the state $|\psi\rangle$ above, calculate the expectation value for the z-component of the spin: $\langle \hat{S}_z \rangle$.

$$\langle \hat{S}_z \rangle = \langle \psi | \hat{S}_z | \psi \rangle = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1/2 \\ -\sqrt{3}/2 \end{pmatrix} = \frac{\hbar}{2}\left(\frac{1}{4} - \frac{3}{4}\right) = -\frac{\hbar}{4}$$

3) [3] Given the state $|\psi\rangle$ above, calculate the uncertainty (standard deviation) of the z-component of spin: $\Delta\hat{S}_z = \sqrt{\langle \hat{S}_z^2 \rangle - \langle \hat{S}_z \rangle^2}$.

$$S_z^2 = \frac{\hbar^2}{4}\hat{I} \quad \text{so} \quad \langle \hat{S}_z^2 \rangle = \frac{\hbar^2}{4}$$

$$\Delta S_z = \sqrt{\frac{\hbar^2}{4} - \left(-\frac{\hbar}{4}\right)^2} = \hbar\sqrt{\frac{1}{4} - \frac{1}{16}} = \hbar\sqrt{\frac{3}{16}} = \frac{\sqrt{3}\hbar}{4}$$

4) [2] The matrix representation of the spin operator along a particular direction $\hat{n}$ is:

$\hat{S}_n = \frac{h}{2}\begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$. What are the eigenvalues of this operator? (You should be able to write down the answer without doing any calculations.)

$\pm \frac{\hbar}{2}$

√3

Consider the following quantum state: $|\psi\rangle = \frac{\sqrt{3}}{2}|+\rangle - \frac{1}{2}|-\rangle$.

1) [2] Circle the correct answer: $|\psi\rangle$ is an eigenstate of a spin operator pointing:

a) in the x-y plane   b) in the x-z plane   c) in the y-z plane   d) along the x axis

2) [3] Given the state $|\psi\rangle$ above, calculate the expectation value for the z-component of the spin: $\langle \hat{S}_z \rangle$.

$$\langle \hat{S}_z \rangle = \langle \psi | \hat{S}_z | \psi \rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 \\ +1/2 \end{pmatrix} = \frac{\hbar}{2}\left(\frac{3}{4} - \frac{1}{4}\right) = \frac{\hbar}{4}$$

3) [3] Given the state $|\psi\rangle$ above, calculate the uncertainty (standard deviation) of the z-component of spin: $\Delta\hat{S}_z = \sqrt{\langle \hat{S}_z^2 \rangle - \langle \hat{S}_z \rangle^2}$.

$$\hat{S}_z^2 = \frac{\hbar^2}{4} \cdot \hat{I} \quad \text{so} \quad \langle \hat{S}_z^2 \rangle = \frac{\hbar^2}{4}$$

$$\Delta \hat{S}_z = \sqrt{\frac{\hbar^2}{4} - \left(\frac{\hbar}{4}\right)^2} = \hbar\sqrt{\frac{1}{4} - \frac{1}{16}} = \hbar\sqrt{\frac{3}{16}} = \frac{\sqrt{3}\,\hbar}{4}$$

4) [2] The matrix representation of the spin operator along a particular direction $\hat{n}$ is:

$\hat{S}_n = \frac{h}{2}\begin{pmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix}$. What are the eigenvalues of this operator? (You should be able to write down the answer without doing any calculations.)

$\pm \hbar/2$

$\sqrt{4}$

Consider the following quantum state: $|\psi\rangle = \frac{1}{2}|+\rangle - \frac{\sqrt{3}}{2}|-\rangle$.

1) [2] Circle the correct answer: $|\psi\rangle$ is an eigenstate of a spin operator pointing:

a) in the x-y plane   b) in the x-z plane   c) in the y-z plane   d) along the x axis

2) [3] Given the state $|\psi\rangle$ above, calculate the expectation value for the z-component of the spin: $\langle \hat{S}_z \rangle$.

$$\langle \hat{S}_z \rangle = \langle \psi | \hat{S}_z | \psi \rangle = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1/2 \\ -\sqrt{3}/2 \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \frac{1}{2} & -\sqrt{3}/2 \end{pmatrix} \begin{pmatrix} 1/2 \\ +\sqrt{3}/2 \end{pmatrix} = \frac{\hbar}{2}\left(\frac{1}{4} - \frac{3}{4}\right) = -\frac{\hbar}{4}$$

3) [3] Given the state $|\psi\rangle$ above, calculate the uncertainty (standard deviation) of the z-component of spin: $\Delta\hat{S}_z = \sqrt{\langle \hat{S}_z^2 \rangle - \langle \hat{S}_z \rangle^2}$.

$$S_z^2 = \frac{\hbar^2}{4} \hat{I} \quad \text{so} \quad \langle \hat{S}_z^2 \rangle = \frac{\hbar^2}{4}$$

$$\Delta S_z = \sqrt{\frac{\hbar^2}{4} - \left(-\frac{\hbar}{4}\right)^2} = \hbar\sqrt{\frac{1}{4} - \frac{1}{16}} = \hbar\sqrt{\frac{3}{16}} = \frac{\sqrt{3}\,\hbar}{4}$$

4) [2] The matrix representation of the spin operator along a particular direction $\hat{n}$ is:

$\hat{S}_n = \frac{h}{2}\begin{pmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$. What are the eigenvalues of this operator? (You should be able to write down the answer without doing any calculations.)

$\pm \frac{\hbar}{2}$