Problem Set #2 - Richardson

Problem Set 2

[Due in class on Tuesday, February 6.]

Question 1

Question 1:

Suppose you face a potential loss L that will occur with probability q. An insurance agent offers to let you buy partial insurance, wherein you can pay αp to insure against proportion α of this loss. But you are restricted to choosing $\alpha \in [0,1]$.

(a) Suppose the insurance is actuarially unfair ($p > qL$).

(i) If you are risk-averse, what can we conclude about the α that you might choose?

(ii) If you are risk-loving, what can we conclude about the α that you might choose?

(a) In this situation, a risk-neutral individual will not buy insurance as it reduces their expected value

Example: L = 100, q = 0.02, p = 3. Zero insurance is (0,.98;100,.02) and full insurance is (3,1)

- (i) A risk-averse individual may purchase a varying amount of insurance, depending on their degree of risk-aversion
- (ii) A risk-loving individual will not purchase insurance since they wouldn't purchase it for actuarially fair, let alone actuarially unfair.

(b) Suppose the insurance is actuarially overfair ($p < qL$).

(i) If you are risk-averse, what can we conclude about the α that you might choose?

(ii) If you are risk-loving, what can we conclude about the α that you might choose?

(b) In this situation, purchasing more insurance will increase expected value, and a risk-neutral individual will buy as much as possible.

- (i) A risk-averse individual will capitalize on the insurance and buy the maximum amount
- (ii) A risk-loving individual will buy a varying amount of insurance, with the most risk loving choosing $\alpha = 0$, as it offers them a chance at paying nothing.

Question 2

Suppose a person chooses Lottery A over Lottery B, but also chooses Lottery D over Lottery C, where:

Lottery A: $(1000, 1)$

Lottery B: $(2000, .2; 1000, .7; 0, .1)$

Lottery C: $(2000, .2; 0, .8)$

Lottery D: $(1000, .3; 0, .7)$

(a) Does this person's behavior violate expected utility (without any restrictions on u)?

(a) To choose these option, Eu(A)>Eu(B), and Eu(D)>Eu(C),

This implies

$$u(1000) > u(2000) * .2 + u(1000) * .7$$
$$u(1000) * .3 > u(2000) * .2$$

Both of these evolve to the same implication

$$u(1000) - u(1000) * 0.7 > u(2000) * .2 \implies u(1000) > u(2000) * .66$$
$$u(1000) * .3 * 3.3 > u(2000) * .2 * 3.3 \implies u(1000) > u(2000) * .66$$

As such, this is internally consistent and does not violate expected utility

(b) Does this person's behavior violate expected utility with more is better?

(b) This does not violate more is better, as the individual may value a certain reward over an uncertain reward. The utility function must be downward sloping and devalue 2000 reward to the point that 2/3rds of u(2000) is less than u(1000)

(c) Does this person's behavior violate expected utility with risk aversion?

Explain your answers.

(c) In order to prefer u(1000) to u(2000)*.66, u(2000) must be at most 3/2 u(1000), while being risk-neutral requires u(2000) to be 4/2 u(1000) and risk-loving to be greater than that. Thus this person appears to be risk-averse

Question 3

Suppose that Liam evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function

$$v(x) = \begin{cases} x^{0.88} & \text{if } x \geq 0 \\ -2.25 * (-x)^{0.88} & \text{if } x \leq 0. \end{cases}$$

Liam owns an asset that yields a lottery $(\$1000, \frac{1}{3}; \$100, \frac{1}{2}; -\$1000, \frac{1}{6})$. If you offer to purchase this asset from Liam for an amount $\$z$, how large would $\$z$ need to be for Liam to accept your offer?

Lets call the lottery in question A.

$V(A) = 1000^{.88} * .33 + 100^{.88} * .5 - 2.25 * (1000)^{.88} * .17 = 5.85$

Liam would sell the lottery for any value z such that $z^{.88} \geq 5.85 \implies \$z \geq \$7.44$

Question 4

Question 4:

Suppose Martin is a risk-averse expected utility maximizer. In contrast, Roberto evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

Consider the following four choice situations:

Choice (i): $(\ \$8000, \frac{1}{8}\ ;\ \$2000, \frac{7}{8}\ )$ vs. $(\ \$2800, 1\ )$

Choice (ii): $(\ -\$800, \frac{2}{5}\ ;\ -\$400, \frac{3}{5}\ )$ vs. $(\ -\$550, 1\ )$

Choice (iii): $(\ -\$1700, \frac{1}{2}\ ;\ \$0, \frac{1}{2}\ )$ vs. $(\ -\$875, 1\ )$

Choice (iv): $(\ -\$200, \frac{4}{5}\ ;\ \$800, \frac{1}{5}\ )$ vs. $(\ \$0, 1\ )$

For each choice, describe for both Martin and Roberto whether we can determine which option they will choose or whether we need more information.

The three properties are

- The carriers of value are changes in wealth -> jeff bezos still likes winning
- Loss aversion: People dislike losses more than they like same-sized gains.
- Diminishing Sensitivity: People pay less attention to incremental differences when changes are further away from the reference point

Choice (i):
Martin will choose either option, with option B chosen for high risk aversion and option A chosen for lower risk aversion.
Since Roberto evaluates probabilites properly ($\pi(p) = p$), he will likely be risk-averse in a bet of all gains will undervalue the $8000 reward and lean towards option B

Choice (ii):
Martin is risk-averse and will not want to risk getting the -
$800 bill, so he will choose option B. A risk - neutral person would also choose this option due t$
$- loving if it gives him the opportuntiy to get the lowest loss of -$
400, even at the risk of getting -$800

Choice (iii):
Martin is risk averse, and will choose option B
Roberto is risk loving in a game of all losses, and will choose option A

Choice (iv):
Martin is risk averse and will choose option B, even though they have the same expected value his utility function will push down the value of $800, resulting in option B having higher expected utility
Roberto appreciates the probabilites of all outcomes properly, so his value function will push the value of each outcome away from the pure expected value model. Since Losses hurt more than gains, Roberto will devalue option A and choose option B.

Question 5

Question 5:

Suppose that Jennifer evaluates gambles according to prospect theory with $\pi(p) = p$ and a value function that has the three properties suggested by Kahneman & Tversky.

(a) If Jennifer chooses lottery $(\$800, .9; \$0, .1)$ over lottery $(\$1400, .5; \$0, .5)$, could she also choose lottery $(\$1400, .25; \$0, .75)$ over lottery $(\$800, .45; \$0, .55)$? Could prospect theory with $\pi(p) \neq p$ explain this pattern of choices? If so, how? If not, why not?

(a) The prospect theory value function is still a function, so for $\pi(p) = p$ the two descisions should be able to be transformed to have the same conclusion.

$$\begin{cases} v(800) * 0.9 > v(1400).5 \implies v(800) > v(1400).556 \\ v(1400) * .25 > v(800).45 \implies v(1400).556 > v(800) \end{cases}$$

However both options cannot be greater than eachother ($(1400).556 \not> (1400).556$), so this statement contradics itself and doesn't agree with prospect theory. With $\pi(p) \neq p$ this is possible, as it could overrate the likelyhood of a 90% chance to occur and overvalue a 25% chance, resulting in both statements being possible.

(b) If Jennifer faces a 2% chance of incurring a loss of \$20,000, would she be willing to purchase full insurance at a premium of \$400? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

(b) $20,000 * .02 = 400$. A risk neutral individual would go either way on this choice.

Prospect theory with $\pi(p) \neq p$ could cause someone to reduce a 2% chance to near zero, resulting in the negative outcome being ignored and the risky lottery being chosen instead of the insurance. This also aligns with the idea that individuals are more risk-loving in a situation of all losses, where an individual will take a risk to avoid all losses (-$400 and -$20000)

(c) If Jennifer faces a 60% chance of incurring a loss of \$2000, would she be willing to purchase full insurance at a premium of \$1200? Does prospect theory with $\pi(p) \neq p$ make a prediction for how a person should behave? If so, what is it? If not, why not?

(c) $2000 * .6 = 1200$ again, a risk neutral individual will go either way

While people tend to appreciate probabilities more accurately with medium probabilites in prospect theory, prospect theory still suggests that an individual will be risk-loving due to their insensitivity to large losses compared to smaller ones, and Jennifer will likely choose to skip on insurance still to try to get the optimal \$0 loss

Question 6

Question 6:

Consider the bet ($\$400, \frac{1}{3}$; $-\$Y, \frac{2}{3}$).

(a) Suppose that Heidi is an expected utility maximizer with $u(x) = \ln x$, and her initial wealth is \$12000.

(i) For what values of Y does Heidi reject a single play of the bet?

(ii) For what values of Y does Heidi accept two independent plays of the bet?

(iii) Is it possible for Heidi to reject the single bet but accept the aggregate bet?

Note: For part (a), report your answers to 3 decimal points.

(a) Remember that expected utility includes current wealth u(w+x)

- (i) $EU(\text{bet}) = \frac{1}{3}u(12000+400) - \frac{2}{3}u(12000-Y)$. Lets evaluate this when expected utility of the the bet is even with the utility of Heidi's current situation $(\$12000, 1)$.

$$\frac{\ln(12400)}{3} + \frac{2\ln(12000-Y)}{3} = \ln(12000)$$
$$Y = -12000\sqrt{30/31} + 12000 = 195.135$$

  Any value greater than 195 causes Heidi to reject the bet, which makes sense since u(x)=ln(x) is a risk-averse function, while risk-neutral would accept at $Y = 200$

- (ii) $\text{bet}^2 = (800, \frac{1}{9}; 400-Y, \frac{4}{9}; -2Y, \frac{4}{9})$. Again, lets compare with the lottery of doing nothing $(\$12000, 1)$

$$1/9 * \ln(12800) + 4/9 * \ln(12400-Y) + 4/9 * \ln(12000-2Y) = \ln(12000)$$
$$Y = -400\left(\sqrt{64 + 225\sqrt[4]{15}} - 23\right) = 195.133$$

  Which explains why you asked for 3 decimal points. Heidi accepts two plays for values of Y less than 195.133

- (iii) Rejection of single bet -> $Y > 195.135$. Accept aggregate bet -> $Y < 195.133$ There's no overlap, so this is not possible.

(b) Suppose that Bruce also has initial wealth \$12000, but he evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ 2.5x & \text{if } x \leq 0. \end{cases}$$

(i) For what values of Y does Bruce reject a single play of the bet?

(ii) For what values of Y does Bruce accept two independent plays of the bet?

(iii) Is it possible for Bruce to reject the single bet but accept the aggregate bet?

(b) Now we can neglect initial wealth thanks to the first principle from Amos and Daniel that I mentioned. In this case the alternative situation $V(\text{do nothing}) = 0$

- (i) $V(\text{bet}) = 1/3 * v(400) + 2/3 * v(-Y)$

$$400 - 2.5Y = 0$$
$$Y = 400/2.5 = 160$$

  Bruce rejects for Y>160
- (ii) $V(\text{bet}^2) = 1/9 * v(800) + 4/9v(400-Y) + 4/9 * v(-2Y)$. I'll assume v(400-Y) is positive based on the context

$$1/9 * 800 + 4/9(400 - Y) + 4/9 * 2.5 * -2Y = 0$$
$$Y = 100$$

Bruce accepts two independent plays for Y<100

- (iii) Again, Y is greater than 160 to reject a single play, but it must be less than 100 to accept two, so it isn't possible

Question 7

Question 7

(a) Suppose that Johnny is an expected utility maximizer with $u(x) = -e^{-0.001x}$, and has initial wealth is \$75,000. Derive how Johnny feels about the following bets:

(i) Johnny will accept $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Johnny will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Johnny will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iv) Johnny will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(a) Hey I remember this one. Remember to calcualte u(w+x) for evaluating lotteries

In [ ]:
```
# This'll whip the math out quick
import numpy as np

bets = np.array([-100,-200,-500,-750])
w = 75000

# u(bet+w) *.5 + u(X+w) *.5 = u(w)
# X = u^-1 (2u(w)-u(bet+w)) - w

def u(x):
    return -np.exp(-0.001*x)

def uinv(x):
    return -1000 * np.log(-1*x)

x = uinv(2*u(w)-u(bets+w)) + w
for i in range(len(x)):
    print(f'Johnny will accept bet #{i+1} if X is greater than {x[i]:.3f}')
# oh shit
```

```
Johnny will accept bet #1 if X is greater than 150111.123
Johnny will accept bet #2 if X is greater than 150250.261
Johnny will accept bet #3 if X is greater than 151046.175
Johnny will accept bet #4 if X is greater than nan
```

```
C:\Users\andre\AppData\Local\Temp\ipykernel_19728\2385166648.py:14: RuntimeWarning: invalid value encountered in log
  return -1000 * np.log(-1*x)
```

Since the value of $2 * u(75000) - u(73500)$ is positive, the log of the negative of the value doesn't exist and there is no value X that Johnny will accept

(b) Like Johnny, Tommy has initial wealth \$75,000. But unlike Johnny, Tommy evaluates gambles according to prospect theory with $\pi(p) = p$ and

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x \leq 0. \end{cases}$$

If we observe that Tommy accepts $(-100, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X > 210$, what can we conclude about Tommy's λ?

(b) Tommy must have a value λ such that

$$-100 * \lambda + 210 = 2 * 0$$
$$\lambda = \frac{-210}{-100} = 2.1$$

(c) Suppose that Tommy has the λ that you found in part **(b)**, and derive how he feels about the following bets:

(i) Tommy will accept $(-200, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(ii) Tommy will accept $(-500, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

(iii) Tommy will accept $(-750, \frac{1}{2}; X, \frac{1}{2})$ if and only if $X >$????

In each case X will be -2.1 times each negative value

(i) X>420

(ii) X>1050

(iii) X>1575

Question 8

Suppose that you have \$1000 to invest, and you can invest it in stocks or bonds. Each month, bonds yield a certain return of 0.8%. Each month, stocks yield a risky return of 1.8% with probability 0.7 and -1.1% with probability 0.3. Assume returns are independent across months.

You choose your portfolio as suggested by Benartzi & Thaler. Let x be the change in your portfolio's value between now and the next time you evaluate your portfolio. Of course x will be risky — that is, your choice will generate a lottery over possible outcomes for x. For any lottery $(x_1, p_1; \ldots; x_N, p_N)$, you evaluate this lottery according to prospective utility

$$\sum_{i=1}^{N} p_i \, v(x_i)$$

where

$$v(x_i) = \begin{cases} x_i & \text{if } x_i \geq 0 \\ 2.25x_i & \text{if } x_i \leq 0. \end{cases}$$

(a) Suppose you plan to evaluate your portfolio after one month. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(a) For all invested in one lottery for a month, we can simply evaluate based on the monthly risky and certain yields. Remember, prospective theory is focused on changes

- All in bonds: $(1000 * .008, 1)$
- All in stocks: $(1000 * .018, .7; 1000 * -.011, 0.3)$

The prospective uitility of each is

- Bonds: $u(8) = 8$
- Stocks: $0.7 * u(18) + 0.3 * u(-11) = .7 * 18 + .3 * 2.25 * -11 = 12.6 - 7.425 = 5.175$

I prefer all bonds

(b) Suppose you plan to evaluate your portfolio after two months. If you invest in all bonds, what is the lottery over x? If you invest in all stocks, what is the lottery over x? Which do you prefer, all bonds or all stocks?

(b) This is similar to question #6

- $\text{bonds}^2 \ (1000 * 1.008^2 - 1000, 1)$

- stocks^2
  $(1000 * 1.018^2 - 1000, 0.7^2; 1000 * 1.018 * .989 - 1000, 0.7 * 0.3 * 2; 1000 * .989^2 - 1$

The prospective utility of each is

- $U(\text{bonds}^2) = 16.064$
- $U(\text{stocks}^2) = .49 * u(36.324) + .42 * u(6.802) + 0.09 * u(-21.879) = .49 * 36.324 +$

In this scenario, I'd prefer all stocks

(c) How does your preference for stocks vs. bonds depend on your evaluation horizon? Discuss the significance of your answer for the equity premium puzzle.

(c) It appears that further time evaluation leads to the low chance of losses being squished by the greatly increased interest (/ classical expected value). Perhaps the equity premium puzzle comes from the inability to understand how the uncertain lottery of stocks is actually more certain as time advances. and random chances compound. which isn't observed at a short-term view.

(d) Suppose you plan to evaluate your portfolio after two months, and suppose further that you consider splitting your $1000 evenly between stocks and bonds. How do you feel about this allocation relative to all bonds or all stocks?

(d) Without calculating, I'm guessing this is inferior to simply investing entirely in stocks. If we replace every $1000 with 500$ in part (b), it will simply scale down each item by half, including the resulting utilites, and the sum of utilities would be the average between the two, which is less.

Question 9

Question 9

In class, we developed a simple model with mug utility and money utility, and we derived implications for the reservation values of buyers, sellers, and choosers in endowment-effect experiments. This question asks you to think through several variants of that model. For all parts, assume that the person has wealth $w = \$15,000$, and that (Total Utility) = (Mug Utility) + (Money Utility).

(a) Let's begin with the case studied in class: Suppose that money utility is $u_m(m) = m$, and that mug utility is $u(c,r) = \mu c + v(c-r)$ where

$$v(x) = \begin{cases} \eta_{\text{mug}} * x & \text{if } x \geq 0 \\ \lambda_{\text{mug}} * \eta_{\text{mug}} * x & \text{if } x \leq 0. \end{cases}$$

(i) Derive the reservation values for buyers, sellers, and choosers as a function of μ, η_{mug}, and λ_{mug}.

(ii) Discuss how and why the three types differ. If it helps, consider the special case when $\mu = 3$, $\eta_{\text{mug}} = 0.5$ and $\lambda_{\text{mug}} = 5$.

(a) That's a lot of variables

- (i) Reservation values are the point where money utility alone overcomes mug utility and money utility. Cash alone is c=0, cash and mug is c=1. Since $u_m(m) = m$, we can neglect their initial wealth and say that their utiltiy for the mug minus their utiltiy for not having their mug is equal to their reservation value
    - Buyers: $r = 0$

$$u(1,0) - u(0,0) = \mu + v(1) - v(0) = \mu + \eta_{\text{mug}}$$

- Sellers: $r = 1$

$$u(1,1) - u(0,1) = \mu - v(-1) = \mu + \lambda_{\text{mug}}\eta_{\text{mug}}$$

- Choosers: $r = 0$

$$u(1,0) - u(0,0) \text{ This is the same as Buyers}$$

- (ii) With the special case, this yields a value for buyers and choosers of 3.5 and sellers 5.5. It appears sellers won't sell to buyers or choosers, since they value their mugs higher than people without them (they're less willing to get rid of them due to λ_{mug})

(b) Now, let's introduce loss aversion over money: Suppose that mug utility is as in part **(a)**, but now suppose that money utility is $u_m(m, r_m) = m + v_m(m - r_m)$ where $r_m = w$ and

$$v_m(x) = \begin{cases} \eta_{\text{money}} * x & \text{if } x \geq 0 \\ \lambda_{\text{money}} * \eta_{\text{money}} * x & \text{if } x \leq 0. \end{cases}$$

(i) Derive the reservation values for buyers, sellers, and choosers as a function of μ, η_{mug}, λ_{mug}, η_{money}, and λ_{money}.

(ii) Discuss how and why the three types differ. If it helps, consider the special case when $\mu = 3$, $\eta_{\text{mug}} = 0.5$, $\lambda_{\text{mug}} = 5$, $\eta_{\text{money}} = 0.3$, and $\lambda_{\text{money}} = 4$.

(b) Even more variables yay. Now $r_m = w$ means our initial value of $15,000 is important.

- (i) Sellers will have a net positive wealth if they sell the mug, while buyers have a negative v_m if they choose to buy the mug. Choosers will choose a positive cash value in exchange for the mug, yielding a positive wealth. Total utility is constant at the indifference point. Unlabeled variables relate to the mug.
  - Buyer: Buy mug $U_{\text{total}} = u_m(m_0, r_m) + u(1,0)$, don't buy mug $U_{\text{total}} = u_m(r_m, r_m) + u(0,0)$

$$m_0 + v_m(m_0 - 15000) + \mu + \eta = 15000 + v(0,0)$$
$$m_0 + (m_0 - 15000) * \lambda_{\text{money}}\eta_{\text{money}} = 15000 - \mu - \eta$$
$$m_0(1 + \lambda_{\text{money}}\eta_{\text{money}}) = 15000 - \mu - \eta + 15000\lambda_{\text{money}}\eta_{\text{money}}$$
$$m_0 = \frac{15000 - \mu - \eta + 15000\lambda_{\text{money}}\eta_{\text{money}}}{(1 + \lambda_{\text{money}}\eta_{\text{money}})}$$

  - Seller:

$$15000 + \mu = m_0 + v_m(m_0 - 15000) - \lambda\eta$$
$$15000 + \mu = m_0 + (m_0 - 15000) * \eta_{\text{money}} - \lambda\eta$$
$$15000 + \mu + \lambda\eta + 15000 * \eta_{\text{money}} = m_0(1 + \eta_{\text{money}})$$
$$m_0 = \frac{15000 + \mu + \lambda\eta + 15000 * \eta_{\text{money}}}{(1 + \eta_{\text{money}})}$$

- Chooser:

$$15000 + \mu = m_0 + v_m(m_0 - 15000) = m_0 + (m_0 - 15000)\eta_{\text{money}}$$
$$15000 + \mu + 15000\eta_{\text{money}} = m_0(1 + \eta_{\text{money}})$$
$$m_0 = \frac{15000 + \mu + 15000\eta_{\text{money}}}{1 + \eta_{\text{money}}}$$

Find the difference between m_0 and 15000 to find the mug indifference value

- (ii) Buyer price $1.59, Seller price$4.23, Chooser price $2.31. The reasoning is similar to part (a), only now choosers do not have the same issue w
\lambda_\text{money} $ as buyers, increasing their indifference value.

(c) Next, instead of assuming loss aversion over money, let's assume diminishing marginal utility from money: Suppose again that mug utility is as in part (a), but now suppose that money utility is $u_m(m) = 15,000 * \ln m$. To simplify things, assume that $\mu = 3$, $\eta_{\text{mug}} = 0.5$, $\lambda_{\text{mug}} = 5$.

(i) Derive the reservation values for buyers, sellers, and choosers.

(ii) Discuss how and why the three types differ.

(c) Okay third time around. Four decimal places.

- (i) Lets copy and paste some stuff.
  - Buyer: Buy mug $U_{\text{total}} = u_m(m_0) + u(1,0)$, don't buy mug $U_{\text{total}} = u_m(r_m) + u(0,0)$

$$15000\ln(m_0) + \mu + \eta = 15000\ln(15000) + v(0,0)$$
$$m_0 = e^{\left(\frac{15000\ln(15000) - \mu - \eta}{15000}\right)}$$
$$m_0 = 15000 - 3.5318$$

  - Seller:

$$15000 \ln(15000) + \mu = 15000 \ln(m_0) - \lambda\eta$$
$$m_0 = e^{\left(\frac{15000 \ln(15000) + \mu + \lambda\eta}{15000}\right)}$$
$$m_0 = 15000 + 5.4688$$

  - Chooser:

$$15000 \ln(15000) + \mu = 15000 \ln(m_0)$$
$$m_0 = e^{\left(\frac{15000 \ln(15000) + \mu}{15000}\right)}$$
$$m_0 = 15000 + 3.0003$$

- (ii) Now buyers are willing to spend more than choosers are willing to. Since Buyers have to pay for the mug (increasing maginal utility) but choosers gain money (decreasing maginal utility), the new money utility function results in a higher value when buying the mug than selling it (although sellers are still mostly influenced by λ_{mug} kicking up their mug value)

(d) Finally, let's keep diminishing marginal utility from money, but now eliminate loss aversion over mugs: Suppose that mug utility is $u(c) = 3.5c$, and that money utility is $u_m(m) = 15,000 * \ln m$.

(i) Derive the reservation values for buyers, sellers, and choosers.

(ii) Discuss how and why the three types differ.

Note: For parts (c) & (d), report your answers to 4 decimal points.

(d) Mug utility is simply 3.5 or 0.

- (i)
  - Buyer:

$$15000 \ln(m_0) + 3.5 = 15000 \ln(15000)$$
$$m_0 = e^{\ln(15000) - 3.5/15000}$$
$$m_0 = 15000 - 3.4996$$

  - Seller:

$$15000 \ln(15000) + 3.5 = 15000 \ln(m_0)$$
$$m_0 = e^{\ln(15000) + 3.5/15000}$$
$$m_0 = 15000 + 3.5004$$

  - Chooser:

$$15000 \ln(15000) + 3.5 = 15000 \ln(m_0)$$
$$\text{same as seller}$$
$$m_0 = 15000 + 3.5004$$

- (ii) They don't differ by much, but sale still doesn't occur between buyers and sellers who don't make rounding errors. Without any utility earned from buying or lost from selling a mug, there isn't much differentiating buyers, sellers, or choosers, and the minute reduction in buyer's indifference point is due to diminishing marginal utility.

Question 10

Nah not tonight

Definitions I pulled up answering these:

Definition: A person is *risk-averse* if for any (risky) lottery $\mathbf{x}$ she prefers to have $EV(\mathbf{x})$ for sure instead of lottery $\mathbf{x}$.

Definition: A person is *risk-neutral* if for any (risky) lottery $\mathbf{x}$ she is indifferent between $EV(\mathbf{x})$ for sure vs. lottery $\mathbf{x}$.

Definition: A person is *risk-loving* if for any (risky) lottery $\mathbf{x}$ she prefers to have lottery $\mathbf{x}$ instead of $EV(\mathbf{x})$ for sure.

Definition: The function $\tilde{u}(x)$ is a *positive affine transformation* of the function $u(x)$ if $\tilde{u}(x) = a \cdot u(x) + b$ for some $a > 0$ and any b.

Definition: $\tilde{u}(x)$ and $u(x)$ *reflect the same preferences* if for any pair of lotteries $\mathbf{x}$ and $\mathbf{y}$

$$EU(\mathbf{x}) > EU(\mathbf{y}) \text{ if and only if } E\tilde{U}(\mathbf{x}) > E\tilde{U}(\mathbf{y}).$$

Benartzi & Thaler's Model

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A (Much) Simplified Example:

Suppose there are two assets, stocks and bonds, and that between τ and $\tau + \Delta$ the returns are:

- For bonds: $(+1\%, 1)$
- For stocks: $(+10\%, \frac{1}{2}; -5\%, \frac{1}{2})$

Suppose further that the person must choose a proportion α of her wealth to invest in stocks, with the remainder invested in bonds. As a function of α, the resulting lottery over $x_{\tau+\Delta}$ is

$$\textbf{Good Outcome:} \qquad \alpha w(.10) + (1-\alpha)w(.01)\ ,\ \frac{1}{2}$$

$$\textbf{Bad Outcome:} \qquad \alpha w(-.05) + (1-\alpha)w(.01)\ ,\ \frac{1}{2}$$

Again, at date τ, person chooses between lotteries over $x_{\tau+\Delta}$.

> Note: Major deviation from the standard approach!

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Evaluating Lotteries

At date t, person chooses her portfolio to maximize her "prospective utility"

$$\sum_{x_{t+\Delta}} \pi(x_{t+\triangle})v(x_{t+\triangle}).$$

Let's use the value function

$$v(x) = x^{\alpha} \qquad \text{if} \quad x \geq 0$$

$$v(x) = -\lambda(-x)^{\beta} \qquad \text{if} \quad x \leq 0$$

The authors assume $\alpha = \beta = .88$ and $\lambda = 2.25$ (Tversky & Kahneman, 1992).

$\pi(x_{t+\triangle})$ reflects probability weighting. The authors use the cumulative form --- including the suggested parameter values --- from Tversky & Kahneman, 1992.

Suppose your Mug Utility is $\boldsymbol{u(c, r)}$, where $\boldsymbol{c}$ is your mug consumption and $\boldsymbol{r}$ is your mug reference point.

- $\boldsymbol{r = 0} \iff$ unendowed (buyers & choosers)
- $\boldsymbol{r = 1} \iff$ endowed (sellers)
- $\boldsymbol{c = 1} \iff$ go home with mug (buy, choose, or keep).
- $\boldsymbol{c = 0} \iff$ go home without mug (don't buy, don't choose, or sell)

Assume $\boldsymbol{u\left(c, r\right) = w(c) + v(c - r)}$, where

$$\boldsymbol{w(c) = \mu * c}$$

$$\textbf{and } \boldsymbol{v(x) = \phi x} \quad \textbf{if } \boldsymbol{x \geq 0}$$

$$\textbf{else } \boldsymbol{v(x) = \lambda \phi x} \quad \textbf{if } \boldsymbol{x < 0.}$$