Quiz # 2, v1 solutions

1) [2] Normalize this ket so that it represents a true quantum state: $|\varphi\rangle = 2|+\rangle - 3|-\rangle$.

$$\langle\varphi|\varphi\rangle = 4 + 9 = 13 \qquad |\varphi_{norm}\rangle = \frac{2}{\sqrt{13}}|+\rangle - \frac{3}{\sqrt{13}}|-\rangle$$

2) [8] A spin-1/2 particle is prepared in the quantum state represented by the ket:

$$|\psi\rangle = \frac{2}{\sqrt{5}}|+\rangle + \frac{i}{\sqrt{5}}|-\rangle$$

a) [2] If the particle is passed through a Stern-Gerlach apparatus oriented along the z-direction, what is the probability that the measurement of $\hat{S}_z$ will produce the result spin up $\left(\frac{+\hbar}{2}\right)$?

$$\mathcal{P}_z\left(+\frac{\hbar}{2}\right) = |\langle+|\psi\rangle|^2 = \left|\frac{2}{\sqrt{5}}\right|^2 = \frac{4}{5} = 0.8$$

b) [2] If the previous measurement resulted in spin up along the z direction, and after that measurement you then measure the y-component of spin, what is the probability that the measurement will produce the result spin up in the y-direction?

$\mathcal{P} = \frac{1}{2}$ because z and y are 90° apart

c) [4] If, instead, we start with the particle in its original quantum state $|\psi\rangle$ and measure the y-component of its spin, what is the probability that the measurement will produce the result spin up along the y direction? Hint: You need to know the y-spin-up state expressed in terms of the z-basis states: $|+\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$. Show all your work for this problem.

$$\mathcal{P}_y\left(+\frac{\hbar}{2}\right) = |{}_y\langle+|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle+| - i\langle-|\right)\frac{1}{\sqrt{5}}\left(2|+\rangle + i|-\rangle\right)\right|^2$$

$$= \frac{1}{10}|2+1|^2 = \frac{9}{10} = 0.9$$

Quiz #2, v2 solutions

1) [2] Normalize this ket so that it represents a true quantum state: $|\varphi\rangle = 3|+\rangle - 2|-\rangle$.

$$\langle\varphi|\varphi\rangle = 9 + 4 = 13 \qquad |\varphi_{norm}\rangle = \frac{3}{\sqrt{13}}|+\rangle - \frac{2}{\sqrt{13}}|-\rangle$$

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2) [8] A spin-1/2 particle is prepared in the quantum state represented by the ket:

$$|\psi\rangle = \frac{2}{\sqrt{5}}|+\rangle + \frac{i}{\sqrt{5}}|-\rangle$$

a) [2] If the particle is passed through a Stern-Gerlach apparatus oriented along the z-direction, what is the probability that the measurement of $\hat{S}_z$ will produce the result spin down $\left(\frac{-\hbar}{2}\right)$?

$$P_z\left(-\frac{\hbar}{2}\right) = \left|\langle -|\psi\rangle\right|^2 = \left|\frac{i}{\sqrt{5}}\right|^2 = \frac{1}{5} = 0.2$$

b) [2] If the previous measurement resulted in spin down along the z direction, and <u>after that measurement</u> you then measure the y-component of spin, what is the probability that the measurement will produce the result spin down in the y-direction?

$P = \frac{1}{2}$ because z and y are 90° apart

c) [4] If, instead, we start with the particle in its original quantum state $|\psi\rangle$ and measure the y-component of its spin, what is the probability that the measurement will produce the result spin down along the y direction? Hint: You need to know the y-spin-down state expressed in terms of the z-basis states: $|-\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle - i|-\rangle)$. Show all your work for this problem.

$$P_y\left(-\frac{\hbar}{2}\right) = \left|{}_y\langle -|\psi\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle +| + i\langle -|\right)\frac{1}{\sqrt{5}}\left(2|+\rangle + i|-\rangle\right)\right|^2$$

$$= \frac{1}{10}\left|2 - 1\right|^2 = \frac{1}{10} = 0.1$$

Quiz #2, v3 solutions

1) [2] Normalize this ket so that it represents a true quantum state: $|\varphi\rangle = 4|+\rangle - |-\rangle$.

$$\langle\varphi|\varphi\rangle = 16 + 1 = 17 \qquad |\varphi_{norm}\rangle = \frac{4}{\sqrt{17}}|+\rangle - \frac{1}{\sqrt{17}}|-\rangle$$

2) [8] A spin-1/2 particle is prepared in the quantum state represented by the ket:

$$|\psi\rangle = \frac{3}{\sqrt{10}}|+\rangle + \frac{i}{\sqrt{10}}|-\rangle$$

a) [2] If the particle is passed through a Stern-Gerlach apparatus oriented along the z-direction, what is the probability that the measurement of $\hat{S}_z$ will produce the result spin up $\left(\frac{+\hbar}{2}\right)$?

$$P_z\left(+\frac{\hbar}{2}\right) = |\langle +|\psi\rangle|^2 = \left|\frac{3}{\sqrt{10}}\right|^2 = \frac{9}{10} = 0.9$$

b) [2] If the previous measurement resulted in spin up along the z direction, and after that measurement you then measure the y-component of spin, what is the probability that the measurement will produce the result spin up in the y-direction?

$P = \frac{1}{2}$ because z and y are 90° apart.

c) [4] If, instead, we start with the particle in its original quantum state $|\psi\rangle$ and measure the y-component of its spin, what is the probability that the measurement will produce the result spin up along the y direction? Hint: You need to know the y-spin-up state expressed in terms of the z-basis states: $|+\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$. Show all your work for this problem.

$$P_y\left(+\frac{\hbar}{2}\right) = |{}_y\langle +|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle +| - i\langle -|\right)\frac{1}{\sqrt{10}}\left(3|+\rangle + i|-\rangle\right)\right|^2$$

$$= \frac{1}{20}|3+1|^2 = \frac{16}{20} = \frac{4}{5} = 0.8$$

Quiz #2, v4 solutions

1) [2] Normalize this ket so that it represents a true quantum state: $|\varphi\rangle = |+\rangle - 4|-\rangle$.

$$\langle\varphi|\varphi\rangle = 1 + 16 = 17 \qquad |\varphi_{norm}\rangle = \frac{1}{\sqrt{17}}|+\rangle - \frac{4}{\sqrt{17}}|-\rangle$$

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2) [8] A spin-1/2 particle is prepared in the quantum state represented by the ket:

$$|\psi\rangle = \frac{3}{\sqrt{10}}|+\rangle + \frac{i}{\sqrt{10}}|-\rangle$$

a) [2] If the particle is passed through a Stern-Gerlach apparatus oriented along the z-direction, what is the probability that the measurement of $\hat{S}_z$ will produce the result spin down $\left(\frac{-\hbar}{2}\right)$?

$$P_z\left(-\frac{\hbar}{2}\right) = |\langle -|\psi\rangle|^2 = \left|\frac{i}{\sqrt{10}}\right|^2 = \frac{1}{10} = 0.1$$

b) [2] If the previous measurement resulted in spin down along the z direction, and <u>after that measurement</u> you then measure the y-component of spin, what is the probability that the measurement will produce the result spin down in the y-direction?

$P = \frac{1}{2}$ because z and y are 90° apart.

c) [4] If, instead, we start with the particle in its original quantum state $|\psi\rangle$ and measure the y-component of its spin, what is the probability that the measurement will produce the result spin down along the y direction? Hint: You need to know the y-spin-down state expressed in terms of the z-basis states: $|-\rangle_y = \frac{1}{\sqrt{2}}(|+\rangle - i|-\rangle)$. Show all your work for this problem.

$$P_y\left(-\frac{\hbar}{2}\right) = |{}_y\langle -|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle +| + i\langle -|\right)\frac{1}{\sqrt{10}}\left(3|+\rangle + i|-\rangle\right)\right|^2$$

$$= \frac{1}{20}|3-1|^2 = \frac{4}{20} = \frac{1}{5} = 0.2$$