Physics 471 – Fall 2023

Homework #5 – due Wednesday, October 4 at 11:30am

Point values for each problem are in square brackets

1. [2] **Commutators:**

Using the matrix representations of the spin operators $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$, show that $[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z$.

2. [6] **Uncertainty:**

Consider a general quantum state $|\psi\rangle = a|+\rangle + b|-\rangle$, where *a* and *b* are complex numbers.

a) [3] Calculate the "expectation" value $\langle\hat{S}_z\rangle$ and the uncertainty $\Delta\hat{S}_z$ in terms of a and b.

b) [3] In what situation is the uncertainty a minimum? A maximum? Write down the values of the expectation value and the uncertainty in those cases. Do these make sense? Explain.

3. [8] **Expectation values and uncertainties:**

a) [3] Given $|\psi\rangle = |-\rangle$, , find $\langle\hat{S}_z\rangle$, $\Delta\hat{S}_z$, $\langle\hat{S}_x\rangle$ and $\Delta\hat{S}_x$. Also, sketch a histogram like Figure 2.8 in the textbook. (One histogram for the Z measurements, and another for the X measurements). Do these calculations make sense to you? Briefly, comment.

b) [2] $|\psi\rangle = |-\rangle_y$. You only need to find $\langle\hat{S}_z\rangle$ and $\Delta\hat{S}_z$, sketch the histogram, and comment.

c) [3] $|\psi\rangle = \frac{1}{\sqrt{5}}(2|+\rangle - i|-\rangle)$. Again find $\langle\hat{S}_z\rangle$, $\Delta\hat{S}_z$, sketch the histogram, and comment.

4. [4] **Compatible measurements and commutators:**

a) [2] Are there any quantum states for which you could simultaneously know with certainty the results of measurements of $\hat{S}_x$ and $\hat{S}_z$? Show this mathematically and describe your answer in words also. Both answers should be very brief!

b) [2] Calculate the commutator $[\hat{S}_x, \hat{S}_z^2]$ using the matrix representations for these operators (in the z-basis). This might initially seem very strange give what you found in part (a). How do you reconcile this result with your answer to part (a)?