PHY 481 - Fall 2023
Homework 03

Due Friday September 22, 2023

Homework 03 emphasizes the electric field and the principle of superposition that will form the basis of much of your understanding of electrostatics. This homework makes use of what you learned from Secs. 1.1-1.4 in Griffiths and adds to it the concepts from Sec. 2.1, which make up the bulk of the assignment. In this assignment, you will also use a Jupyter notebook to determine the electric field of a point charge and a dipole as we build up the architecture to solve electric field problems numerically.

1 Electric field of a ring of charge

In this problem, we will calculate the electric field of a uniformly charged ring of radius R. The ring lies on $xy-$plane and we would like to find the electric field at point P which is at a distance z above the center of the ring.

1. Draw a picture showing the infinitesimal charge dq on the ring, the vectors $\vec{r}$, $\vec{r'}$, and $\vec{\mathscr{r}}$.
2. Write $\vec{r}$, $\vec{r'}$, $\vec{\mathscr{r}}$, $\hat{\mathscr{r}}$, and dl' using the cylindrical coordinates s, θ, and ϕ.
3. One of our "Ten Curvilinear Coordinate Commandments" is "Thou shalt always re-express Griffiths script-r in terms of rectangular unit vectors." Write $\hat{\mathscr{r}}$ in Cartesian unit vectors. (It can contain the angle ϕ but no $\hat{s}$ or $\hat{\phi}$)
4. Write the electric field equation $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda}{\mathscr{r}^2}\hat{\mathscr{r}}\, dl'$ by putting it all together.
5. Solve the integrals for x, y, and z components of the electric field. Did you find any of them as zero? Does it make sense from the symmetry point of view? Write the final electric field vector.

2 Checking your result

In this class, you will often produce new formulae that describe some situation for which you might not have developed intuition yet. So, one question you should always be asking yourself is: **How do I believe the physics/math that I've just done?!?** In this problem, you will develop some techniques for checking your results against the intuition that you already hold.
If done correctly, the problem above should give the result:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda 2\pi R z}{(z^2+R^2)^{3/2}}\hat{z} \tag{1}$$

1. Check the units of this expression to verify it is correct.
2. What are at least two other independent checks that you could do to see if your result makes sense?
3. Perform those two independent checks. Comment on if you believe this result based on these checks. Why or why not?

3 Electric field of a disk of charge

We will apply the same ideas in the previous problems to a case where the charge is distributed over a surface.
Consider a thin disk of radius R with a uniform charge density, $-\sigma$:

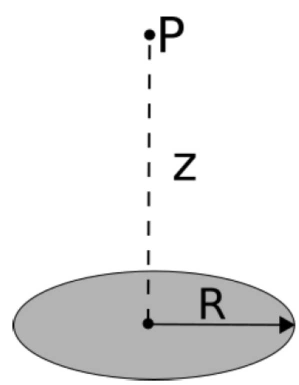

Figure 1: Disk of charge

1. Find the electric field at point P, which is a distance z above the center of the disk, by integrating across the surface of the disk. *Yes, we know that this field is well-known, but the practice of setting up and doing these kinds of integrals is important.* The functional form of your solution is a bit complicated and it might be tough to see how if its correct - *you can certainly look up the answer to check it, but you won't always be able to do that in this class (and in life)!*

2. If you were very far from this disk, what would you expect the field to look like? Use your intuition from PHY 184. Explicitly check the limiting form of your solution at very large z (i.e., when $z >> R$). *By "limiting form", we mean "how it behaves as a function of distance."* So, don't just say "it goes to zero" (if that's what you think happens). Tell us how, functionally it vanishes (like $1/z$? like e^{-z}? Something else?).

3. If you were very close to the disk, what would expect the field too look like? Again, use your intuition from PHY 184. Explicitly check the limiting form of your solution at very small z (i.e., when $z << R$).

4. Sketch a qualitatively correct graph of the component of the electric field in the z-direction along the center line. Be sure to include both the positive and negative z-axis in your graph. Your answers to parts 2 and 3 might help you here.

4 Python: Numerically calculating the electric field

In this problem, we will lay the groundwork for determining the electric field of a distribution of charge numerically. This is a different kind of numerical integration, which uses superposition as the main element. But to get started, we need to learn how to represent vectors using Python. So this problem introduces that and asks you to compute the electric field of a point charge and a dipole.

You will do this work in a Jupyter notebook. You can download the notebook HW3_Calculate_Electric_Field.ipynb from D2L. As you work through this problem, you will work through the following activities:

1. Observe how vector calculations can be done using the "numpy" library. For example, what "numpy" function allows you to take a cross product? How about a dot product?

2. Use the "numpy" library to compute the electric field of a point charge and plot the electric field vectors.

3. Use the "numpy" library to compute the electric field of a dipole and plot the electric field vectors.