Physics 471 – Fall 2023

Homework #10 – due Wednesday, November 15

**1.** [7] Consider a "half-infinite" square well in which the potential energy is $\infty$ for $x < 0$, zero for $0 < x < a$, and finite (with a value $V_0$) for $x > a$.

a) [2] The energy eigenstates are almost exactly the same as one set of the solutions to the finite well problem (i.e., either McIntyre Eq. 5.80 or 5.81, with corresponding eigenvalue Eq 5.83 or 5.85). Which of those equations are the right ones here? Briefly, explain. There *is* going to be one minor difference from the McIntyre solution (in normalization.) What is that difference?

b) [3] Give a qualitative (graphical) argument that the energy eigenvalues of this finite well are close to the usual results for an infinite well where $V_0$ goes to $\infty$ at $x = 0$ and $x = a$.
Are the finite well energies a little smaller or a little larger than the infinite well energies? Why? As you go higher and higher in (allowed) energy states, are the <u>differences</u> in energy eigenvalues between this finite well and the infinite well getting smaller or larger? Why?

c) [2] Assume that $V_0$ is large enough so that there are at least 4 bound states in the well. Draw sketches of the four lowest-energy wavefunctions. Your sketches won't be quantitatively accurate, because I haven't given you values for $a$ and $V_0$, but they should be qualitatively accurate, i.e. with the right number of nodes and some appreciation for how far the wavefunction extends into the "classically forbidden region" as the energy increases.

**2.** [6] Consider a particle of mass $m$ that is in the ground state of an infinite square well potential of width $L$., i.e. the particle is trapped in the interval $0 < x < L$. Suddenly, the right wall of the potential is moved to the position $x = 2L$. The change in the potential is so sudden that the wavefunction of the particle does not change at first. But the particle is no longer in an energy eigenstate because the eigenstates have all changed. So if we measure the energy of the particle in the new, wider, well, we are no longer guaranteed to get a single answer.

a) [2] Draw pictures of the four lowest-energy wavefunctions of the new potential, directly under a picture of the initial wavefunction. We know that the new energy eigenstates form a complete basis, therefore the initial state can be written as a linear superposition of the new energy eigenstates. By looking at your pictures, can you estimate which of the eigenstates will have the largest coefficient in the superposition?

b) [4] Now calculate the probabilities that a measurement of the particle's energy will give the results $E_1$, $E_2$, $E_3$ or $E_4$ in the new potential. (It may be tempting to write a general formula valid for all $n$. If you want to try that, I suggest you treat the $n = 2$ case separately.) Compare your answers with your guess in the previous part. Also, explain using your pictures why one of those four probabilities is zero.

**3.** [4] A particle in an infinite square well at $t = 0$ has a probability density that is uniform in the *left half* (0< x < L/2), and 0 in the right half. (This is a bit unphysical, because $\psi(x, t = 0) \neq 0$ at the left wall of the potential well, but it makes the math easier for this question.) Assume that $\psi(x, t = 0)$ is a positive, real function.

a) [1] Sketch $\psi(x, t = 0)$ and write a mathematical formula for it (e.g. a "piecewise" formula specifying it in the regions 0< x < L/2 and L/2 < x < L). Make sure your formula is normalized!

b) [3] Find a general formula (valid for all $n$) for the probability that a measurement of energy yields $E_n$, and evaluate this formula for the first 4 cases ($n$ = 1 to 4). Explain what happens when $n$ = 4 (Explain the "math" answer using a graph!)

**4.** [3] In last week's homework, you calculated the product $\Delta x \Delta p$ for the second energy eigenstate of the infinite square well. Your answer should have been greater than $\hbar/2$ because of the Uncertainty Principle. McIntyre writes the Uncertainty Principle in equation (2.98), but to use that equation we need to know the commutator: $[\hat{x}, \hat{p}]$.

Calculate that commutator using the position representations of the operators $\hat{x}$ and $\hat{p}$. Since one of those operators contains a derivative, the way to calculate the commutator is to have it act on a wavefunction $\psi(x)$. Note that the derivative acts on <u>everything</u> to the right of it!