Physics 471 – Fall 2023

Homework #6 – due Wednesday, October 11 at 11:30am

Point values for each problem are in square brackets

1. [3] **Schrodinger equation in matrix notation**

The Schrodinger equation in operator/ket notation is written in Equation (3.1) of McIntyre. Consider a 2-state system, and use as your basis states the eigenstates of the Hamiltonian: call them $|E_1\rangle$ and $|E_2\rangle$. You can write a general time-dependent state as a linear superposition of the basis states, with time-dependent coefficients, $c_1(t)$ and $c_2(t)$, as shown in Equation (3.3).

Write out the Schrodinger equation in matrix form. The Hamiltonian will be a 2×2 matrix, and the state of the system will be represented by a column vector. From your matrix equation show that $c_1(t)$ and $c_2(t)$ obey Equation (3.8).

2. [7] **Time dependence of operators that don't commute with H:**

Consider a two-state quantum system with Hamiltonian $\widehat{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$, with $E_1 > E_2$.

We also have a physical observable A described by the operator $\hat{A} = \begin{pmatrix} 0 & \mu \\ \mu & 0 \end{pmatrix}$, where μ is real and positive.

a) [2] What are the possible measurement outcomes for the operator $\hat{A}$, and what are their associated eigenstates? (Please show the calculation; do not just write down the answers.)

b) [2] Next let's assume that the initial state of the system is $|\psi(t=0)\rangle = |a_1\rangle$, which is the eigenstate corresponding to the larger of the two eigenvalues of $\hat{A}$. Write an expression for $|\psi(t)\rangle$. You may use ket notation or column vector notation.

c) [2] How does the expectation value of $\hat{A}$ depend on time? Make a sketch of $\langle\hat{A}\rangle$ as a function of time.

d) [1] What is the expectation value of the energy, $\langle\widehat{H}\rangle$, as a function of time? Briefly discuss your result – does it make sense?

(continued on back)

3. [10] **Spin precession in a magnetic field**

At time t = 0, the state of an electron spin is $|\psi(t=0)\rangle = |+\rangle_n$ with $\theta = \frac{\pi}{2}$ and $\phi = -\frac{\pi}{4}$.

The system is allowed to time-evolve in a uniform magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$.

a) [1] Write the initial state in the usual z-basis i.e. $|\psi(0)\rangle = \binom{a}{b}$, and find a and b.

Suggestion: Leave the factor $e^{i\phi} = e^{-i\pi/4}$ in that form; don't rewrite it as $\frac{1-i}{\sqrt{2}}$.

b) [2] Follow the usual procedure to find $|\psi(t)\rangle$ (in the usual S_z-basis).

c) [3] What is the probability that the electron will be measured to have spin up in the y-direction after a time t? Make a graph of this probability as a function of time. It is always useful to check your answers – this is a probability, so convince yourself (and us) that your answer always falls in the allowed range for probabilities.

d) [1] Will the electron ever get back to its original starting state? (If so, when? If not, why not?)

e) [3] Calculate the expectation values of the three spin operators, $\langle\widehat{S_x}\rangle$, $\langle\widehat{S_y}\rangle$, and $\langle\widehat{S_z}\rangle$ as a function of time. (Show all your work.) Draw graphs of these quantities as a function of time and use the graphs to describe in your own words the behavior of the average spin vector $\langle\widehat{\boldsymbol{S}}\rangle$. From $\langle\widehat{S_z}\rangle$, you can easily find the expectation value of the energy, $\langle\widehat{H}\rangle$.