QUIZ #4 SOLUTIONS (v1)

At time $t = 0$, the state of an electron spin is $|\psi(t=0)\rangle = |+\rangle_x = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$.

The system is allowed to time evolve in a uniform magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$. In our usual z-spin basis, the Hamiltonian of the system is represented by the matrix:

$$\hat{H} = \frac{\hbar\omega_0}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \text{ where } \omega_0 = \frac{eB_0}{m_e}.$$

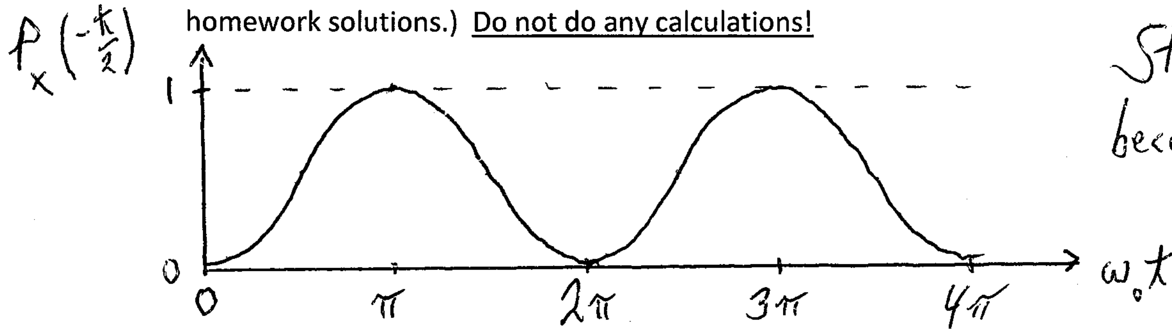
1) [1] What are the two eigenstates of $\hat{H}$ (kets) and what are their energies, E_+ and E_-? (You should know this without doing any calculations!)

$$|E_+\rangle = |+\rangle \doteq \begin{pmatrix}1\\0\end{pmatrix},\quad E_+ = \frac{\hbar\omega_0}{2} \qquad |E_-\rangle = |-\rangle \doteq \begin{pmatrix}0\\1\end{pmatrix},\quad E_- = -\frac{\hbar\omega_0}{2}$$

2) [2] Write down an expression for $|\psi(t)\rangle$. You may use either ket notation or column vector notation – your choice.

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_0 t/2}|+\rangle + e^{+i\omega_0 t/2}|-\rangle\right) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega_0 t/2}\\ e^{+i\omega_0 t/2}\end{pmatrix}$$

3) [3] At time t, we measure the x-component of the electron spin, $\widehat{S_x}$. Draw a graph of the probability vs time to obtain the result $\left(-\frac{\hbar}{2}\right)$ from that measurement. Label both axes on the graph: the labels should show the largest and smallest values of the function being plotted, as well as the time scale. (For simplicity, plot the quantity $\omega_0 t$ on the horizontal axis as I did in the homework solutions.) <u>Do not do any calculations!</u>



Starts at 0 because $|\psi(t=0)\rangle = |+\rangle_x$

4) [4] Calculate the time-dependent probability you plotted in the previous problem. <u>For this question, you must show all your work</u>. Use the front side of the paper if you need more space.

$$P_x\left(-\frac{\hbar}{2}\right) = \left|{}_x\langle -|\psi(t)\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega_0 t/2}\\ e^{i\omega_0 t/2}\end{pmatrix}\right|^2$$

$$= \left|\frac{1}{2}\left(e^{-i\omega_0 t/2} - e^{i\omega_0 t/2}\right)\right|^2 = \left|-i\sin\left(\frac{\omega_0 t}{2}\right)\right|^2 = \sin^2\left(\frac{\omega_0 t}{2}\right) = \underline{\underline{\frac{1}{2}\left(1-\cos(\omega_0 t)\right)}}$$

QUIZ # 4 SOLUTIONS (v2)

At time $t = 0$, the state of an electron spin is $|\psi(t=0)\rangle = |-\rangle_x = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$.

The system is allowed to time evolve in a uniform magnetic field $\boldsymbol{B} = B_0\hat{\mathbf{z}}$. In our usual z-spin basis, the Hamiltonian of the system is represented by the matrix:

$$\hat{H} = \frac{\hbar\omega_0}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ where } \omega_0 = \frac{eB_0}{m_e}.$$

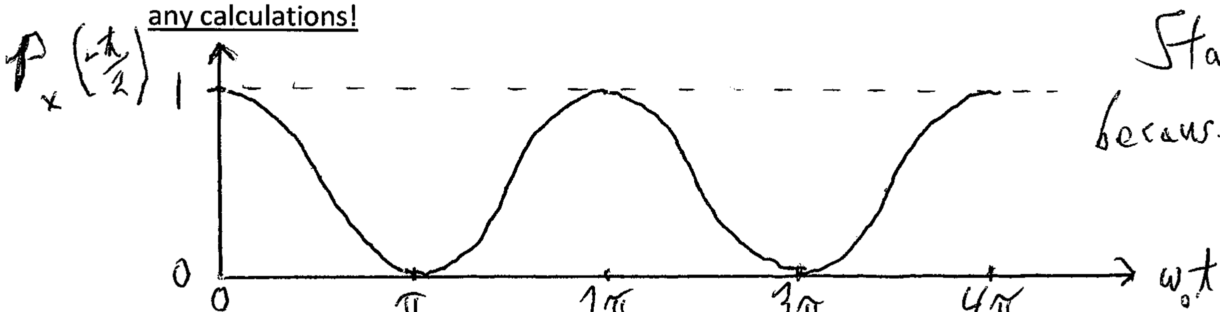
1) [1] What are the two eigenstates of $\hat{H}$ (kets) and what are their energies, E_+ and E_-? (You should know this without doing any calculations!)

$$|E_+\rangle = |+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad E_+ = \frac{\hbar\omega_0}{2} \qquad |E_-\rangle = |-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad E_- = \frac{-\hbar\omega_0}{2}$$

2) [2] Write down an expression for $|\psi(t)\rangle$. You may use either ket notation or column vector notation – your choice.

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_0 t/2}|+\rangle - e^{+i\omega_0 t/2}|-\rangle\right) \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega_0 t/2} \\ -e^{+i\omega_0 t/2} \end{pmatrix}$$

3) [3] At time t, we measure the x-component of the electron spin, $\widehat{S_x}$. Draw a graph of the probability vs time to obtain the result $\left(-\frac{\hbar}{2}\right)$ from that measurement. Label both axes on the graph: the labels should show the largest and smallest values of the function being plotted, as well as the time scale. For simplicity, plot the quantity $\omega_0 t$ on the horizontal axis. <u>Do not do any calculations!</u>



Starts at 1 because $|\psi(t=0)\rangle = |-\rangle_x$

4) [4] Calculate the time-dependent probability you plotted in the previous problem. <u>For this question, you must show all your work</u>. Use the front side of the paper if you need more space.

$$P_x\left(-\frac{\hbar}{2}\right) = \left|{}_x\langle -|\psi(t)\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega_0 t/2} \\ -e^{+i\omega_0 t/2} \end{pmatrix}\right|^2$$

$$= \left|\frac{1}{2}\left(e^{-i\omega_0 t/2} + e^{+i\omega_0 t/2}\right)\right|^2 = \left|\cos\left(\frac{\omega_0 t}{2}\right)\right|^2 = \cos^2\left(\frac{\omega_0 t}{2}\right) = \frac{1}{2}\left(1 + \cos(\omega_0 t)\right)$$