

1 Let  $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$   $y = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$   $\vec{v} = ax + by$

$\vec{v} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & | & 9 \\ 2 & 4 & | & 6 \end{bmatrix}$

$R_1 \rightarrow -4R_1 + R_2$

$= \begin{bmatrix} -2 & 0 & | & -30 \\ 2 & 4 & | & 6 \end{bmatrix}$

$R_1 \rightarrow \frac{R_1}{-2}$

$R_2 \rightarrow R_1 + R_2$

$= \begin{bmatrix} 1 & 0 & | & 15 \\ 0 & 4 & | & -24 \end{bmatrix}$

$R_2 \rightarrow \frac{R_2}{4}$

$= \begin{bmatrix} 1 & 0 & | & 15 \\ 0 & 1 & | & -6 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$(a, b) = (15, -6)$

$\boxed{v = 15x - 6y}$   $(15 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 6 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix})$

Subject: \_\_\_\_\_

Date: \_\_\_\_\_

2  $X_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$   $X_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$   $X_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$a \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + b \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 3 \end{array} \right]$$

$R_2 \rightarrow -2R_1 + R_2$

$$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & -3 \\ 3 & 1 & 3 \end{array} \right]$$

$R_3 \rightarrow R_3 - 3R_1$

$$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & -5 & -9 \end{array} \right]$$

$R_3 \rightarrow 5R_2 + R_3$

$$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 12 \end{array} \right]$$

$R_3 \rightarrow \frac{R_3}{12}$

$$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array} \right]$$

equation 3 can't be solved system has no solution

Subject: \_\_\_\_\_

Date: \_\_\_\_\_

$$\boxed{3} \quad (*) \quad V_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \quad W = \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$C \cdot V_1 = W$$

$$C \cdot \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$C \cdot \begin{bmatrix} -2 \\ 3 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad (C = 4)$$

$$\boxed{W = 4 \vec{V}_1}$$

$$(*) \quad V_1 = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} \quad V_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \quad W = \begin{bmatrix} 4 \\ -6 \\ 10 \end{bmatrix}$$

$$W = a \vec{V}_1 + b \vec{V}_2$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left[ \begin{array}{cc|c} 2 & 0 & 4 \\ 0 & 2 & -6 \\ 5 & 0 & 10 \end{array} \right]$$

$$R_3 \rightarrow -3R + R_3 \quad R_1 \rightarrow \frac{R_1}{2} \quad R_2 \rightarrow \frac{R_2}{2}$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ -3 & 0 & -2 \end{array} \right]$$

$$R_3 \rightarrow R_1 + R_3$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array} \right]$$

Good!

$$W = 2 \vec{V}_1 - 3 \vec{V}_2$$



Subject: \_\_\_\_\_

Date: \_\_\_\_\_

4

1	2	3	4	5	6
c	d	b	f	e	a

$$(1) v_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \rightarrow 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow 3X = Y$$

$$(6) 2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} \rightarrow X = \frac{1}{2} X$$

5 Let  $S = \{v_1, v_2, v_3, v_4, v_5\}$

Put Vectors in a form of a matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 5 & 1 & 7 \\ 2 & 1 & -1 & 4 & 0 \\ -1 & 1 & 5 & -1 & 2 \end{bmatrix}$$

$R_2 \rightarrow -2R_1 + R_2$   
 $R_3 \rightarrow -2R_1 + R_3$   
 $R_4 \rightarrow R_1 + R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & -1 & -3 & 2 & -4 \\ 0 & 2 & 6 & 0 & 4 \end{bmatrix}$$

$$R_3 \rightarrow R_1 + R_3$$

$$R_4 \rightarrow 2R_3 + R_4$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & -1 & 3 \\ 0 & 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 4 & -4 \end{bmatrix}$$

$R_3 \rightarrow \frac{-1}{2} R_3$   
 $R_4 \rightarrow \frac{-1}{4} R_4$

Subject: \_\_\_\_\_

Date: \_\_\_\_\_

$$\begin{bmatrix} \textcircled{1} & 1 & 1 & 1 & 2 \\ 0 & \textcircled{1} & 3 & -1 & 3 \\ 0 & 0 & 0 & \textcircled{1} & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

↑      ↑                      ↑

dimension = 3

basis is :  $\{V_1, V_2, V_4\}$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \\ 1 \end{bmatrix} \right\}$$