Amr Eid mohamed hanafy Abd ElGawad
group:5

Subject: Date:

$\boxed{1}$ Find The determinant of matrix M

$$M = \begin{bmatrix} 17 & -11 \\ 6 & -3 \end{bmatrix} \quad |M| = 17 * -3 - (-11 * 6)$$

$$= -51 + 66 = 15$$

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 4 & -5 \end{bmatrix} = 1*(3*-5-(4*1)) - (1*((2*-5)-3)$$

$$+ 2*(2*4 - 3*3)$$

$$= -15 - 4 + 13 - 2 = -8$$

$\boxed{2}$ Find The inverse matrix A^{-1} To The matrix A

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$$

$$|A| = -3 * 3 - (-2 \times 3) = -9 + 6 = -3$$

$$A_{(minors)} = \begin{bmatrix} 3 & 3 \\ -2 & -3 \end{bmatrix} \quad \text{cofactors}(A) = C = \begin{array}{c} \begin{array}{cc} + & - \end{array} \\ \begin{bmatrix} 3 & -3 \\ 2 & -3 \end{bmatrix} \end{array} \begin{array}{c} \\ - \quad + \end{array}$$

$$A_{adj} = C^T = \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj(A) = \frac{1}{-3} \cdot \begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & \frac{-2}{3} \\ 1 & 1 \end{bmatrix}$$

Subject: Date:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 1*(1-1) - 0(0-1) + 1(0-1)$$

$$|A| = -1$$

$$A_{minors} = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\text{Cofactors } A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A_{adj} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Subject: ______ Date: ______

$\boxed{3}$ Find The rank of matrix M

* $M = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 2 & 4 & 3 & 2 \end{bmatrix} \longrightarrow R_1 \to \frac{R_1}{3}$

$= \begin{bmatrix} 1 & \frac{1}{3} & 0 & \frac{-1}{3} \\ 2 & 4 & 3 & 2 \end{bmatrix} \to R_2 \to -2R_1 + R_2$

$\begin{bmatrix} 1 & \frac{1}{3} & 0 & \frac{-1}{3} \\ 0 & \frac{10}{3} & 3 & \frac{8}{3} \end{bmatrix} \quad R_2 \to R_2 * \frac{3}{10}$

$\begin{bmatrix} 1 & \frac{1}{3} & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{9}{10} & \frac{8}{10} \end{bmatrix} \quad \text{Rank} = \underline{\underline{2}}$

* $M = \begin{bmatrix} 5 & 2 & 3 \\ 7 & 2 & 2 \\ 9 & -1 & 1 \end{bmatrix} \quad R_1 \to \frac{R_1}{5}$

$= \begin{bmatrix} 1 & \frac{2}{5} & \frac{3}{5} \\ 7 & 2 & 2 \\ 9 & -1 & 1 \end{bmatrix} \quad R_2 \to -7R_1 + R_2$

$= \begin{bmatrix} 1 & \frac{2}{5} & \frac{3}{5} \\ 0 & \frac{-8}{10} & \frac{-11}{5} \\ 9 & -1 & 1 \end{bmatrix} \quad \begin{matrix} R_3 \to -9R_1 + R_3 \\ R_2 \to \frac{-10}{8} R_2 \end{matrix}$

Subject: Date:

$$= \begin{bmatrix} 1 & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{11}{4} \\ 0 & \frac{-23}{5} & \frac{-22}{5} \end{bmatrix} \rightarrow R_3 \rightarrow \frac{-5}{23} * R_3$$

$$= \begin{bmatrix} 1 & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{11}{4} \\ 0 & 1 & \frac{22}{23} \end{bmatrix} \quad R_3 \rightarrow R_3 - R_2$$

$$\begin{bmatrix} 1 & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{11}{4} \\ 0 & 0 & -\frac{165}{92} \end{bmatrix} \quad R_3 \rightarrow \frac{-92}{165} * R_3$$

$$\begin{bmatrix} 1 & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & \frac{11}{4} \\ 0 & 0 & 1 \end{bmatrix} \quad \text{Rank} = 3$$

Subject: Date:

$\boxed{4}$ Find The solution set of The following system of Linear equations

(a) $x_1 + 4x_2 + 3x_3 - x_4 = 5$

$x_1 - x_2 + x_3 + 2x_4 = 6$

$4x_1 + x_2 + 6x_3 + 5x_4 = 9$

$$\begin{bmatrix} 1 & 4 & 3 & -1 & 5 \\ 1 & -1 & 1 & 2 & 6 \\ 4 & 1 & 6 & 5 & 9 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$R_2 \longrightarrow R_1 - R_2$

$R_3 \rightarrow 4R_1 - R_2$

$$\begin{bmatrix} 1 & 4 & 3 & -1 & 5 \\ 0 & 5 & 2 & -3 & -1 \\ 0 & 15 & 6 & -9 & 11 \end{bmatrix} \quad R_2 \rightarrow \frac{R_2}{5}$$

$$\begin{bmatrix} 1 & 4 & 3 & -1 & 5 \\ 0 & 1 & \frac{2}{5} & \frac{-3}{5} & \frac{-1}{5} \\ 0 & 15 & 6 & -9 & 11 \end{bmatrix}$$

Subject: Date:

$R_1 \to -4R_2 + R_1$

$R_3 \to -15R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & \frac{7}{5} & \frac{7}{5} & \frac{29}{5} \\ 0 & 1 & \frac{2}{5} & \frac{-3}{5} & \frac{-1}{5} \\ 0 & 0 & 0 & 0 & 14 \end{bmatrix} \Leftarrow$$

System inconsistent

(b) $x_1 - 2x_2 + x_3 - x_4 = 3$

$2x_1 - 4x_2 + x_3 + x_4 = 2$

$x_1 - 2x_2 - 2x_3 + 3x_4 = 1$

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 3 \\ 2 & -4 & 1 & 1 & 2 \\ 1 & -2 & -2 & 3 & 1 \end{bmatrix}$$

$R_2 \to -2R_1 + R_2$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 3 \\ 0 & 0 & -1 & 3 & -4 \\ 0 & 0 & -3 & 4 & -2 \end{bmatrix}$$

Subject: Date:

$R_2 \rightarrow -R_2$, $R_1 \rightarrow R_2 + R_1$

$R_3 \rightarrow -3R_2 + R_3$

$$\begin{bmatrix} 1 & -2 & 0 & 2 & -1 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & -5 & 10 \end{bmatrix}$$

$R_3 \rightarrow \frac{R_3}{-5}$, $R_2 \rightarrow \frac{2R_3}{5} + R_2$

$R_1 = \frac{2R_3}{5} + R_1$

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$X_4 = -2$, $X_3 = -2$

$X_1 - 2X_2 = 3$

$X_1 = 2X_2 + 3$

$X_2 \rightarrow$ Free

Subject: Date:

(c) $x_1 + 2x_2 + 3x_3 = 1$

$2x_1 - x_2 + x_3 = 2$

$3x_1 + x_2 + x_3 = 4$

$5x_2 + 2x_3 = 1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & -1 & 1 & 2 \\ 3 & 1 & 1 & 4 \\ 0 & 5 & 2 & 1 \end{bmatrix}$$

$R_2 \to -2R_1 + R_2$

$R_3 \to -3R_1 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -5 & -5 & 0 \\ 0 & -5 & -8 & 1 \\ 0 & 5 & 2 & 1 \end{bmatrix} \quad R_2 \to \frac{R_2}{-5}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -8 & 1 \\ 0 & 5 & 2 & 1 \end{bmatrix}$$

الأقصى

Subject: Date:

$R_1 \to -2R_2 + R_1$, $R_3 \to 5R_2 + R_3$

$R_4 \to -5R_2 + R_4$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & -3 & 1 \end{bmatrix}$$

$R_3 \to \frac{-R_3}{3}$

$R_4 \to \frac{-R_4}{3}$

$R_2 \to -R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{-1}{3} \\ 0 & 0 & 1 & \frac{-1}{3} \end{bmatrix}$$

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{-1}{3} \\ 0 & 0 & 1 & \frac{-1}{3} \end{bmatrix}$$

$\boxed{X_1 = 1\frac{1}{3}}$

$\boxed{X_2 = \frac{1}{3}}$

$\boxed{X_3 = \frac{-1}{3}}$

Subject: Date:

$\boxed{5}$

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$$

$(2-\lambda)(2-\lambda) - 1 = 0$

$\lambda^2 - 4\lambda + 4 - 1 = 0$

$\lambda_2 - 4\lambda + 3 = 0$

$(\lambda - 1)(\lambda - 3) = 0$ $\boxed{\lambda_1 = 1 \quad \lambda_2 = 3}$

eign vector for $\lambda = 1$ $(A - 1I) \cdot V = 0$

$$(A - I) \cdot V = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Subject:

Date:

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad R_2 \rightarrow R_1 + R_2$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \quad \boxed{V_1 - V_2 = 0}$$

$$\boxed{\begin{matrix} V_1 = V_2 \\ \lambda = 1 \end{matrix}} \qquad \boxed{V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

For $\lambda = 3$

$(A - 3I) \cdot V = 0$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \cdot V = 0 \qquad R_2 \rightarrow -R_1 + R_2$$

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$-V_1 - V_2 = 0 \qquad \boxed{V_1 = -V_2}$

$$\boxed{\lambda_2 = 3 , \quad V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

Subject:

Date:

Trace (A) = 2+2 = 4   ($\lambda_1 = 1$ ($\lambda_2 = 3$

Trace = $\lambda_1 + \lambda_2$

deT(A) = 2(2) - (-1 * -1) = 3 = $\lambda_1 \cdot \lambda_2$

Scanned with CamScanner

Subject:

Date:

$$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$$\begin{bmatrix} 5-\lambda & -4 \\ -4 & 5-\lambda \end{bmatrix} = (A - \lambda I)$$

$(5-\lambda)\cdot(5-\lambda) - 16 = 0$

$\lambda^2 - 10\lambda + 9 = 0$

$(\lambda - 1)(\lambda - 9) = 0$

$$\boxed{\lambda_1 = 1 \quad , \lambda_2 = 9}$$

eign vectors for $\lambda_1 = 1$

$(A - \lambda I) \cdot V = 0$

$$\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \cdot V = 0$$

$$\begin{bmatrix} 4 & -4 & 0 \\ -4 & 4 & 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

الأقصى

Subject: Date:

$$\begin{bmatrix} 4 & -4 & 0 \\ -4 & 4 & 0 \end{bmatrix} \qquad R_2 = R_1 + R_2 \qquad R_1 = \frac{R_1}{4}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R_2 \rightarrow R_2 + 4R_1$$

$V_1 - V_2 = 0 \qquad V_1 = V_2$

$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ when $\lambda = 1$

eigh vector when $\lambda_2 = 9$

$$\begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} = 4\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

eign vector same as $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

$V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Trace $= 5 + 5 = 10$

$\lambda_1 + \lambda_2 = 1 + 9 = 10 \qquad$ Trace $= \lambda_1 + \lambda_2$

$\det(A^2) = 5(-4) - (-4 * 4) = 9$

$\det(A^2) = \lambda_1 * \lambda_2 = 9$

Subject: Date:

$$(A+4I) = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(A-\lambda I) = 0$$

$$\det\left(\begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right) = 0$$

$$(6-\lambda)\cdot(6-\lambda) - (-1\cdot -1) = 0$$

$$\lambda^2 - 12\lambda + 36 - 1 = 0$$

$$\lambda^2 - 12\lambda + 35 = 0$$

$$(\lambda_1 - 5)(\lambda_2 - 7) = 0$$

$$\boxed{\lambda_1 = 5 \quad , \quad \lambda_2 = 7}$$

For $\lambda_1 = 5$ eign vector

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot V = 0$$

الأقصى

Subject: Date:

$R_2 \to R_1 + R_2$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \cdot v = \qquad v_1 = v_2$$

$$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

for $\lambda_2 = 7$ eignVector

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \cdot V = 0 \qquad R_2 \to -R_1 + R_2$$

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} \cdot V_2 \qquad V_1 = -V_2$$

$$V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$