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**FACTOR INVESTING WITH RISK PARITY PORTFOLIOS**

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Abstract <p>This thesis investigates factor investing and risk parity methods by constructing seven risk parity portfolios. We find that both single-factor portfolios and multi-factor risk parity portfolios outperform the market and our benchmarks. The methods produce higher absolute returns and better risk-adjusted returns with lower volatilities and drawdowns. Therefore, the presented methodologies may benefit investors by providing more efficient portfolios and greater risk management. We use long-only factor tilt indices to construct the portfolios as this is one of the simplest ways to implement such strategies for both institutional and individual investors. Thus, our methods are both practical and realistic.</p> <p>In addition, we show that there are significant diversification benefits in combining factors into multi-factor portfolios. Mainly, volatilities and drawdowns are significantly decreased when six factor tilt indices are combined using the methods described in this thesis.</p> <p>Furthermore, traditional diversification methods can lead to skewed portfolio risk profiles where equity risk is significantly overweighted. Risk-based diversification methods offer an alternative approach where diversification is based on the risk contribution of each asset. This results in more balanced portfolio risk profiles. Therefore, we also analyze the compositions of the constructed portfolios and find that the majority overweight low volatility assets.</p>			
Keywords <b>Factor investing, risk parity, risk budgeting, portfolio management, diversification, asset allocation</b>			
Additional information			

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## 1 INTRODUCTION

Risk factor investing is an investing approach where capital allocation is based on risk factors instead of asset classes. Since various factor risks are believed to underlie all assets, exposure to them cannot be diversified away. Therefore, the goal of factor investing is to allocate capital more effectively based on the individual needs and preferences of the investor. Furthermore, factor investing attempts to capture risk premiums resulting from various market anomalies that have been observed. (Ang 2014)

The first risk factor was the capital asset pricing model's (CAPM) market risk factor (Sharpe 1964, Lintner 1965 & Mossin 1966). Subsequently, the CAPM has been expanded to include additional factors such as size and value (Fama & French 1996), momentum (Carhart 1997), and profitability and investment (Fama & French 2016). Other risk factors that have been found to explain returns are volatility (Ang, Hodrick, Xing & Zhang 2006) and liquidity (Pástor & Stambaugh 2003).

Risk parity is a diversification method which approaches diversification from the point of view of risk contribution of each asset instead of diversification across traditional asset classes. Traditional diversification across various asset classes leads to skewed risk contributions in the final portfolio. Specifically, equities typically contribute the majority of the total portfolio risk. The risk parity methods underweight high risk assets and overweight low risk assets. This allows the risk parity methods to capture the low risk anomaly.

Much of the literature on factor investing is focused on analyzing the characteristics of individual factors and comparing the performance of factor portfolios to asset portfolios. This thesis also analyzes the characteristics of the individual MSCI factor tilt indices, but we complement the literature by analyzing the performance of multi-factor portfolios constructed using risk parity methods in order to see whether combining factors provides superior performance to individual factors. We find that factor tilt indices outperform the market and that multi-factor risk parity portfolios provide better risk-adjusted returns than single-factor portfolios. Our results indicate that investors can successfully implement factor investing using simple long-only

factor tilt indices and risk parity methods, and that these strategies outperform the market and our benchmarks.

The aim of this thesis is to review the literature on various risk factors and risk parity, and to analyze the performance of multi-factor risk portfolios constructed using risk parity methods. The thesis is organized as follows. Section 2 discusses the theoretical background of risk factors and portfolio construction using risk parity. Section 3 describes the data and methodology. Section 4 discusses the empirical findings of the portfolio analysis. Section 5 concludes.



## 2 THEORETICAL BACKGROUND

### 2.1 Risk Factors as Underlying Sources of Risk

#### 2.1.1 Risk Factors in Asset Pricing

The first model to include a risk factor is the capital asset pricing model (CAPM) published by Sharpe (1964), Lintner (1965), and Mossin (1966). The CAPM's risk factor is the market factor. Essentially, the market factor, often called the market beta, measures the sensitivity of individual assets to changes in the market's performance. For example, a market beta value of 2 indicates that when the overall market's excess return changes by 2%, the asset's excess return changes by 4%. This means that the asset's excess returns are more volatile than the market. In contrast, a negative beta means that the asset's returns move in the opposite direction of the market's return. The CAPM is extended by Fama and French (1996) to include two additional factors: size (SMB or small-minus-big) and value (HML or high-minus-low). Furthermore, Carhart (1997) adds a fourth factor, the momentum factor. Recently, Fama and French (2016) published a five-factor model which adds a profitability factor and an investment factor. Other important factors are: liquidity (Pástor & Stambaugh 2003), volatility (Ang, Hodrick, Xing & Zhang 2006), quality (Asness, Frazzini & Pedersen 2014), and carry (Kojien, Moskowitz, Pedersen & Vrugt 2012).

The CAPM (Sharpe 1964, Lintner 1965 & Mossin 1966) is defined as:

$$E(R_i) - R_f = \beta_i [E(R_M) - R_f] \quad (1)$$

where  $E(R_i)$  is the expected return of the asset,  $\beta_i$  is the beta coefficient of the market factor,  $E(R_M)$  is the expected return of the market portfolio,  $R_f$  is the risk-free rate, and  $E(R_M) - R_f$  is the market excess return, that is, the market factor. The expected return of the market portfolio can be proxied with a broad market index and the risk-free rate is typically represented by short-term government bonds or bills.

The CAPM (Sharpe 1964, Lintner 1965 & Mossin 1966) is estimated with the following time-series regression:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{M,t} - R_{f,t}) + \varepsilon_t \quad (2)$$

where  $\alpha_i$  is the intercept term,  $\beta_i$  is the beta coefficient or slope, and  $\varepsilon_t$  is the error term. The subscript  $t$  denotes time. The intercept term, or alpha, is the part of the return which is unexplained by the model. A positive (negative) alpha means that the returns are higher (lower) than estimated by the model. The error term contains all omitted factors and measurement errors which affect the return.

The two most common methods to test new factors, or risk premiums, are by using a two-step portfolio sorting approach and Fama-MacBeth (1973) regressions. The explanatory power of factors in asset pricing is investigated by forming long-short portfolios and applying regressions. For example, a value factor mimicking portfolio can be constructed by buying stocks with a high book-to-market<sup>1</sup> ratio and selling stocks with a low book-to-market ratio. The stocks are sorted into quintiles or deciles based on the characteristic which is being investigated, and portfolios are formed from each quintile or decile. Thus, the performance of each quintile can be compared, and various tests can be applied to test whether there are any performance differences. For example, tests for return differences and different Sharpe ratios are common. Additionally, a spread portfolio (the top portfolio less the bottom portfolio) can be formed and analyzed. Regressions are applied to the spread portfolio returns to test whether other common factors can explain the return difference. If statistically and economically significant alpha remains even after controlling for the common factors, it can be concluded that there is a factor effect. (Fama & French 1996.)

The first step in the Fama-MacBeth (1973) approach is to estimate the factor loadings (i.e. quantity of risk) with the time-series regression in (2). The second step is to use the estimated betas to estimate the risk premium (i.e. price of risk). The

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<sup>1</sup> The book-to-market (B/M) ratio is calculated as: *Book value of equity/Market value of equity*.

pricing of factors can be estimated with the following rolling cross-sectional regression at each time period:

$$E(R_i) - R_f = \beta_i' \lambda + a_i \quad (3)$$

where  $\lambda$  corresponds to the risk factor premium,  $\beta_i'$  is a vector of coefficients<sup>2</sup> estimated with the time-series regression in (3), and  $a_i$  is the residual or pricing error. Since  $\lambda$  and  $a_i$  are calculated at each point in time, the averages of the variables are used in the statistical tests. We conclude that a factor is priced if  $\lambda > 0$ . (Fama & MacBeth 1973.)

Whereas the CAPM (Sharpe 1964, Lintner 1965 & Mossin 1966) has only the market factor, Arbitrage Pricing Theory (APT) presents the first multi-factor model. However, it does not explicitly attempt to categorize and identify specific factors, but instead, simply states that they are different across markets and that they vary over time. APT's factors are not observable macroeconomic (e.g. interest rates, Inflation, GDP, growth) or fundamental factors (e.g. size, value, dividend yield, industry), but they are unobservable statistical factors which are estimated from the return data with factor analysis or principal component analysis. Thus, APT models estimate both the factors and the loadings. (Ross 1976.) Common model estimation methods include the generalized method of moments (GMM), ordinary least squares (OLS), and maximum likelihood estimations.

The APT model can be defined as:

$$x_i = a_i + \beta_i' f + \varepsilon_i \quad (4)$$

where  $x_i$  denotes returns,  $a_i$  is the pricing error,  $\beta_i'$  is a vector of factor loadings,  $f$  are the factors, and  $\varepsilon_i$  are the residuals. (Ross 1976.)

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<sup>2</sup> Note that in this model they are the right-hand side variables used for the regression.

The Fama-French Three-Factor Model (Fama & French 1996) includes three risk factors: the market factor, size, and value, which are used to explain portfolio returns. The market factor is the market excess return (market return less the risk-free rate). The size (SMB) factor is based on the historical evidence that small firms (measured by market capitalization) tend to outperform large firms. Similarly, the value (HML) factor is based on the evidence that high value firms (measured by the book-to-market ratio) tend to outperform low-value, or growth, firms. Companies with high expected profitability or investments tend to perform better than firms with low expected profitability or investments (Fama & French 2016).

The Fama-French Three-Factor Model defines the expected excess return of a portfolio  $i$  as:

$$E(R_i) - R_f = b_i[E(R_M) - R_f] + s_iE(SMB) + h_iE(HML) \quad (5)$$

where  $E(R_i) - R_f$  is the expected excess return of portfolio  $i$ ,  $E(R_M) - R_f$  is the expected market factor premium,  $E(SMB)$  is the expected size factor premium,  $E(HML)$  is the expected value factor premium, and  $b_i$ ,  $s_i$ , and  $h_i$  are the factor loadings to the factors. The loadings are estimated with the following time-series regression:

$$R_{i,t} - R_{f,t} = a_i + b_i(R_{M,t} - R_{f,t}) + s_iSMB + h_iHML + \varepsilon_t \quad (6)$$

where  $a_i$  is the intercept,  $b_i$ ,  $s_i$ , and  $h_i$  are the factor coefficients or slopes,  $(R_{M,t} - R_{f,t})$ ,  $SMB$ , and  $HML$  are the factors, and  $\varepsilon_t$  is the error term.

Previous research has identified and categorized several risk factors. Risk factors can be categorized into macroeconomic (or fundamental) factors, investment factors, and dynamic (or style) factors. Examples of macroeconomic risk factors are economic growth, interest rates, inflation, and political risk (Chen, Roll & Ross 1986). These factors are not directly tradeable, but all assets have exposure to them in varying degrees. The investment factors are the equity factor and the bond factor. Dynamic (or style) factors include size, value, momentum, volatility, liquidity, and credit risk,

for example. Investment and dynamic factors are tradeable. (Ang 2014.) The next section summarizes the empirical evidence of the most common factors and presents the economic explanations for their existence.

### 2.1.2 Factor Theory and Empirical Evidence

Risk factor theory attempts to identify various risk factors which affect the performance of asset classes. The goal is to better understand what affects the performance of stocks, and thus, improve asset management. The theory states that the return on assets is due to the exposure to these underlying risk factors. More specifically, risk premiums<sup>3</sup> are due to bearing downside risk when returns are low or negative, referred to as bad times. (Ang, Chen & Xing 2006.)

Fama and French (1995) show that the HML factor proxies for relative distress. Poorly performing companies tend to have high book-to-market ratios, and therefore, their loadings to the HML factor are positive. Conversely, companies with strong financial performance tend to have low book-to-market ratios and negative loadings to the HML factor. Thus, the HML factor compensates for the additional risk of poorly performing companies. Similarly, the SMB factor proxies for the relatively higher riskiness of small companies relative to large companies. Small companies have positive loadings and large companies have negative loadings to the SMB factor. (Fama & French 1995.)

Jegadeesh and Titman (1993) find that the performance of stocks, whether positive or negative, tends to persist over several months. This characteristic is often called momentum. Plausible explanations for the momentum effect include both overreactions to long-term expectations of firm performance and underreactions to expectations of short-term performance information. Carhart (1997) finds that a large part of mutual funds' abnormal returns can be explained by a momentum factor. The momentum factor is constructed based on the past performance of stocks. Typically, the stocks past 12-month performance is analyzed and then the stocks are divided

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<sup>3</sup> Risk premium refers to the expected future return difference, whereas excess return refers to the observed historical return difference (Arnott & Bernstein 2002).

into winners and losers (WML or winners-minus-losers). Value and momentum risk premiums are robust across asset classes, markets, time periods, and they are negatively correlated with each other. In addition, value is positively correlated with liquidity and momentum is negatively correlated with liquidity. (Asness, Moskowitz & Pedersen 2013.)

Pástor and Stambaugh (2003) find that stocks with a high loading to the liquidity factor have higher average returns compared to low-liquidity stocks. Liquidity refers to the ease of selling assets quickly, at a low cost, and without changing the asset price significantly. Theory dictates that risk-averse investors will demand a premium for stocks which face the risk of low prices and large costs when liquidity decreases. Similarly, if two assets have the same expected returns, the investor will prefer the one with higher liquidity (i.e. lower liquidity risk). Additionally, they find that the liquidity factor explains half of the momentum factor's alpha, and that small stocks are less liquid and have high loadings to the liquidity factor.

Ang et al. (2006) find that stocks with high idiosyncratic volatilities tend to underperform low volatility stocks. Their reasoning is that high volatility stocks may also have exposure to aggregate volatility risk, and therefore, have lower returns. However, they find that aggregate volatility does not completely explain the low returns of high volatility stocks.

Asness, Frazzini and Pedersen (2014) find that high quality stocks, defined by being safe, profitable, growing, well-managed, and high-yielding, tend to outperform low quality (i.e. risky, unprofitable, shrinking, and low-yielding) stocks. In addition, they find negative correlation between the size and quality factors, and this is due to smaller stocks being riskier in general. Koijen, Moskowitz, Pedersen and Vrugt (2013) find that high-yielding assets outperform low-yielding assets. The carry factor is robust across asset classes and global.

The returns of fixed income securities (i.e. bonds) can be explained by exposure to three factors: level, slope, and curvature. The level factor is the most important as it explains over 90% of the movement in term structure of interest rates. The level factor is comparable to the equity market factor, or beta, because it represents the

tendency of all bond yields (or returns) to move up or down together. Exposure to the level factor is called duration, which represents the sensitivity to level changes in interest rates. The slope factor represents the changes in the long-end of the yield curve compared to changes in the short-end, and the exposure to it is the term spread factor. The curvature factor represents the movement of the middle of the yield curve independently from the other two factors. These three factors are systematic to all bonds, and therefore, bonds have very little idiosyncratic risk. The risk premiums for long-term bonds are countercyclical because they are high during recessions since agents do not want exposure to risk. Conversely, short-term bonds are procyclical. The opposite is true in times of economic growth. (Ang 2014.)

Risk factors have been shown to be able to explain asset returns (Fama & French 1996, Carhart 1997, Ang et al. 2006, Pástor & Stambaugh 2003). Therefore, the risk factors represent risk premia that investors demand for exposure to these risks. Additionally, these risk premia are considered as anomalies of the CAPM because they cannot be explained by differences in market betas. Risk factor investing seeks to generate returns from exposure to these risk premia directly. Therefore, factor investing seeks to exploit these documented anomalies. These strategies are sometimes referred to as smart beta or alternative beta strategies. Risk factors are investable assets and factor index exchange-traded funds are offered by several sponsors. Furthermore, empirical research has found that these factors are robust and persistent, and factor investing assumes that these characteristics remain in the future.

Additionally, factors can be used to form benchmarks against which the performance of asset managers can be compared to. For example, in order for an active manager to have superior investing skills and knowledge, the manager must be able to produce superior returns compared to the factor benchmarks. Therefore, the emergence of factor benchmarks increases the returns which are expected from active asset management. (Fama & French 1996.) Empirical research has found that active managers are aware of the market anomalies and that they adjust their portfolios based on the most common factors. For example, Fama and French (2010) show that mutual funds do not outperform the Fama-French three-factor model after fees. However, hedge funds are able to generate alpha after controlling for the Fung

Hsieh (2004) factors (Joenväärä, Kosowski & Tolonen 2016). The next section presents how the literature on risk factors can be utilized in the asset management industry to improve performance and manage risk. In addition, we introduce some practical considerations that should be taken into account when considering factor investing.

### 2.1.3 Factor Investing

Factor theory states that there are underlying risk factors that affect all assets in varying degrees. These factor risks cannot be diversified away, and therefore, investors require positive risk premiums for the exposure to these risk factors and particularly the low, or negative returns, that occur during bad times. Factor investing recognizes the risk factors which affect all assets, and determines the optimal allocation of capital based on the individual requirements of the investor. Factor investing strategies are systematic and rules-based methods to harvest the risk premiums.

Diversification is a risk management approach which attempts to reduce an investor's exposure to idiosyncratic risk. Traditionally, this is done by allocating an investor's capital between various asset classes. A typical approach is to divide an investor's capital between equity and fixed income investments. Furthermore, an investor can diversify within asset classes. For example, an investor can invest in the stocks of different companies and in various bonds. Factor investing takes a different approach to diversification because assets have exposure to the same risk factors. Because these underlying risk factors cannot be diversified away, the goal is to find the optimal level of exposure to each factor. Therefore, factor investing is risk-based investing. Conventionally, bad times have been defined as periods of low or negative returns on assets. With risk factors, however, the story might not be as simple. Therefore, more research is needed to define bad times for each risk factor. This will better our understanding on the underlying causal relationship between changes in risk factor conditions and changes in asset performance. (Ang 2014.)

The correlations between assets are also an important consideration for diversification. When assets are not perfectly correlated (i.e. under 1), combining



them into portfolios improves the risk-return relationship. Koedjik, Slager, and Stork (2016) find that the correlations between different factor portfolios are under 1. Therefore, they find significant diversification benefits by combining the factor portfolios into one multi-factor portfolio. In addition, they find that the factor portfolio returns remain stable over time and across U.S., European, and other markets. Finally, the results are not dependent on a single factor.

Another consideration is an investor's time horizon. Factor investing can be done for both tactical (i.e. short-term) and strategic (i.e. long-term) asset allocation purposes. Tactical factor bets can be made based on a manager's beliefs that certain factors will outperform over a period, and more aggressive tilts to these factors can be made. To contrast, more balanced factor exposures can be allocated if the manager does not want to emphasize any specific factor. (Ang 2014.)

Even though it is recognized that factor investing outperforms in the long-term, some factors may underperform during certain periods. Additionally, the performance of factors can vary significantly during times of financial distress. However, the low, or even negative, correlation between factors is the key characteristic that results in significant diversification benefits over long investment horizons. Another source of concern is the fact that the growing interest in factor investing may cause mispricing of the factors. Due to these reasons, investors with long horizons, such as institutions and pension funds, may be able to implement factor investing most efficiently. (Ben-Ami 2016.) Furthermore, it is important to recognize how a specific factor is valued. That is, an expensive factor would not be expected to provide high returns. (Moore 2016.)

In practice, there are at least five steps that an investor should consider when implementing factor investing:

- 1) Identify the factors that the current portfolio has exposure to.
- 2) Identify the goals and liabilities of the current portfolio.
- 3) Decide which factors should be included into the portfolio to meet the needs of the investor.

- 4) Decide how to access these factors: passive indices (i.e. long-only tilts) or more active solutions (i.e. long/short approaches).
- 5) Decide how to construct (i.e. asset or factor allocation and rebalancing) and manage (i.e. in-house or outsource) the portfolio.

Factor investing can provide another benefit over other types of investing in the form of enabling the investor to understand and control their risk exposure more transparently. This may enhance the (active) management of large portfolios, especially pension funds, who are subject to strict regulation and constraints. For example, a pension fund concerned with meeting their liabilities can focus on a tilt to low-volatility, and more aggressive funds can allocate to value and momentum. However, even though factor investing may be presented as a more objective and systematic investment strategy than stock picking, there is still room for significant subjectivity in how the factors are defined, constructed, and how the strategy is implemented. (Moreolo 2016.) In addition, factor investing can help to mitigate agency issues between an asset manager and an investor, because the investor has greater control over which factors he wants exposure to, and to what extent.

The theoretical work and empirical research on risk factors started with the CAPM's market factor, but at the time, it was not possible for an investor to invest in the factor. Later, market index funds allowed an investor to get exposure to the market factor and the equity premium. In recent years, several low-cost indices have been developed which allow an investor to capture the value, size, momentum, liquidity, and volatility factors, for example. Empirical research has found that factor investing can provide advantages when compared to more conventional methods and this conclusion remains stable over time and across markets. The next section introduces an alternative way to construct portfolios through risk contributions. We also illustrate how this method results in true risk diversification.

## 2.2 Risk-Based Portfolio Construction

### 2.2.1 The Risk Budgeting Approach

The Markowitz (1952) mean-variance optimization model is the standard model to minimize variance for a given level of return. This is done by combining assets which are not perfectly positively correlated with each other into a portfolio. Then, an investor's capital is allocated between the resulting mean-variance efficient portfolio and a risk-free asset, such as treasury bills, based on their level of risk aversion. The mean-variance model depends on expectations of returns and standard deviations, and therefore, is sensitive to input errors. The 2008 subprime crisis demonstrated that allocating capital between assets is not truly effective diversification as correlations tend to increase in crisis times. Therefore, the asset management industry has become increasingly focused on risk management instead of returns after the crisis (Bruder & Roncalli 2012).

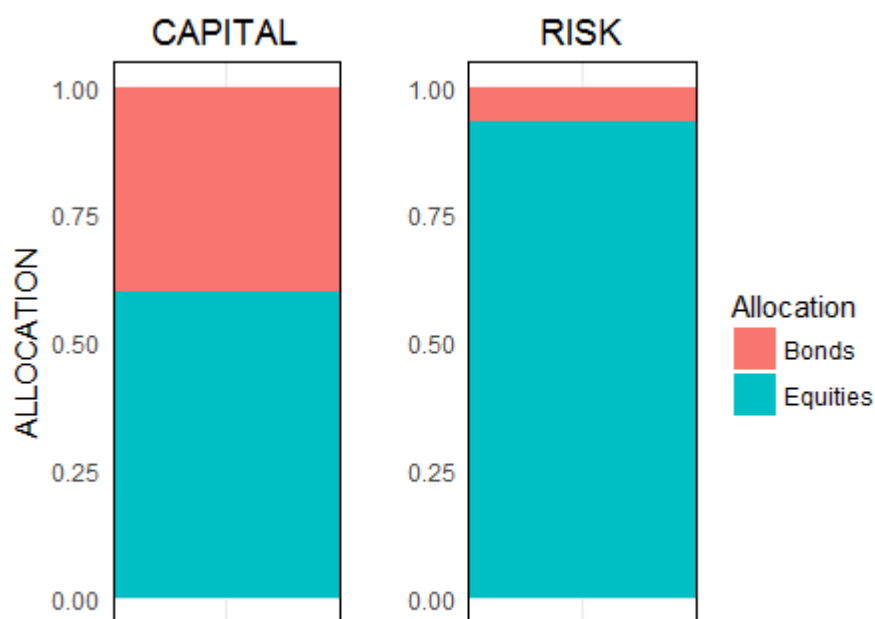
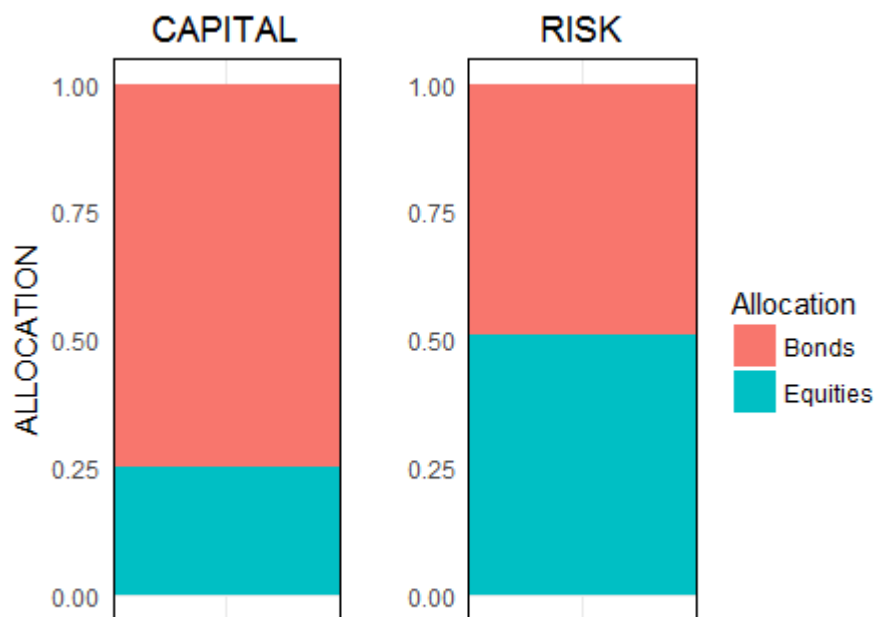


Figure 1. Capital allocation and risk allocation of a 60/40 portfolio (adapted from Dudley 2011).

Traditional diversification approaches such as the classic 60/40 portfolio of equities and bonds (60% in equities and 40% in bonds) can lead to skewed portfolio risk profiles. In fact, in a 60/40 portfolio, equities contribute over 93% of the total portfolio risk. Furthermore, the correlation between the 60/40 portfolio and equities is 0.98 which means that the portfolio is still very sensitive to the movements of the overall equity market. Figure 1 illustrates this difference between capital allocation and risk allocation. These same issues are true for other capital allocation models such as the endowment model which includes alternative asset classes such as private equity and real estate. In order to achieve roughly equal risk allocation (51% for equities and 49% for bonds), that is, true risk diversification, the asset weights should be approximately 25/75 (25% in equities and 75% in bonds) as illustrated in figure 2. The correlation between the 25/75 portfolio and equities is 0.76. (Dudley 2011.)



**Figure 2. Capital allocation and risk allocation of a 25/75 portfolio (adapted from Dudley 2011).**

The 2008 financial crisis illustrated this weakness of the traditional diversification approach. Many professionals believe that the solution to this problem is another diversification approach: risk budgeting. Risk budgeting is an allocation method where the weight allocated to a particular asset is determined by its risk contribution to the total risk of the portfolio instead of by capital allocation. Whereas risk budgeting is a general term referring to many allocation methods which are based on

the risk contributions of assets, risk parity methods are specific cases where different ways to allocate risk are applied. (Bruder & Roncalli 2012.)

The goal of the risk parity techniques is to maintain total portfolio volatility constant through changes in economic conditions. In practice, this results in portfolios with higher concentrations in low volatility assets such as bonds compared to traditional asset class diversification techniques. Consequently, this tilts the portfolio's risk towards fixed income risks, such as interest rate risk. Therefore, the returns of risk parity portfolios can be increased by leveraging the low volatility assets. (Mariathasan 2011). Gosh (2011) concludes that risk parity produces superior diversification than traditional methods because it also diversifies the risk of incorrect timing of the investments. It is important to note that risk parity is not a risk-minimization approach such as the mean-variance portfolio (Markowitz 1952), and therefore it is not located on the efficient frontier. However, risk parity portfolios can be mean-variance optimal if the portfolio components have equal Sharpe ratios and are uncorrelated (Qian 2005).

It is important to note that one of the major assumptions of risk parity portfolios constructed from equities and bonds is that the correlation between the two is negative. Historically, the correlation between equities and bonds has been negative during only two periods: 1955–1965 and 2000–2010. A change to a positive correlation would mean that the total risk of the portfolio increases, and therefore, the leverage in the bonds would have to be reduced, making the portfolios less efficient. However, there is no consensus on whether the recent negative correlation is the new norm or an abnormality. (Turner 2014.) To contrast, portfolios implementing various style factors; such as value, momentum, or carry, have close to zero correlation with real bond yields in the long-term (Ilmanen & Palazzolo 2014).

Bruder and Roncalli (2012) state that the risk budgeting approach does not require any return forecasts and thus can be considered a more robust approach. Allocation is based on the risk contribution that is budgeted for each of a portfolio's assets. They show that the risk budgeting portfolio exists and that it is unique. Furthermore, its volatility is between the minimum variance portfolio and the equally-weighted portfolio.

Maillard, Roncalli and Teiletche (2010) show that the underlying component in risk budgeting is the risk contribution of a particular asset to the total risk of the portfolio. The risk contribution ( $RC$ ) can be calculated by multiplying the weight allocated to the asset by its marginal risk contribution ( $MRC$ ).

The marginal risk contribution of asset  $i$  can be calculated as:

$$MRC_i = \frac{\Sigma w}{\sqrt{w' \Sigma w}} \quad (7)$$

where  $\Sigma$  is the variance-covariance matrix of the asset returns and  $w$  is the  $(N \times 1)$  weight vector.

The risk contribution of asset  $i$  can be calculated as:

$$RC_i = w_i \times MRC_i \quad (8)$$

where  $w_i$  is the weight of asset  $i$ . The next section discusses why risk parity methods are appropriate for constructing factor portfolios.

### 2.2.2 Risk Parity Portfolios with Risk Factors

Roncalli and Weisang (2016) analyze risk factor contributions in portfolio diversification. They show that a connection exists between the risk contributions of economic risk factors and assets for total risk. They approach risk factor diversification as an optimization problem for portfolio construction. Since risk factors are the underlying sources of risk, seeking maximum diversification of them leads to portfolios which are close to an equally-weighted portfolio. Thus, they named the method risk factor parity (Qian [2005] is credited with coining the term risk parity). See Roncalli and Weisang (2012) and Bruder and Roncalli (2012) for detailed derivation of the theoretical frameworks for risk parity and risk parity for factor portfolios.

Roncalli and Weisang (2016) find that their risk parity portfolios using the principal component analysis (PCA) risk factors, equal risk contribution (ERC), and factor-weighted portfolios, have smaller risk in terms of volatility, maximum drawdown, and kurtosis than their asset-weighted portfolio. Their portfolios are rebalanced monthly, and they find that the risk parity portfolios have higher turnover than the asset-weighted portfolio. Of the two risk parity portfolios, the factor-weighted portfolio performs better, provides the highest diversification, and has the highest turnover (43% for the factor-weighted and 7% for the ERC). They note that the connection between economic risk factors and asset performance varies across time, and thus, the results are highly dependent on the study period. Finally, they add that the risk parity framework is suitable for factor investing by using the market risk factors, such as value, size, and momentum instead of the economic risk factors. The next section presents eight risk-based portfolio construction methods.

### 2.2.3 A Practical Diversification Approach

Mesomeris, Wang, Salvini and Avettand-Fenoel (2012) argue that the 2008 financial crisis revealed a problem in the conventional asset class diversification approach. This diversification strategy can lead to hidden concentrations of risk in particular factors. Thus, they classify the diversification failures of the crisis as "user error". Firstly, risk parity approaches may offer superior diversification and risk management because they attempt to identify the underlying sources of risk for a portfolio. Secondly, risk factors have lower correlations with each other compared to the correlations between asset classes. Thirdly, risk factor correlations are more stable across time and changes in economic conditions such as regime shifts or financial crises.

Mesomeris et al. (2012) explain how risk factor-based asset allocation could be used for strategic asset allocation over the long-run for risk (volatility) management and to increase portfolio performance. In addition, factor allocation can provide advantages in tactical asset allocation for short-term investment strategies and hedging. However, in order to benefit from these strategies, the risk-return profiles of the risk factor premiums have to be understood. Strategic and tactical risk parity policies can be used especially by institutional investors with long investment horizons, such as

pension funds. Long investment horizons mitigate the effects of incorrect market timing and adoption of the risk parity policies.

Even though risk parity refers to one approach where each of a portfolio's components contribute an equal amount to the overall risk, there exist extensions of the basic approach where the risk budgets are determined differently. Mesomeris et al. (2012) present five risk-based portfolio construction methods: Inverse Volatility (Asness, Frazzini & Pedersen 2012), Equal Risk Contribution (Maillard, Roncalli & Teiletche 2010), Alpha Risk Parity (Alvarez 2011), Maximum Diversification (Choueifaty & Coignard 2008), and Diversified Risk Parity (Meucci 2009; Lohre, Opfer & Orszag 2014). Other risk parity methods include Beta Risk Parity and Systematic risk Parity (Kahra 2016), and Inverse Variance.

Inverse Volatility (IV) weights each factor or asset in inverse proportion to its volatility. The method allocates an equal volatility budget for each asset. However, the method does not consider the correlations between the factors or assets and it underweights high volatility components. Thus, this can result in suboptimal allocation if one component has negative correlation with another component.

The Inverse Volatility portfolio asset weights are calculated as:

$$w_i^{IV} = \frac{\frac{1}{\sigma_i}}{\sum_{i=1}^n \frac{1}{\sigma_i}} \quad (9)$$

where  $w_i^{IV}$  is the weight of asset  $i$  in the IV portfolio and  $\sigma_i$  is the standard deviation of asset  $i$ . (Asness, Frazzini & Pedersen 2012.)

Inverse Variance (IV2) is similar to the inverse volatility portfolio, but it uses the asset's variance to calculate the weight.

The Inverse Variance portfolio asset weights are calculated as:



$$w_i^{IV2} = \frac{\frac{1}{\sigma_i^2}}{\sum_{i=1}^n \frac{1}{\sigma_i^2}} \quad (10)$$

where  $w_i^{IV2}$  is the weight of asset  $i$  in the IV2 portfolio and  $\sigma_i^2$  is the variance of asset  $i$ .

Equal Risk Contribution (ERC) allocates an equal risk budget for each of a portfolio's components. Thus, each component has an equal marginal contribution to risk, which is calculated by multiplying the weight of the asset, the standard deviation of the portfolio, and the correlation between the asset and the portfolio. See Maillard, Roncalli and Teiletche (2010) for the theoretical properties of the ERC portfolio. Mainly, they find that the volatility of the ERC portfolio is located between the minimum variance portfolio and equally-weighted portfolio. Thus, the ERC method considers the correlations between the assets.

The Equal Risk Contribution portfolio asset weights are calculated as:

$$w_i^{ERC} = \arg \min_w \sum_{i=1}^n \sum_{j=1}^n \left( w_i \text{cov}(r_i, r_p) - w_j \text{cov}(r_j, r_p) \right)^2 \quad (11)$$

where  $w_i^{ERC}$  is the weight of asset  $i$  in the ERC portfolio,  $w_i$  is the weight of asset  $i$ , and  $\text{cov}(r_i, r_p)$  is the covariance between asset  $i$ 's return and portfolio  $p$ 's return. (Maillard, Roncalli & Teiletche 2010.)

Alpha Risk Parity (ARP) extends the ERC technique and the risk budget is allocated in proportion to a component's forecasted alpha.

The Alpha Risk Parity portfolio asset weights are calculated as:

$$w_i^{ARP} = \arg \min_w \sum_{i=1}^n \sum_{j=1}^n \left( \frac{w_i \text{cov}(r_i, r_p)}{\alpha_i} - \frac{w_j \text{cov}(r_j, r_p)}{\alpha_j} \right)^2 \quad (12)$$

where  $w_i^{ARP}$  is the weight of asset  $i$  in the ARP portfolio and  $\alpha_i$  is the alpha of asset  $i$ . (Alvarez 2011.)

The Beta Risk Parity (BRP) and Systematic Risk Parity (SRP) methods extend the Alpha Risk Parity technique by allocating the risk budget in proportion to the systematic risk of each component in the portfolio. BRP and SRP are equivalent. (Kahra 2016.)

The Beta Risk Parity portfolio asset weights are calculated as:

$$w_i^{BRP} = \arg \min_w \sum_{i=1}^n \sum_{j=1}^n \left( \frac{w_i \text{cov}(r_i, r_p)}{\beta_i^2} - \frac{w_j \text{cov}(r_j, r_p)}{\beta_j^2} \right)^2 \quad (13)$$

where  $w_i^{BRP}$  is the weight of asset  $i$  in the BRP portfolio and  $\beta_i$  is asset  $i$ 's beta risk. (Kahra 2016.)

The Systematic Risk Parity portfolio asset weights are calculated as:

$$w_i^{SRP} = \arg \min_w \sum_{i=1}^n \sum_{j=1}^n \left( \frac{w_i \text{cov}(r_i, r_p)}{\beta_i^2 \text{var}(r_p)} - \frac{w_j \text{cov}(r_j, r_p)}{\beta_j^2 \text{var}(r_p)} \right)^2 \quad (14)$$

where  $w_i^{SRP}$  is the weight of asset  $i$  in the SRP portfolio and  $\beta_i^2 \text{var}(r_p)$  is asset  $i$ 's systematic risk. (Kahra 2016.)

Maximum Diversification (MD) maximizes the distance between the weighted average volatility of the portfolio's components and the total portfolio's volatility. This is achieved by concentrating the allocation to components with low or negative correlations, or to uncorrelated components.

The Maximum Diversification portfolio asset weights are calculated as:

$$w_i^{MD} = \arg \max_w \frac{\sum_{i=1}^n w_i \sigma_i}{\sqrt{w' \Sigma w}} \quad (15)$$

where  $w_i^{MD}$  is the weight of asset  $i$  in the MD portfolio,  $\Sigma$  is the variance-covariance matrix of the asset returns, and  $w$  is the  $(N \times 1)$  weight vector. (Choueifaty & Coignard 2008.)

Diversified Risk Parity (DRP) uses principal component analysis to determine the uncorrelated components of a portfolio. These uncorrelated components are used to form principal portfolios. Then, the final portfolio is constructed from the principal portfolios by optimizing the diversification distribution.

The covariance matrix is decomposed as

$$\Sigma = E\Lambda E' \quad (16)$$

where  $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_n)$  is a diagonal matrix of  $\Sigma$ 's eigenvalues in descending order and  $E$  represents the eigenvectors of  $\Sigma$ . The eigenvectors define a set of  $n$  uncorrelated principal portfolios. The returns for these principal portfolios can be calculated as  $\tilde{R} = E'R$  and they have variance  $\lambda_i$ . Thus, the final portfolio can be defined as the weights  $w$  in the original assets or as its weights  $\tilde{w} = E'w$  in the principal portfolios. In order to optimize the diversification distribution, we can calculate each principal portfolio's contribution,  $p_i$ , to the final portfolio's variance with:

$$p_i = \frac{\tilde{w}_i^2 \lambda_i}{\sum_{i=1}^n \tilde{w}_i^2 \lambda_i} \quad (17)$$

The Diversified Risk Parity portfolio asset weights are calculated as:

$$\tilde{w}_i^{DRP} = \arg \max_w \exp(-\sum_{i=1}^n p_i \ln p_i) \quad (18)$$

where  $p_i$  is the principal component portfolio's contribution to the final portfolio's total risk. (Meucci 2009; Lohre, Opfer & Orszag 2014.) The next section addresses the most common criticisms of risk parity methods.

#### 2.2.4 Risk Parity and Active Asset Management

One source of criticism for risk parity techniques is that they do not attempt to control for portfolio performance in terms of returns, which has been the traditional purpose of active asset management (Roncalli 2014). A second source of criticism is the fact that using volatility as a measure of risk assumes that the asset's returns are normally distributed (Boudt et al. 2013).

However, Roncalli (2014) presents a method where a portfolio manager can modify the final weights of the risk budgeted portfolios, and thus, incorporate their own knowledge. Compared to traditional mean-variance portfolios which are aggressive in their allocation, even the modified risk parity portfolios are significantly more conservative or defensive strategies. Nonetheless, Roncalli (2014) demonstrates that risk parity methods, especially the ERC, can be modified to include forecasts and expectations to accommodate more active investment strategies.

Risk parity portfolios tend to produce lower absolute returns than traditional portfolio diversification methods since they allocate large weights to low volatility assets which often have low returns, such as fixed income securities. Due to the low volatility, however, they tend to perform well in terms of Sharpe ratios. Therefore, leverage can be used to scale the returns (and risk) higher while still maintaining the Sharpe ratio. However, using leverage brings its own set of risks and costs. In addition, the access and willingness to use leverage varies between investors. (Asness, Frazzini & Pedersen 2012.) However, Dudley (2011) notes that creating leverage through exchange-traded derivatives can reduce the risks associated with more traditional borrowing methods. Finally, investors, especially institutions, can use risk parity methods as a part of a multi-strategy portfolio, instead of using risk parity for their whole portfolio. Additionally, risk parity portfolios can be used as benchmarks, or starting points, for other portfolio strategies. (Mariathasan 2011.) This thesis does not analyze any active risk parity methods nor the use of leverage.

### **3 DATA AND METHODOLOGY**

This thesis analyzes risk parity portfolios constructed from risk factor tilt indices. The main focus is on the performance and properties of the portfolios.

The main research questions are:

- 1) How do multi-factor risk parity portfolios perform compared to the benchmarks?
- 2) Do multi-factor risk parity portfolios generate more efficient risk-adjusted returns compared to the benchmarks?

The key assumptions of this analysis are:

- 1) The risk factors exist, and are not simply a result of data mining.
- 2) The risk factors are priced and are assumed to provide risk premia.
- 3) The risk factors are tradable (several sponsors offer factor index ETFs).
- 4) The correlations between factor returns are not positively perfect.
- 5) Risk parity portfolios are efficient only if the components have equal Sharpe ratios and they are uncorrelated.

The empirical literature reviewed in section 2 supports these assumptions. However, our results do not support assumption 5.

#### **3.1 Econometric Techniques**

##### **3.1.1 Basics**

This section summarizes the basic financial mathematics used to calculate asset returns and their summary statistics. Table 1 summarizes the basic return calculations.

**Table 1. Return calculations.**

	Return	Excess Return	
Asset	Simple:		
	$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$	$R_t^e = R_t - R_{f,t}$	$P_t$ = asset price at time $t$ $R_f$ = risk-free return $R_t$ = vector of returns $r_f$ = log risk-free return
	Log:		
	$r_t = \ln(1 + R_t) = \ln P_t - \ln P_{t-1}$	$r_t^e = r_t - r_{f,t}$	
Portfolio	simple:		$w$ = vector of weights $R$ = vector of returns
	$R_p = w'R$	$R_p^e = R_p - R_f$	
Transformations	Log to simple:		
	$R_t = e^{r_t} - 1$		
	Simple to log:		
	$r_t = \ln(1 + R_t)$		

Note: Logarithmic asset returns cannot be aggregated into portfolio returns directly. Logarithmic asset returns must first be transformed into simple returns and then aggregated into portfolio returns. The portfolio returns can then be transformed back into logarithmic returns. Logarithmic returns can be cumulated by summing the return of each period.

Source: adapted from Hurn, Martin, Phillips and Yu (2015).

For the sake of consistency and comparability, all presented summary statistics and performance measures are calculated from logarithmic returns. However, it should be noted that logarithmic returns have lower means and higher variance than simple returns, and they are not a direct measure of monetary returns (see Hudson and Gregoriou [2015] for a further discussion on the differences between using logarithmic and simple returns). The difference between log and simple returns is small when the return is small, but the same is not true for large returns. As an example, consider a simple return of 1%. Using the return transformation from table 1, the transformed logarithmic return is  $r_t = \ln(1 + 0.01) \approx 0.995\%$ . However, if the simple return is 80%, the transformed logarithmic return is approximately 58.78%. Since the logarithmic returns calculated in this thesis are monthly and relatively small, they can be considered to approximate the simple (i.e. true

monetary) returns for an investor that has implemented the strategies (e.g. a monthly mean logarithmic return of 6% equals approximately 6.18% as a simple return).

Table 2 summarizes the calculation of the basic summary statistics of returns. These measures are used to describe the statistical properties of the data.

**Table 2. Summary statistic calculations.**

Variance	$var(R_p) = \sigma_p^2 = w' \Sigma w$	$w$ = vector of weights $\Sigma$ = variance-covariance matrix
Standard Deviation	$\sigma_p = \sqrt{\sigma_p^2}$	
Covariance	$cov(R_x, R_y) = \sigma_{xy} = w_x' \Sigma w_y$	$w_x$ = weight of asset $x$ $w_y$ = weight of asset $y$ $\Sigma$ = variance-covariance matrix
Correlation	$corr(R_x, R_y) = \rho_{xy} = \frac{cov(R_x, R_y)}{\sigma_x \sigma_y} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$	

Source: adapted from Hurn, Martin, Phillips and Yu (2015).

### 3.1.2 DCC-GARCH Model Definition

The time-varying covariances and correlations are estimated with the dynamic conditional correlation (DCC) GARCH model (Engle 2002). We apply the first order DCC-GARCH(1,1) model because it relaxes the assumption that the correlations are constant over time. The method is chosen due to its flexibility and simplicity. We assume a multivariate normal distribution for the errors and that the conditional variance matrix ( $H_t$ ) is positive definite. For a more detailed specification of the model see Engle (2002).

The multivariate<sup>4</sup> DCC-GARCH model can be defined as:

$$r_t = \phi_0 + \phi_1 r_{t-1} + u_t \quad (19)$$

$$H_t = S_t R_t S_t \quad (20)$$

$$u_t \sim N(0, H_t) \quad (21)$$

where  $r_t$  is a vector of log returns and it is specified as an AR(1) model:  $\phi_0 + \phi_1 r_{t-1}$ , where the conditional mean is a function of lagged returns  $r_{t-1}$  and the first-order autocorrelation coefficient,  $\phi_1$ .  $u_t$  is the normally distributed vector of error terms with zero mean and variance  $H_t$ .

$H_t$  is the  $(N \times N)$  conditional variance matrix.  $S_t$  is the  $(N \times N)$  diagonal matrix

of the conditional standard deviations and it is defined as  $S_t = \begin{bmatrix} \sqrt{h_{1,t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{N,t}} \end{bmatrix}$

where  $h_{i,t} = \alpha_{0,i} + \alpha_{1,i} u_{i,t-1}^2 + \beta_{1,i} h_{i,t-1}$ .

$R_t$  is the  $(N \times N)$  conditional correlation matrix and it is defined as  $R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2}$  where  $Q_t$  has a GARCH(1,1) specification  $Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha z_{t-1} z'_{t-1} + \beta Q_{t-1}$ ,  $z_t = \frac{u_{i,t}}{\sqrt{h_{i,t}}}$  are the standardized residuals,  $\alpha$  and

$\beta$  are unknown scalars, and  $\bar{Q} = \frac{1}{T} \sum_{t=1}^T \begin{bmatrix} z_{1,t}^2 & z_{1,t} z_{2,t} & \cdots & z_{1,t} z_{N,t} \\ z_{2,t} z_{1,t} & z_{2,t}^2 & \cdots & z_{2,t} z_{N,t} \\ \vdots & \vdots & \ddots & \vdots \\ z_{N,t} z_{1,t} & z_{N,t} z_{2,t} & \cdots & z_{N,t}^2 \end{bmatrix}$  is the

unconditional covariance matrix. The subscript  $t$  denotes time. (Hurn, Martin, Phillips & Yu 2015.)

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<sup>4</sup> The univariate GARCH(1,1) model's conditional mean,  $r_t$ , can be calculated in the same way, the variance is defined as  $h_t = \alpha_{0,i} + \alpha_{1,i} u_{i,t-1}^2 + \beta_{1,i} h_{i,t-1}$ , and the distribution is  $u_t \sim N(0, h_t)$ . For a detailed specification of the GARCH model see Bollerslev (1986).



### 3.1.3 DCC-GARCH Model Estimation

The DCC-GARCH model parameters can be estimated by maximum likelihood. For a sample of  $t = 1, 2, \dots, T$  observations, the general form of the log-likelihood function of a multivariate GARCH model is defined as:

$$\log L = \frac{1}{T} \sum_{t=1}^T \log L_t = \frac{1}{T} \sum_{t=1}^T \log f(r_{1,t}, r_{2,t}, \dots, r_{N,t}) \quad (22)$$

where  $f(r_{1,t}, r_{2,t}, \dots, r_{N,t})$  is an  $N$ -dimensional multivariate probability distribution.

With the assumption of a multivariate normal distribution the function is defined as:

$$f(r_{1,t}, r_{2,t}, \dots, r_{N,t}) = \left(\frac{1}{2\pi}\right)^{N/2} |H_t|^{-1/2} \exp(-0.5 u_t' H_t^{-1} u_t) \quad (23)$$

where  $H_t$  is the conditional variance matrix of the returns and  $u_t$  is the  $(N \times 1)$  vector of errors at time  $t$  and they are calculated as:

$$\begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{N,t} \end{bmatrix} = \begin{bmatrix} r_{1,t} \\ r_{2,t} \\ \vdots \\ r_{N,t} \end{bmatrix} - \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \\ \vdots \\ \mu_{N,t} \end{bmatrix} \quad (24)$$

$$u_t = r_t - \mu_t$$

where  $r_t$  is an  $(N \times 1)$  vector of returns and  $\mu_t$  is the  $(N \times 1)$  vector of conditional means of the returns. (Hurn, Martin, Phillips & Yu 2015.)

Assuming multivariate conditional normality, the log-likelihood function is:

$$\begin{aligned} \log L_t &= \log f(r_{1,t}, r_{2,t}, \dots, r_{N,t}) \\ &= -0.5N \log(2\pi) - 0.5 \log |H_t| - 0.5 u_t' H_t^{-1} u_t \end{aligned} \quad (25)$$

### 3.2 Data Description

The U.S. factor tilt indices and the U.S. equity index are collected from MSCI. The factor tilt indices are constructed based on a factor score which is then used to tilt the parent index to provide exposure to the risk factor. This method offers high capacity and investability, but due to being long-only, they still have high exposure to the market beta. MSCI provides factor tilt indices for six factors: value, quality, momentum, size, dividend, and volatility. Table 3 summarizes the factor tilt indices and the empirical evidence supporting the risk premia that they have earned historically. The MSCI indices are total return indices with net dividends, that is, the dividends are reinvested after deducting the withholding taxes. The market capitalization weighted MSCI USA equity index is used as a proxy for the market portfolio.

Bonds are represented by the market capitalization weighted Citigroup U.S. Government Bond Index and it is composed of investment grade (minimum credit quality of A- by S&P and A3 by Moody's) U.S. sovereign bonds of all maturities (minimum maturity of 1 year). The index is a total return index. The bond index is used in the risk parity portfolios to represent the bond risk factors as these are not available separately.

The risk-free rate is represented by the U.S. 3-month treasury bill secondary market rate from the Federal Reserve Bank of St. Louis' FRED database. The risk-free rate is used to calculate excess returns. All data is monthly and in USD. The sample period is November 1998 to March 2017. This dataset was chosen because the indices are tracked by several exchange-traded funds, and therefore, the analyzed portfolios can be effectively implemented by any investor.<sup>5</sup>

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<sup>5</sup> The factor and equity data are obtained from MSCI: <https://www.msci.com/end-of-day-data-search>. The bond data is obtained from Datastream. For more information see: <https://www.yieldbook.com/m/indices/single.shtml?ticker=WGBI>. The treasury bill data is obtained from the FRED database: <https://fred.stlouisfed.org/series/TB3MS>.

**Table 3. MSCI factor descriptions.**

Factor	Description and Empirical Evidence
Value	High value stocks (as measured by the B/M ratio) outperform low value stocks (Fama & French 1995).
Quality	Stocks with low leverage, stable earnings, and high profitability (i.e. high quality stocks) outperform low quality stocks (Asness, Frazzini & Pedersen 2013).
Momentum	Winner stocks outperform loser stocks in the short-term (Jegadeesh & Titman 1993).
Size	Small market capitalization stocks outperform large market capitalization stocks (Fama & French 1995).
Dividend	Stocks with persistently high and increasing dividend yields outperform low dividend yield stocks (Fama & French 1988).
Volatility	Low volatility stocks outperform high volatility stocks (Ang, Hodrick, Xing & Zhang 2006).

Source: MSCI.

### 3.3 Benchmarks

The benchmark portfolios are the equally-weighted factor portfolio, the 60/40 policy portfolio, and the market proxy portfolio (market capitalization weighted U.S. equity index). The 60/40 portfolio allocated 60% to the market proxy portfolio and 40% to bonds.

The equally-weighted (EW) portfolio weights are calculated as:

$$w_i^{EW} = \frac{1}{N} \tag{26}$$

where  $w_i^{EW}$  is the weight of asset  $i$  in the EW portfolio and  $N$  is the number of assets in the portfolio.

### 3.4 Performance Measures

The Sharpe (1994) ratio is a common measure for the risk-adjusted returns of assets or portfolios. The Sharpe ratio is defined as:

$$SR = \frac{R_p - R_f}{\sigma_{R_p - R_f}} = \frac{R_p^e}{\sigma_{R_p^e}} \quad (27)$$

where  $R_p$  is the annualized return of portfolio  $p$ ,  $R_f$  is the annualized risk-free rate,  $R_p^e$  is the annual portfolio excess return, and  $\sigma_{R_p - R_f} = \sigma_{R_p^e}$  is the annual standard deviation of the portfolio excess returns.

The information ratio is a measure of the excess return relative to a benchmark (Goodwin 1998). The Information ratio is defined as:

$$IR = \frac{R_p - R_b}{\sigma_{R_p - R_b}} = \frac{\alpha_p}{\sigma_{\alpha_p}} \quad (28)$$

where  $R_b$  is the return of benchmark  $b$ ,  $\alpha_p$  is the excess return of a portfolio relative to a benchmark, and  $\sigma_{R_p - R_b} = \sigma_{\alpha_p}$  is the standard deviation of the difference in returns between the portfolio and the benchmark.

The (maximum) drawdown is a risk measure which measures the (maximum) loss during a period. Drawdowns can be calculated in various ways depending on the purpose. Commonly, the value tells the percentage loss from peak equity to lowest trough. In addition, a time series of drawdowns can be used to determine the recovery period after a substantial financial loss. For our purposes, the drawdowns are calculated as a time series of losses from the cumulative return time series.

## 4 EMPIRICAL RESULTS

### 4.1 Asset Analysis

This section analyzes the properties and the performance of the individual assets. The performance of the individual assets is presented to demonstrate how single-factor portfolios perform in our sample. The results are then compared with the multi-factor portfolios in the following section. The main purpose of this comparison is to determine whether there are diversification benefits from combining factor tilt indices into multi-factor portfolios.

Figure 3 presents the time series of returns for each asset during the sample period. We find that bonds have the lowest volatility and returns. The factor tilt index returns behave very similarly to the market, and each other, over the sample. Therefore, they can be expected to have high correlations. The largest negative returns occur at the same time for all series in 2002 and 2009 except for bond returns, which seem to be unaffected by those crisis periods. In addition, the asset return series illustrate that the volatility is not constant. There is significant volatility clustering; periods of low volatility tend to be followed by low volatility, and high volatility is followed by high volatility. Finally, this volatility clustering occurs at the same points for each asset.

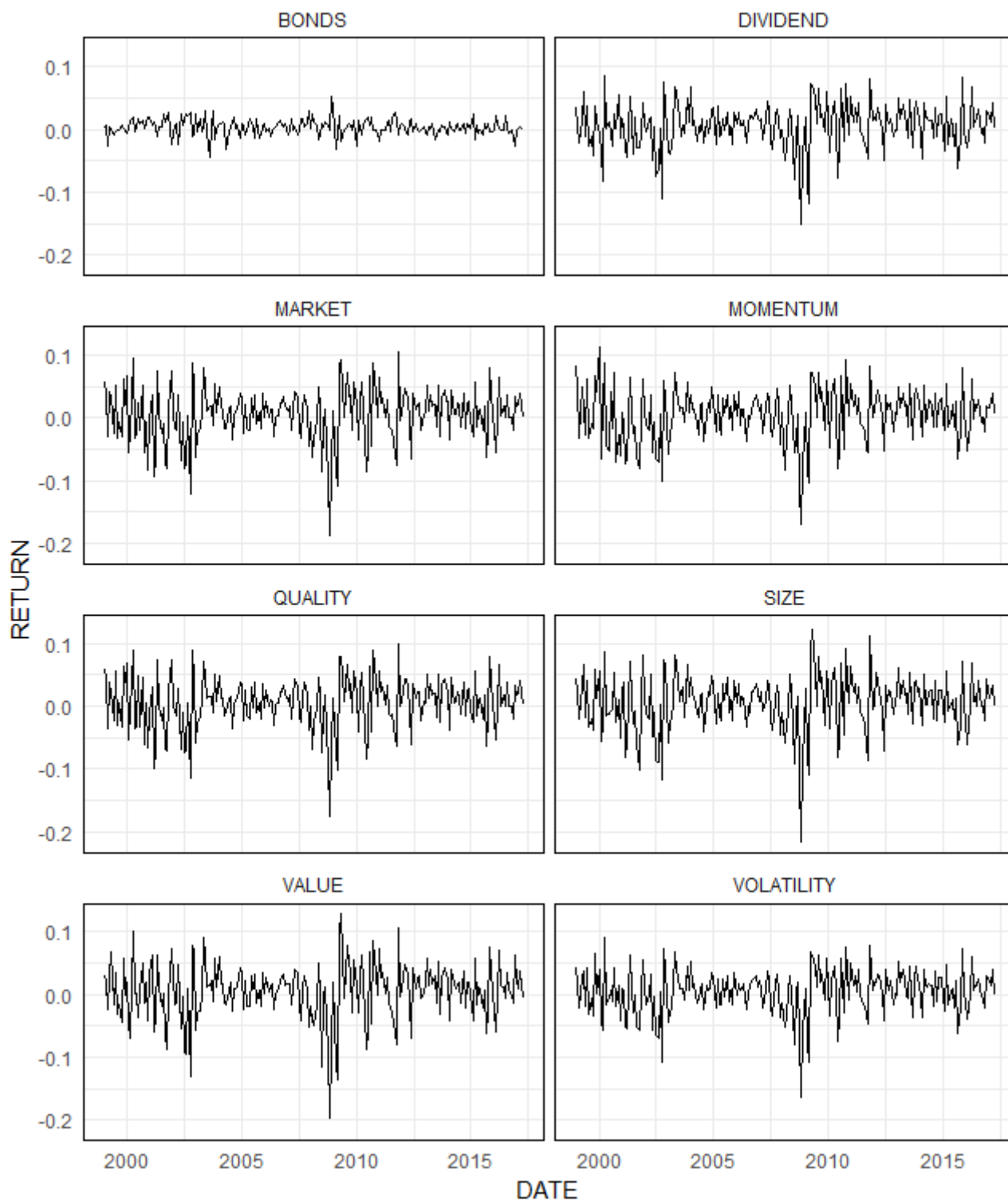


Figure 3. Time series of asset returns, December 1998 to March 2017.

Table 4 reports the estimated correlations between the asset excess returns. Bonds have negative correlations with all other assets ranging from  $-0.32$  to  $-0.21$ . The factor tilt indices have high correlations of over 0.9 with the market. The lowest correlation is between the dividend tilt and the market with a correlation of 0.92. Among the factor tilt indices, the correlations range from 0.86 to 0.97. The lowest correlations are between dividend and momentum (0.86) and momentum and value (0.89). Since the factor indices are not perfectly correlated, we can expect diversification benefits from combining them into multi-factor portfolios. In addition, bonds can provide significant diversification due to the negative correlations.

**Table 4. Correlations between asset excess returns.**

	Dividend	Momentum	Quality	Size	Value	Volatility	Bonds	Market
Dividend	1	0.86	0.91	0.92	0.94	0.96	-0.21	0.92
Momentum		1	0.97	0.94	0.89	0.93	-0.28	0.97
Quality			1	0.97	0.93	0.96	-0.29	0.99
Size				1	0.98	0.96	-0.32	0.98
Value					1	0.96	-0.31	0.96
Volatility						1	-0.26	0.97
Bonds							1	-0.31
Market								1

This table reports the correlations between the excess returns of the individual assets. Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI Factor Tilt indices. Bonds is the Citigroup U.S. Government Bonds Index and Market is the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is December 1998 to March 2017.

Figure 4 presents the DCC-GARCH(1,1) estimated time-varying correlations between the factor tilt index excess returns. We find that there is significant variation in the correlations between the factor indices. All pairs have the lowest correlations in 2000–2001. For example, dividend and momentum have correlations under 0.5 and momentum and value have correlations under 0.65 during that period. The correlations are significantly higher after that period, but with varying degrees of variation.

Figure 5 presents the time-varying correlations between the factor index and the market and bond excess returns. We find that the correlations between the factor tilt indices and the market are similar to the correlations among the factor tilt indices. There are large declines in 2000–2001, and the correlations are more stable after that.

The time-varying correlations between the factor tilt indices and bonds are mostly negative during the period, but they exhibit large variation ranging between  $-0.6$  and  $0.05$ . The lowest correlations are in 2003, 2008, and 2013. This finding is significant as it shows that we cannot assume a constant negative correlation between the factor tilt indices and bonds as the simple correlation matrix in table 4 would suggest.

One area of interest is the change in correlations during times of financial distress. We observe large declines in the correlations between the factor indices, and the market, in 2000–2001, but not during the financial crisis in 2009. Since our sample includes only two periods of financial distress, we cannot conclude that factor tilt indices would offer diversification benefits during crisis times in general. However, this would have been the case in 2000–2001. A longer history of correlations would be needed to draw more generalizable conclusions.

Overall, the factor indices have high correlations with each other and the market. However, this is not a surprising result due to the long-only construction of the indices which means that they are expected to be sensitive to the movements of the market. Our analysis shows that there can be significant time-variation in the correlations of excess returns. Therefore, we suggest that the time-variation of correlations be considered in financial analysis.



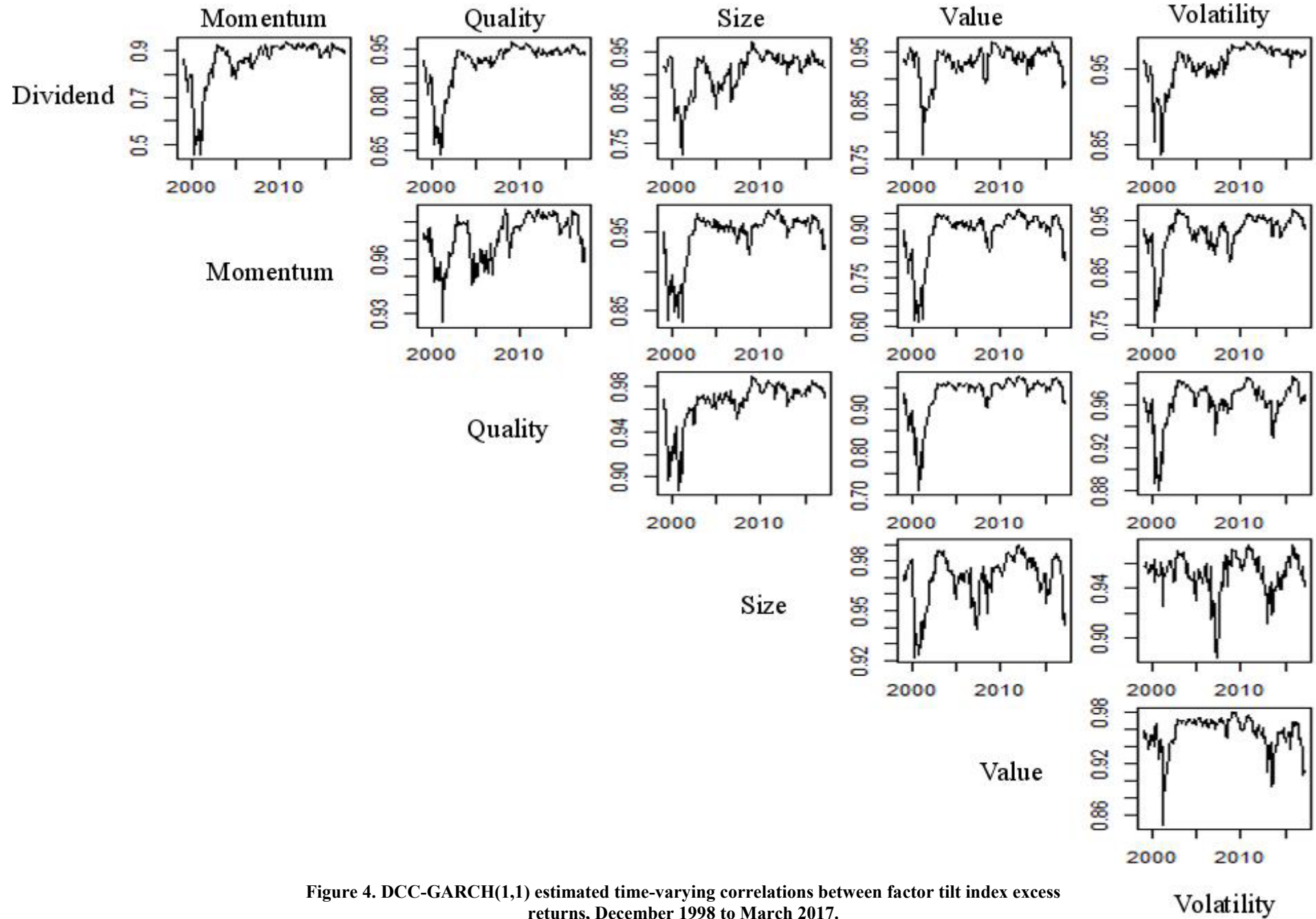


Figure 4. DCC-GARCH(1,1) estimated time-varying correlations between factor tilt index excess returns, December 1998 to March 2017.

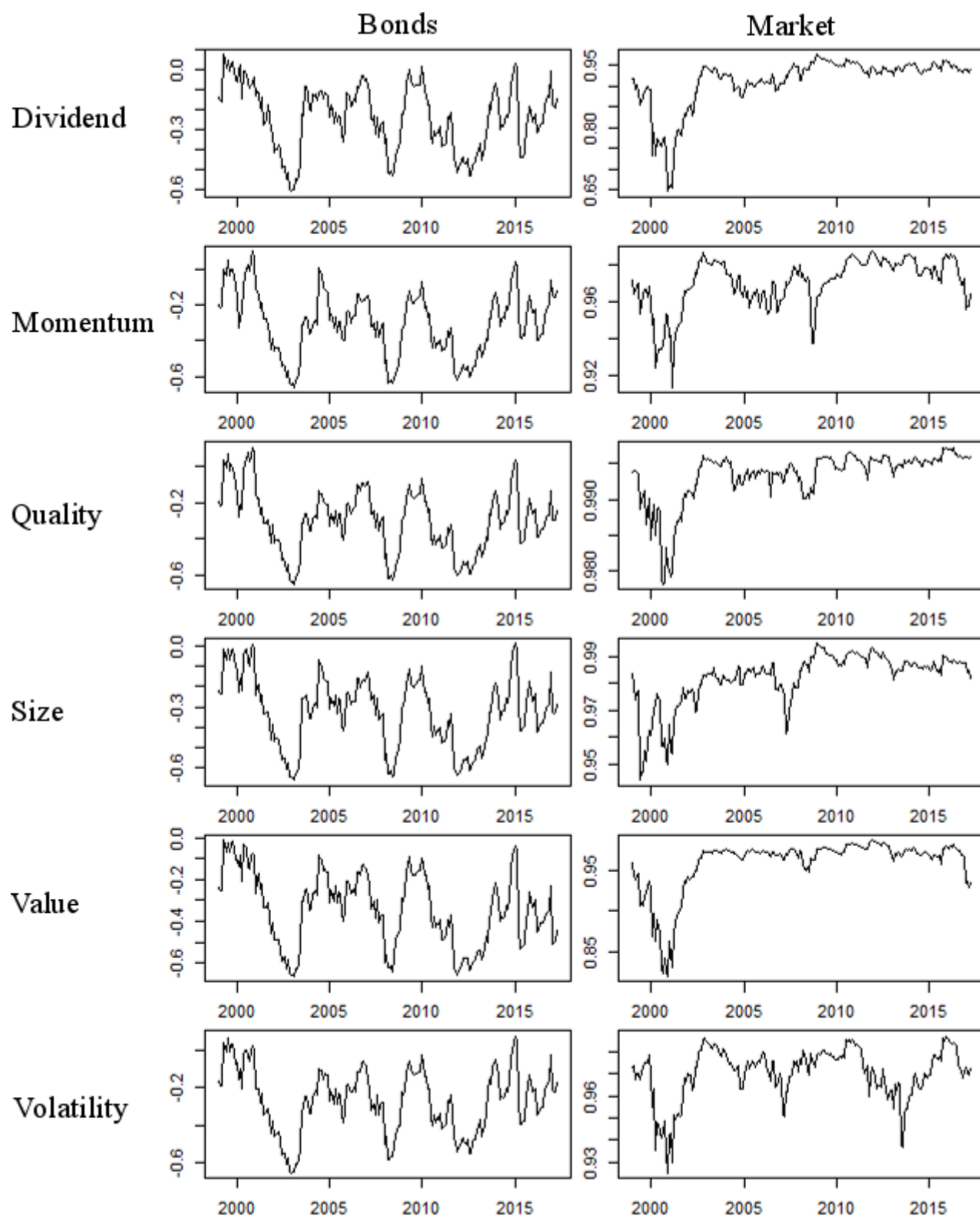


Figure 5. DCC-GARCH(1,1) estimated time-varying correlations between factor tilt index, bond, and market excess returns, December 1998 to March 2017.

Table 5 reports the summary statistics of the assets. We find that all factor tilt indices generate higher returns than the market. Dividend provides the highest mean returns of 6.47% followed by size with 6.40%. The market generates a return of 5.08% and bonds 4.29%. In addition, we find that the returns are non-normal based on the Jarque-Bera test with p-values under 0.0001. The returns are negatively skewed and have long tails which is typical for asset returns. Figure 6 plots the cumulative returns for the assets over the period. We find that the returns follow a similar pattern and the largest losses are experienced in 2002 and 2009, except for bonds which show less volatility.

All factor tilt indices have a higher Sharpe ratio than the market. We find that bonds have a Sharpe ratio of 0.57, the market has 0.22, and dividend has 0.37 which is the highest of the factor tilt indices. In addition, all indices have a positive Information ratio. Size has the highest Information ratio of 0.46 and is followed by quality with 0.38.

In addition, four of the factor tilt indices (dividend, momentum, quality, volatility) have lower volatilities and drawdowns than the market. This indicates that the higher returns are not compensation for higher risk. For size and value, however, the returns may be correlated to their higher risk measured by volatility and drawdowns. Dividend appears to be the best performing factor tilt index as it has the highest returns and Sharpe ratio and lowest volatility and drawdown. For example, dividend has an annual volatility of 12.47%, the market has 14.98%, and bonds have 4.40%. Dividend has a drawdown of -44.93%, the market has -53.40%, and bonds have -5.08%. Figure 7 plots the drawdowns for the assets. We find that the drawdowns follow a similar pattern and the largest drawdowns are experienced at the same points.

Based on these results we can conclude that factor tilt indices can provide higher returns for lower risk compared to a broad market capitalization weighted index. In addition, since both the volatilities and drawdowns are lower, the higher returns are not simply due to exposure to higher risk. In addition, we find that the factor tilt indices behave very similarly in terms of returns and drawdowns. It should be noted that our results do not support assumption 5 about equal Sharpe ratios and non-

correlation. Therefore, the risk parity portfolios constructed in the next section may not be efficient. However, since the factor returns are not perfectly correlated, we can expect some diversification benefits from combining them into multi-factor portfolios.

**Table 5. Summary Statistics of the Assets.**

	Dividend	Momentum	Quality	Size	Value	Volatility	Bonds	Market
Return	6.47%	6.09%	5.76%	6.40%	5.78%	5.47%	4.29%	5.08%
Excess return	4.67%	4.29%	3.95%	4.60%	3.97%	3.66%	2.49%	3.27%
Volatility	12.47%	14.65%	14.37%	15.58%	15.69%	12.54%	4.40%	14.98%
Sharpe	0.37	0.29	0.27	0.29	0.25	0.29	0.57	0.22
Information	0.23	0.28	0.38	0.46	0.16	0.10		
Drawdown	-44.93%	-52.35%	-48.96%	-55.10%	-59.37%	-47.54%	-5.08%	-53.40%
Skewness	-0.76	-0.61	-0.67	-0.82	-0.74	-0.78	-0.15	-0.71
Kurtosis	1.87	0.91	1.20	2.27	1.92	1.92	1.50	1.41
Normality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	-14.97%	-17.04%	-17.54%	-21.56%	-19.75%	-16.40%	-4.48%	-18.81%
Maximum	8.48%	11.18%	9.80%	12.20%	12.82%	9.09%	5.27%	10.38%

This table reports the key statistics of the individual assets. All values are calculated from logarithmic returns. We have not considered transaction costs. The returns are the annual means and the volatilities are the annual standard deviations of the returns. The Sharpe ratios are calculated as the average annual return in excess of the risk-free rate divided by the annual standard deviation of the excess return. The Information ratios are calculated as the average annual return in excess of the market divided by the standard deviation of the excess return. Drawdown measures the largest loss over the period as a percentage. Normality is the p-value of the Jarque-Bera test of normality with a null hypothesis of normality. Kurtosis is the excess kurtosis. Dividend, Momentum, Quality, Size, Value, and Volatility are the MSCI Factor Tilt indices. Bonds is the Citigroup U.S. Government Bonds Index and Market is the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is December 1998 to March 2017.

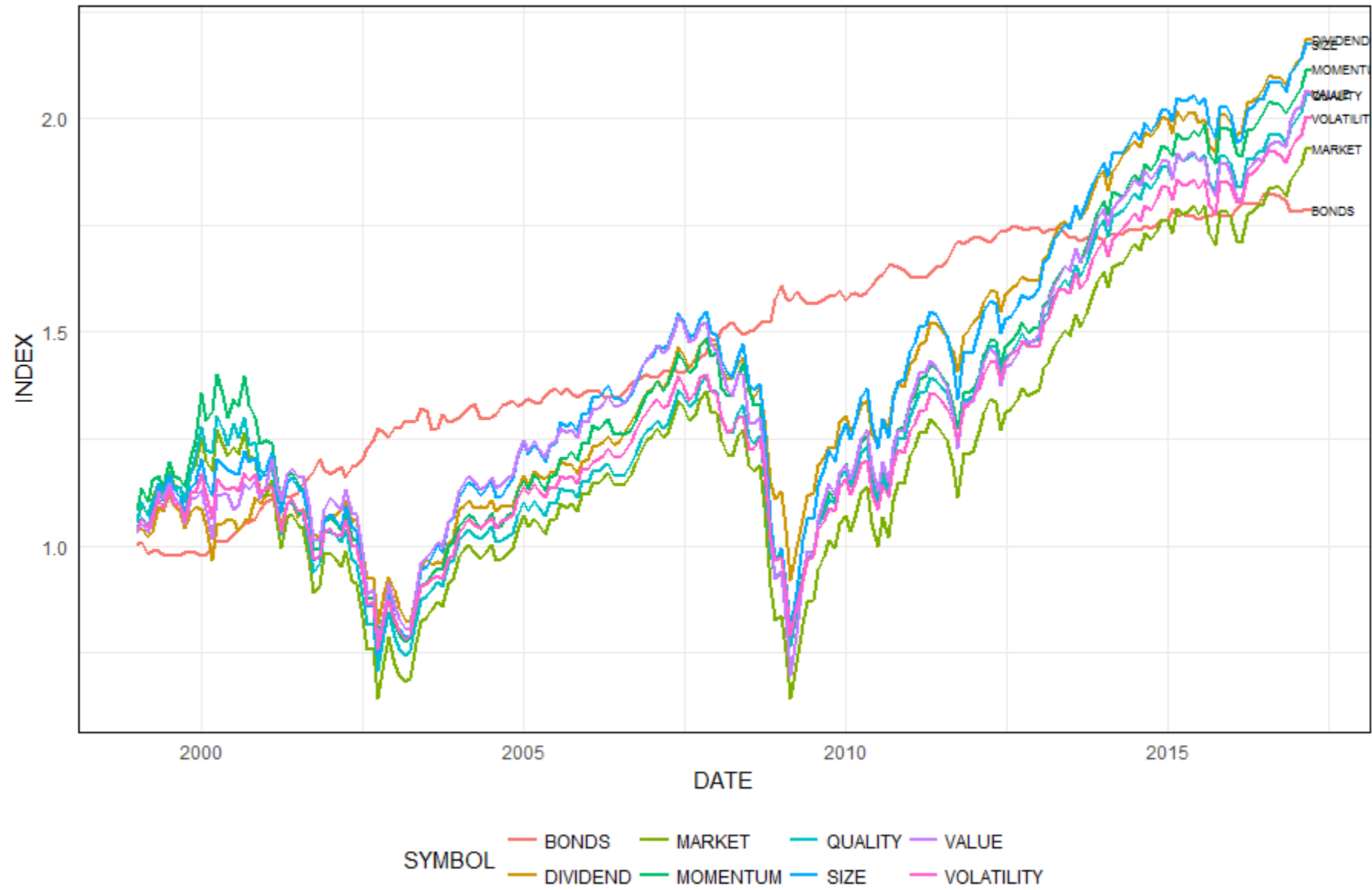


Figure 6. Asset cumulative returns, December 1998 to March 2017.

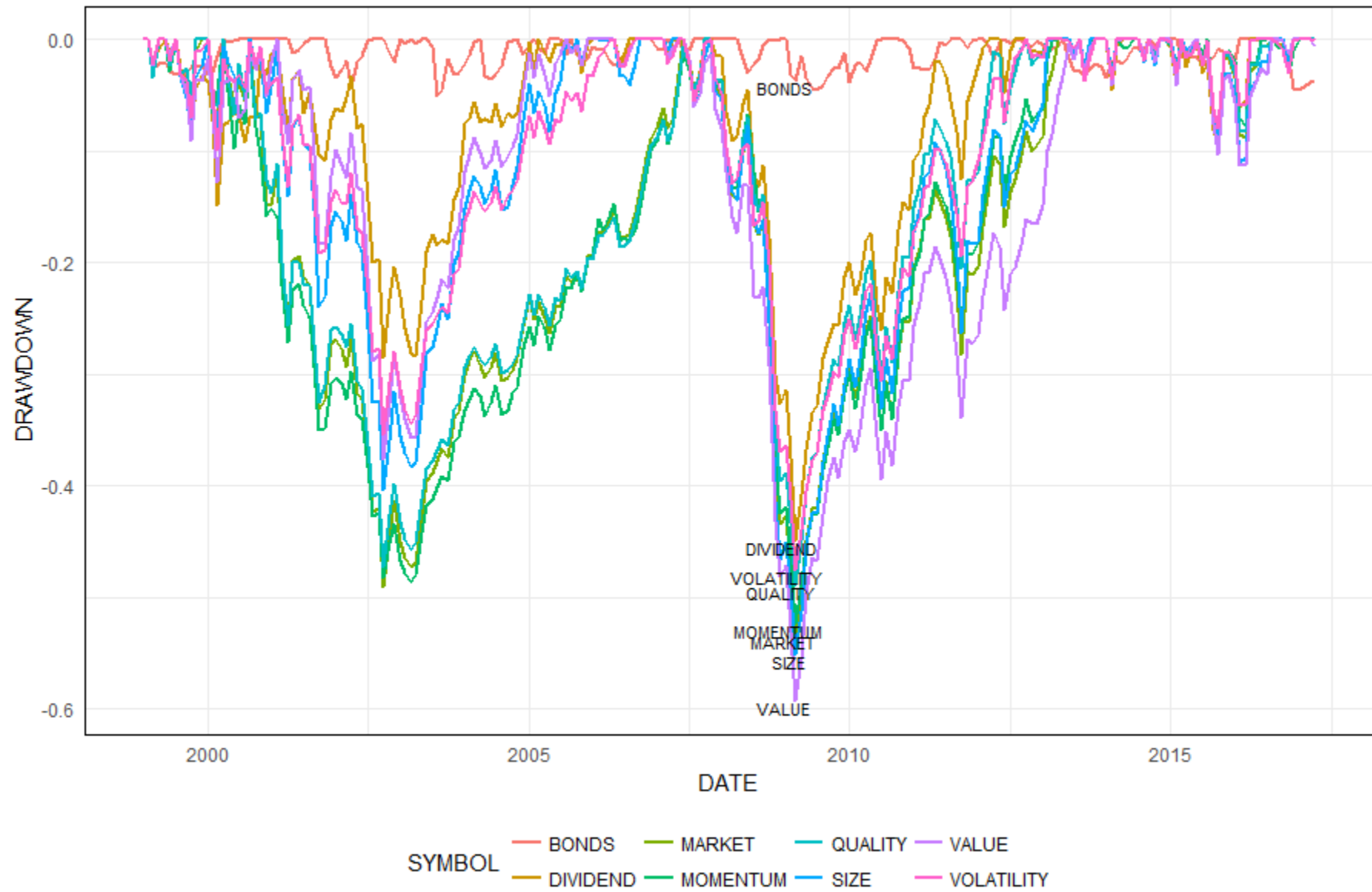


Figure 7. Asset drawdowns, December 1998 to March 2017.

## 4.2 Portfolio Analysis

This section analyzes the properties and the performance of the portfolios. The initial portfolio weights are estimated with a rolling 60-month window. The portfolios are rebalanced monthly and we do not consider any transaction costs. Since our results in the previous section do not support the assumption that the portfolio components have equal Sharpe ratios and that they are uncorrelated, we cannot assume that the risk parity portfolios are efficient.

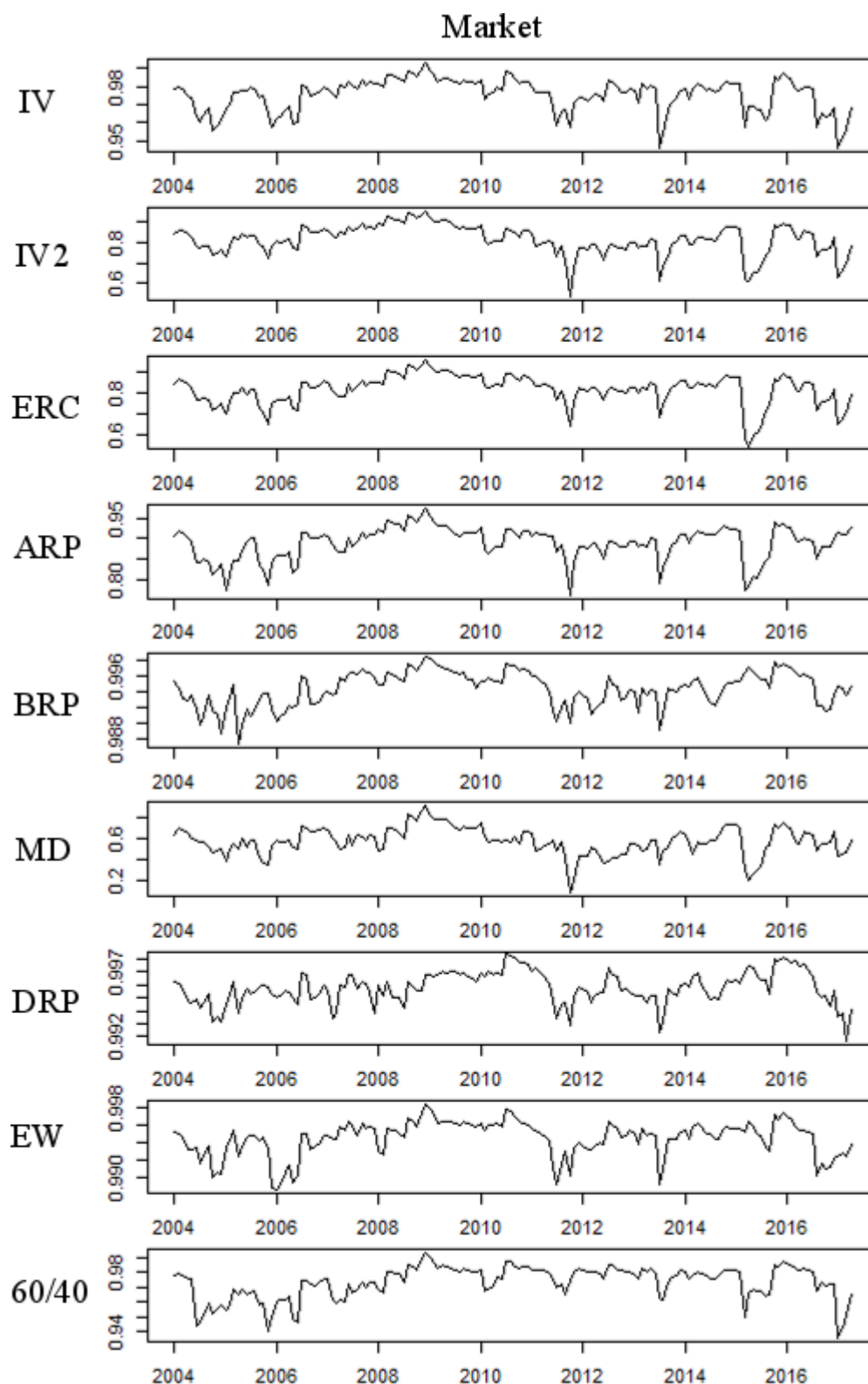
Table 6 reports the estimated correlations between the portfolio, market, and bond excess returns. We find that the portfolios are mostly highly correlated with each other, the market, and the benchmarks. The lowest correlations are between the market and DRP at 0.63, DRP and MD at 0.64, and MD and BRP 0.65. The correlations between the portfolios and bonds are mostly low or negative ranging from -0.27 to 0.56.

**Table 6. Correlations between portfolio, market, and bond excess returns.**

	IV	IV2	ERC	ARP	BRP	MD	DRP	EW	60/40	Market	Bonds
IV	1	0.93	0.94	0.96	0.99	0.77	0.98	0.99	0.99	0.98	-0.08
IV2		1	0.99	0.96	0.86	0.93	0.85	0.87	0.91	0.84	0.23
ERC			1	0.97	0.87	0.93	0.87	0.89	0.93	0.85	0.25
ARP				1	0.91	0.86	0.91	0.92	0.95	0.9	0.12
BRP					1	0.65	1	1	0.98	1	-0.24
MD						1	0.64	0.68	0.76	0.63	0.56
DRP							1	1	0.98	1	-0.25
EW								1	0.99	1	-0.2
60/40									1	0.98	-0.07
Market										1	-0.27
Bonds											1

This table reports the correlations between the excess returns of the risk parity portfolios, the benchmark portfolios, the market, and bonds. The portfolios are constructed from MSCI indices. Bonds is the Citigroup U.S. Government Bonds Index and Market is the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is December 2003 to March 2017.





**Figure 8. DCC-GARCH(1,1) estimated time-varying correlations between portfolio and market excess returns, December 2003 to March 2017.**

Figure 8 presents the time-varying correlations between the portfolios and the market excess returns. We find that the correlations demonstrate significant time-variation. The correlations seem to behave similarly over the period. The highest correlation peaks occur during the financial crisis period after 2008. In addition, the lowest points occur at roughly the same points in 2011, 2013, 2015, and 2017. Most notably, the correlations between the market and MD fluctuate between zero and over 0.8. Additionally, large changes in the correlations occurs for IV2, ERC, and ARP.

Table 7 reports the summary statistics of the portfolios. The returns are non-normal with negative skewness and long tails. BRP and DRP are the best performing portfolios in term of returns with average annual returns of 8.12% and 7.93%, respectively. In addition, they are the only risk parity portfolios to outperform the EW portfolio which generated 7.64% annually. For comparison, the market generated an average annual return of 7.23% with a volatility of 13.92% for the same period. Figure 9 plots the cumulative returns of the portfolios and we find that the portfolios behave very similarly over the period. The largest losses occur during the financial crisis in 2009.

Even though IV, IV2, ERC, ARP, and MD generate lower returns, their volatilities are also very low compared to the rest. Thus, they produce higher Sharpe ratios. Interestingly, MD generates the lowest returns at 4.56% annually, however, with a volatility of only 3.90%, it has the highest Sharpe ratio of 0.86. In fact, MD has lower volatility and a higher Sharpe ratio than bonds in table 5. BRP and DRP are the only risk parity portfolios to produce a positive Information ratio, which indicates that the other portfolios underperform the market in terms of returns. They underperform the market by roughly 1-2.5%, but the portfolios' volatilities are over two times lower. Finally, it should be noted that the 60/40 policy portfolio performs very similarly to the risk parity portfolios with a Sharpe ratio of 0.64.

A higher maximum drawdown may indicate that the higher return for the portfolio is due to higher risk. This is indeed the case in our sample. BRP, DRP, and EW generate the highest returns (8.12%, 7.93%, 7.64%, respectively), and the highest maximum drawdowns (−50.48%, −51.30%, −44.41%, respectively). Note however, that BRP has marginally higher returns and a lower maximum drawdown when

compared with DRP. In a case where the maximum drawdown is not higher even though the returns are, there may be a behavioral explanation for the higher return instead. In terms of drawdowns, IV2, ERC, ARP, and MD have significantly lower drawdowns than the other portfolios. Figure 10 plots the drawdowns over the period and the largest drawdowns occur in 2009.

By combining the factor tilt indices into multi-factor risk parity portfolios, we can generate similar returns with significantly lower volatilities and drawdowns. Thus, we are able to generate returns with more efficient risk-return profiles and over two times higher Sharpe ratios. These results indicate that combining factor tilt indices into multi-factor portfolios results in significant diversification benefits.

We can conclude that BRP and DRP are able to outperform the benchmarks in terms of returns, but they also have higher volatilities and drawdowns. In contrast, the other risk parity portfolios produce lower returns and lower volatilities, and thus achieve higher Sharpe ratios. Finally, we are able to generate better risk-adjusted returns by combining factor tilt indices into multi-factor portfolios.

**Table 7. Summary Statistics of the Portfolios.**

	Risk Parity Portfolios							Benchmarks	
	IV	IV2	ERC	ARP	BRP	MD	DRP	EW	60/40
Return	6.48%	4.83%	5.20%	5.57%	8.12%	4.56%	7.93%	7.64%	6.31%
Excess return	5.28%	3.63%	3.99%	4.37%	6.92%	3.35%	6.73%	6.44%	5.10%
Volatility	8.52%	5.44%	5.15%	6.07%	12.97%	3.90%	13.15%	11.23%	8.00%
Sharpe	0.62	0.66	0.77	0.71	0.53	0.86	0.51	0.57	0.64
Information	−0.18	−0.28	−0.24	−0.22	0.35	−0.25	0.29	0.03	−0.20
Drawdown	−36.03%	−23.32%	−19.82%	−24.86%	−50.48%	−10.71%	−51.30%	−44.41%	−31.74%
Skewness	−1.29	−1.72	−1.37	−1.56	−1.07	−1.22	−1.08	−1.07	−1.07
Kurtosis	4.47	6.26	5.09	5.39	3.23	4.13	3.40	3.39	3.79
Normality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	−12.00%	−7.85%	−6.87%	−7.99%	−17.36%	−4.53%	−17.90%	−15.12%	−10.92%
Maximum	6.01%	4.50%	4.65%	4.60%	8.90%	3.46%	9.24%	7.89%	6.05%

This table reports the key statistics of the portfolios. All values are calculated from logarithmic returns. We have not considered transaction costs. The returns are the annual means and the volatilities are the annual standard deviations of the returns. The Sharpe ratios are calculated as the average annual return in excess of the risk-free rate divided by the annual standard deviation of the excess return. The Information ratios are calculated as the annual return in excess of the market divided by the standard deviation of the excess return. Drawdown measures the largest loss over the period as a percentage. Normality is the p-value of the Jarque-Bera test of normality with a null hypothesis of normality. Kurtosis is the excess kurtosis. The portfolios are constructed from the MSCI factor tilt indices and the Citigroup U.S. Government Bonds Index. The market is represented by the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is December 2003 to March 2017.

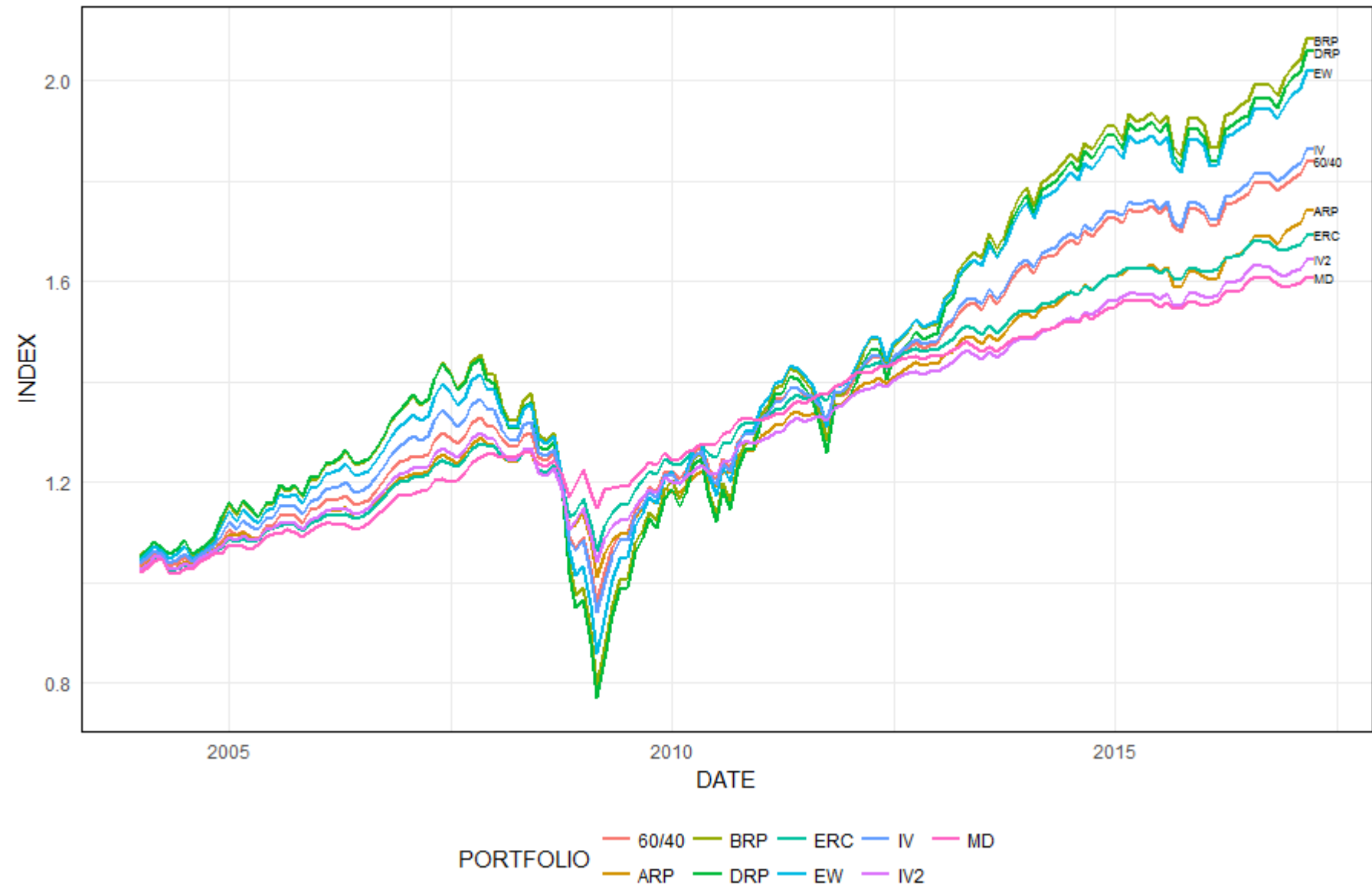


Figure 9. Portfolio cumulative returns, December 2003 to March 2017.

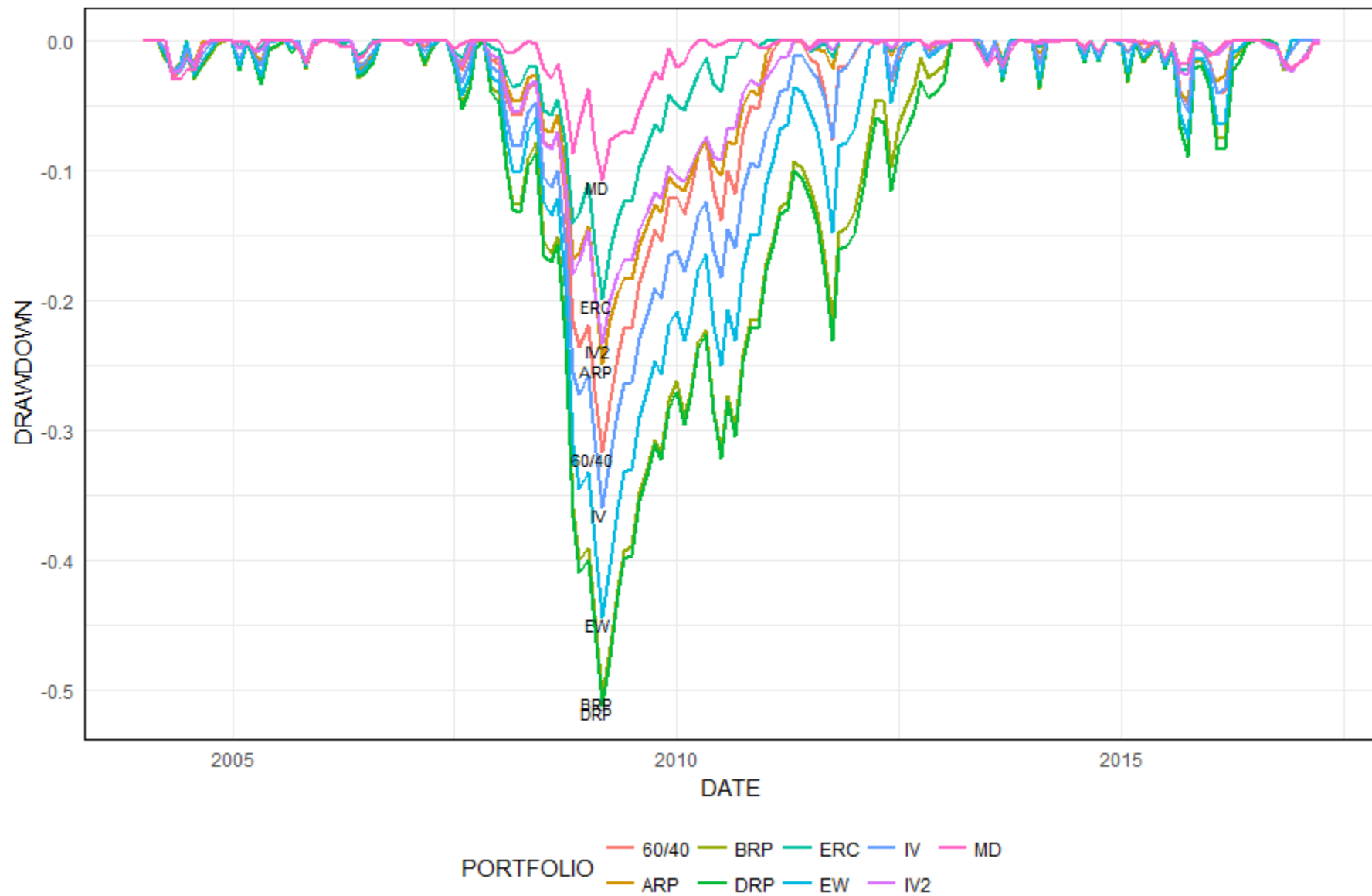


Figure 10. Portfolio drawdowns, December 1998 to March 2017.

### 4.3 Portfolio Composition

This section analyzes the composition of multi-factor risk parity portfolios. The main point of interest is to see whether some factors are overweighted relative to others because risk parity portfolios tend to overweight low volatility securities, such as fixed income. In addition, we will analyze how the weightings change over the sample period, especially during the financial crisis in 2008–2009.

Figures 11–14 plot the time-series of the portfolio weights for the risk parity portfolios. IV, IV2, ERC, ARP, and MD allocate the largest weights to bonds. In fact, for most of the period, more than 50% is allocated to bonds in IV2, ERC, ARP, and MD. To contrast, BRP and DRP have very small allocations to bonds.

IV allocated 25–30% to bonds and the rest is allocated to the factor indices roughly equally. For IV2 and ERC, the allocations are very similar with 50–60% being allocated to bonds and rest roughly equally to the factor indices. ARP shows several large spikes where the allocation to bonds is decreased significantly. MD allocates 75% to bonds and the allocation of the remaining 25% varies significantly. Prior to 2014, momentum and size have the largest allocations. After 2014, the remaining 25% is allocated to value.

BRP allocates the largest weights to volatility and dividend and they account which account for nearly 50% of the portfolio. The remaining half is allocated roughly equally to the other factor indices. DRP allocates roughly 30% to volatility, 25% to value, 20% to size, and 5–10% to the other factors. Bonds receive roughly 1%.

IV, IV2, ERC, BRP, and DRP have relatively stable allocations whereas ARP and MD show more variation. In addition, the portfolios do not show unusually large changes in allocation during the financial crisis in 2008–2009. IV2, ERC, ARP, and MD seem to overweight bonds and the other portfolios seem more balanced.

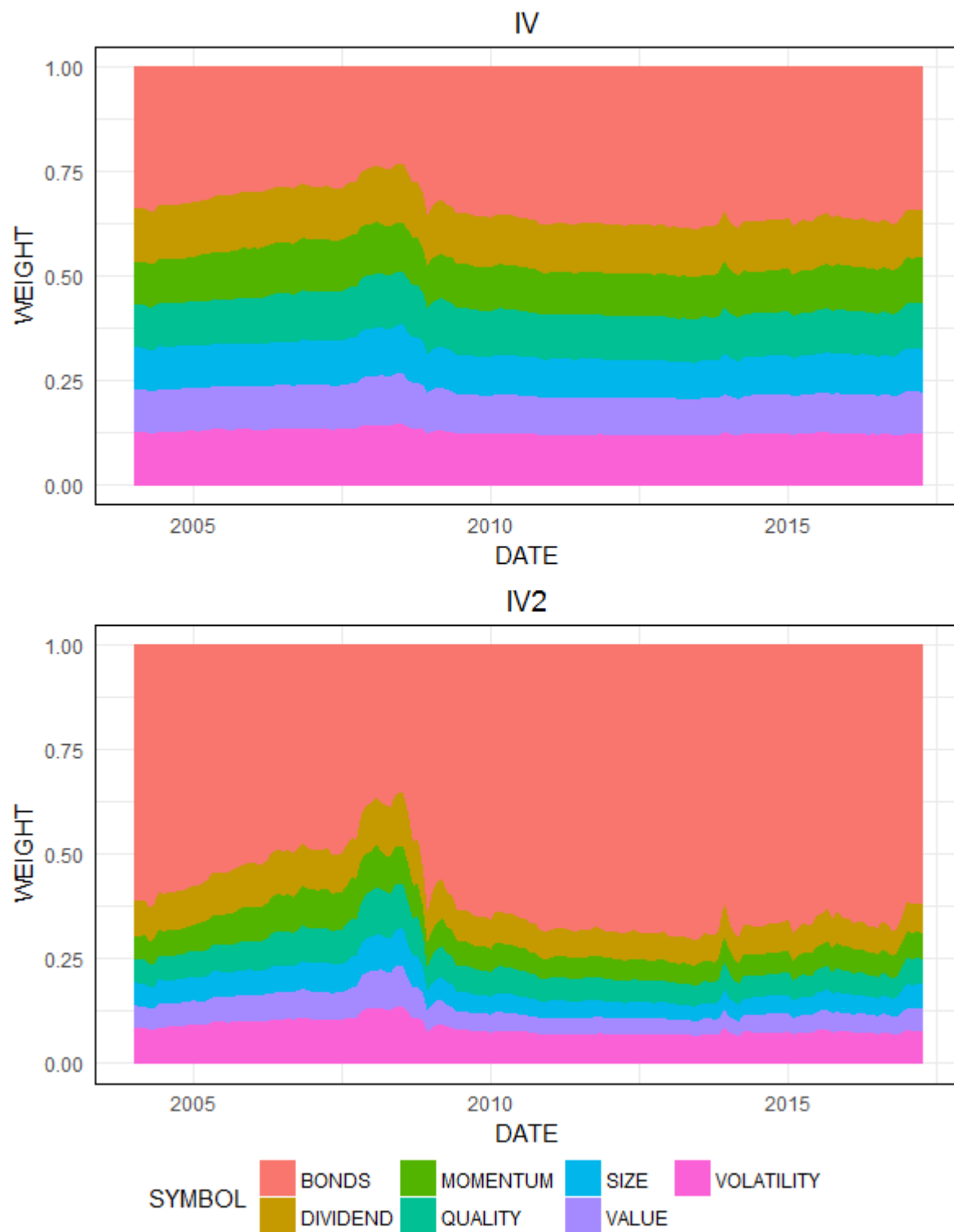


Figure 11. Time series of IV and IV2 portfolio weights, December 2003 to March 2017.



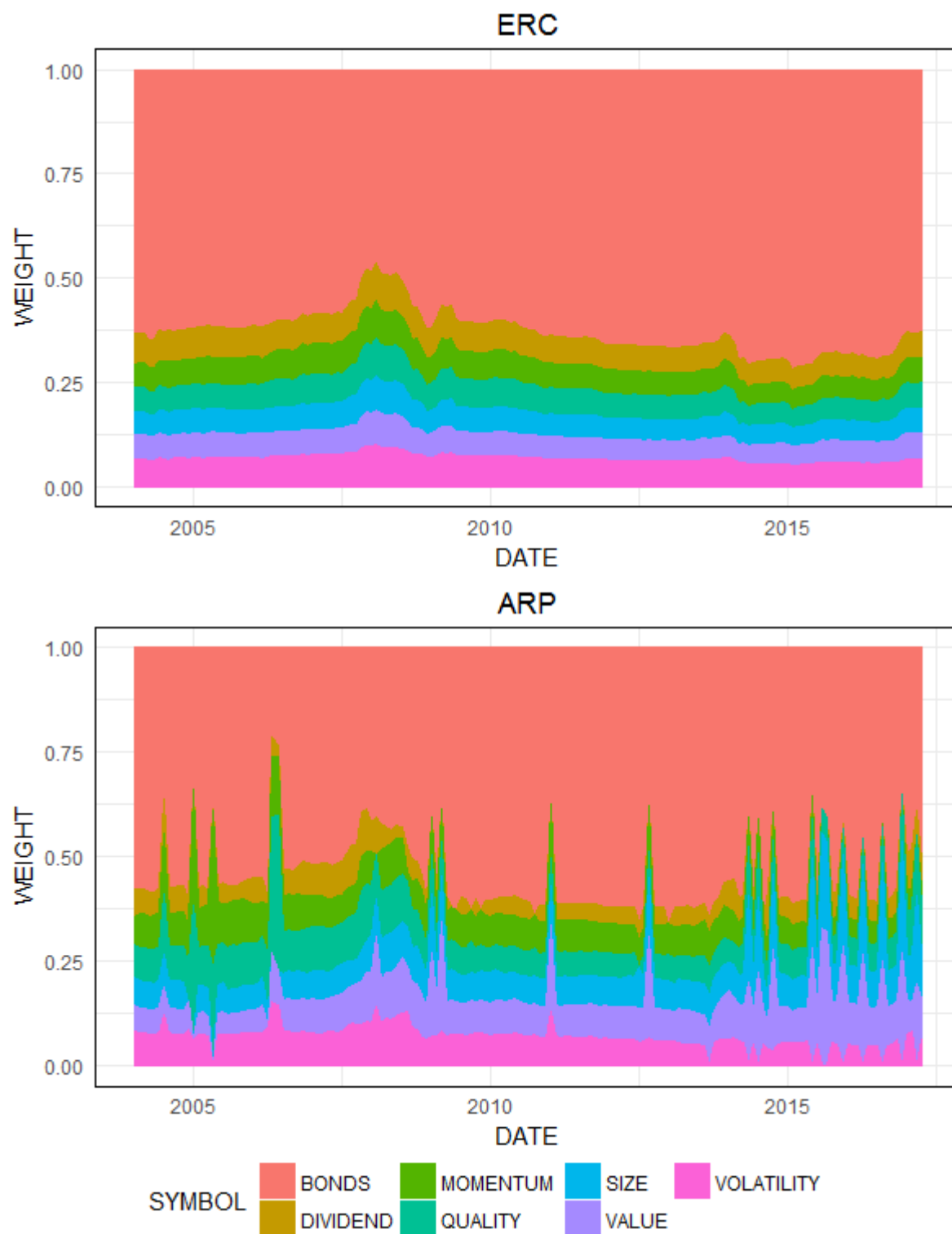
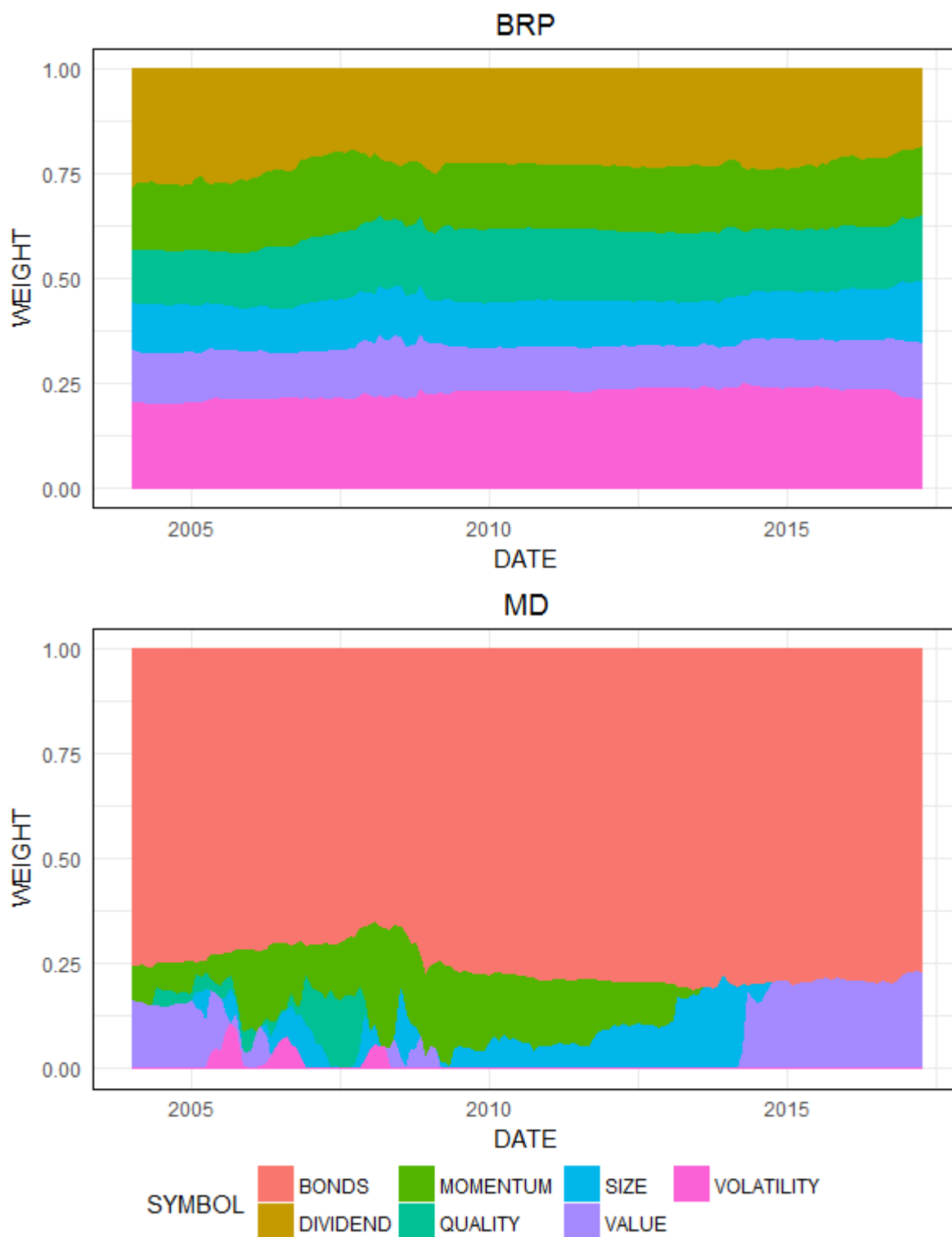
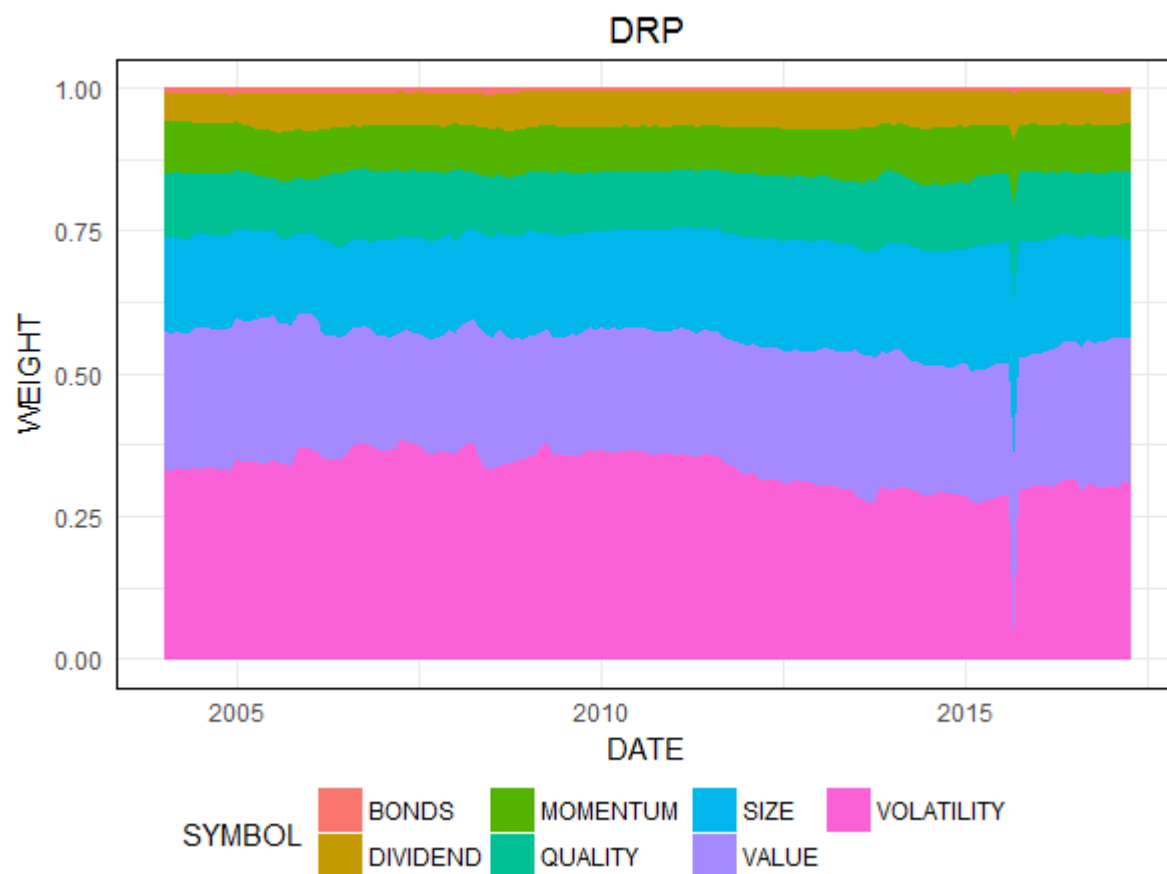


Figure 12. Time series of ERC and ARP portfolio weights, December 2003 to March 2017.



**Figure 13. Time series of BRP and MD portfolio weights, December 2003 to March 2017.**



**Figure 14. Time series of DRP portfolio weights, December 2003 to March 2017.**

#### **4.4 Robustness tests**

The robustness of the results is tested by constructing the portfolios with rolling 12-month, 36-month, and 100-month windows. The results of the robustness tests are discussed in this section. The results of the 36-month estimation are presented in this section (see appendices 1 and 2 for the detailed results of the 12-month and 100-month estimations).

The results of the robustness tests show that our results are not dependent on the estimation window length. The multi-factor portfolios are able to generate higher risk-adjusted returns in all cases and the results remain similar both qualitatively and quantitatively. BRP and DRP are able to outperform the benchmarks in terms of absolute returns and the other risk parity portfolios provide higher Sharpe ratios and lower drawdowns. In addition, the portfolios continue to outperform the single factor tilt indices.

**Table 8. Results of the robustness test using a rolling 36-month estimation window.**

	Risk Parity Portfolios							Benchmarks	
	IV	IV2	ERC	ARP	BRP	MD	DRP	EW	60/40
Return	6.09%	5.27%	5.24%	4.83%	6.80%	4.67%	6.70%	6.59%	5.64%
Excess return	4.86%	4.04%	4.01%	3.60%	5.58%	3.44%	5.47%	5.36%	4.41%
Volatility	8.46%	5.09%	4.96%	5.88%	13.43%	3.91%	13.66%	11.58%	8.20%
Sharpe	0.57	0.79	0.81	0.61	0.41	0.88	0.40	0.46	0.54
Information	−0.02	−0.08	−0.09	−0.14	0.34	−0.12	0.34	0.12	−0.08
Drawdown	−33.85%	−17.88%	−15.89%	−22.77%	−50.53%	−8.90%	−51.47%	−44.41%	−31.74%
Skewness	−1.01	−1.06	−0.91	−1.48	−0.97	−0.84	−0.99	−0.95	−0.90
Kurtosis	2.98	3.20	3.20	5.62	2.47	2.99	2.64	2.59	2.90
Normality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	−11.21%	−6.22%	−5.85%	−8.44%	−17.36%	−4.37%	−18.01%	−15.12%	−10.92%
Maximum	5.88%	4.22%	4.57%	4.40%	8.88%	3.43%	9.29%	7.89%	6.05%

This table presents the results of the robustness test where the portfolios are estimated using a rolling 36-month estimation window. All values are calculated from logarithmic returns. We have not considered transaction costs. The returns are the annual means and the volatilities are the annual standard deviations of the returns. The Sharpe ratios are calculated as the average annual return in excess of the risk-free rate divided by the annual standard deviation of the excess return. The Information ratios are calculated as the annual return in excess of the market divided by the standard deviation of the excess return. Drawdown measures the largest loss over the period as a percentage. Normality is the p-value of the Jarque-Bera test of normality with a null hypothesis of normality. Kurtosis is the excess kurtosis. The portfolios are constructed from the MSCI indices and the Citigroup U.S. Government Bonds Index. The market is represented by the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is December 2001 to March 2017.

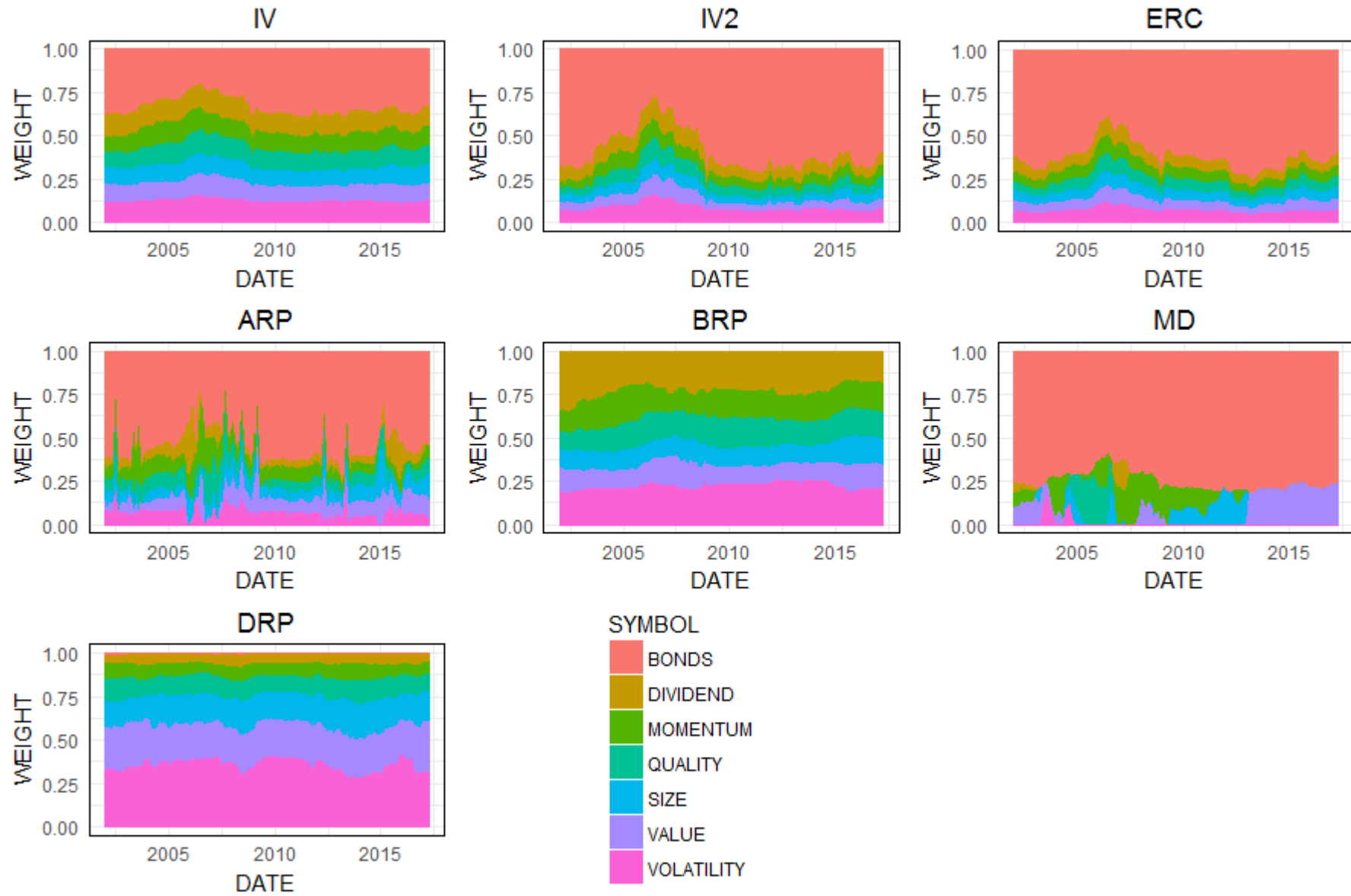


Figure 15. Portfolio weights estimated with a rolling 36-month window, December 2001 to March 2017.

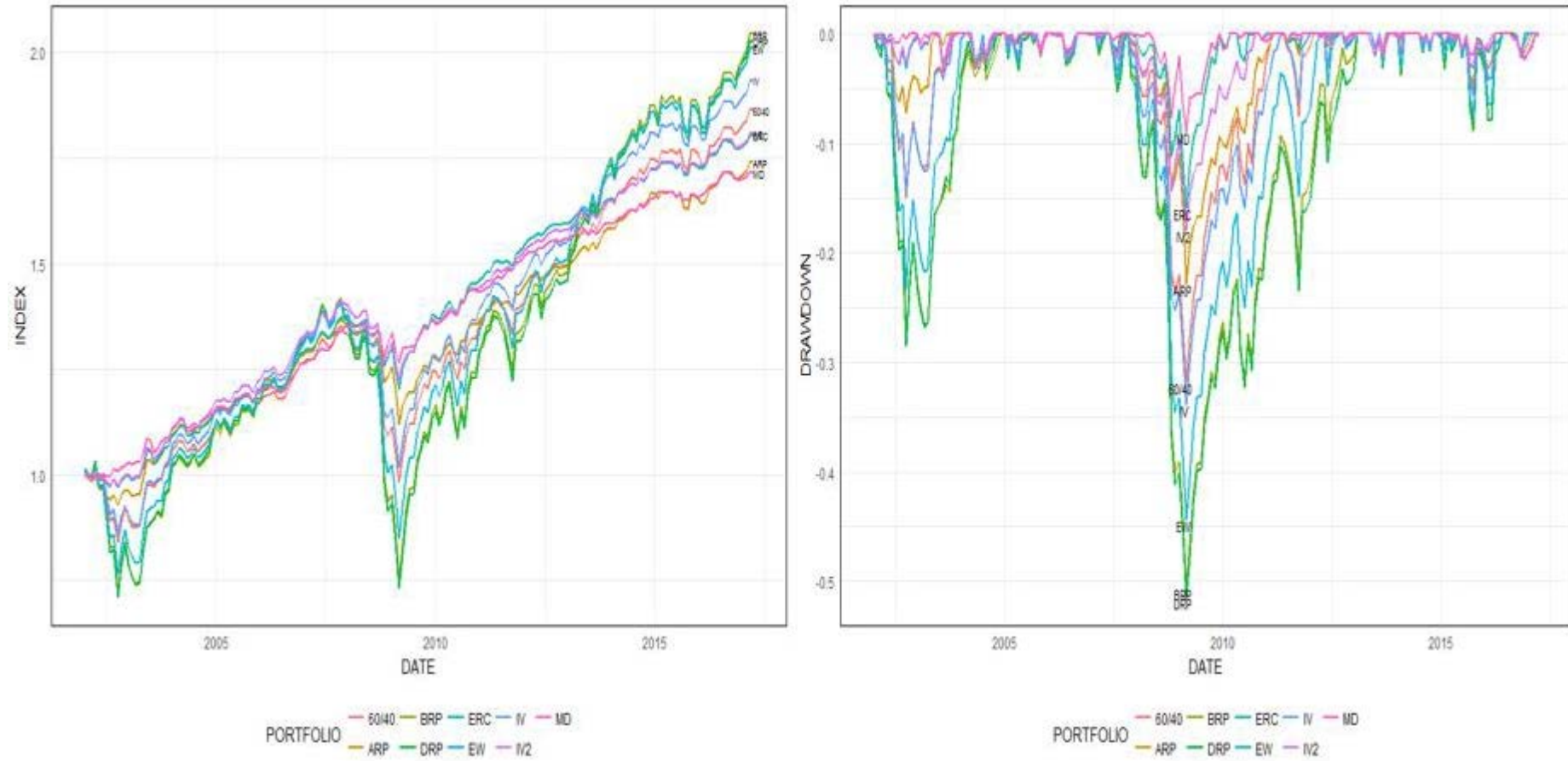


Figure 16. Performance of portfolios estimated with a rolling 36-month window, December 2001 to March 2017.

## 5 CONCLUSIONS

The purpose of this thesis is to analyze the performance and properties of multi-factor risk parity portfolios. We find that both single-factor tilt indices and multi-factor risk parity portfolios generate higher returns and exhibit lower volatilities and drawdowns than the market. Our results indicate that a simple long-only single-factor or multi-factor (tilt) strategy can be profitable for investors and it is one of the simplest ways to implement factor investing. Furthermore, our results show that we can generate higher risk-adjusted returns using risk parity methods to construct multi-factor portfolios.

Factor investing and risk parity portfolios offer broad possibilities to manage the risk exposures of investors and tailor the portfolios to specific needs. Factor portfolios can be used for tactical and strategic asset allocation, and risk management purposes. Factor-based solutions can also offer transparent and cost-effective investment vehicles for both individual and institutional investors.

Creating leverage through derivatives can reduce the risks associated with borrowing (Dudley 2011). Therefore, further research can analyze leveraged risk parity portfolios with the use of derivatives. Another area of interest is the effect of active management on the performance of risk parity portfolios. The performance of actively managed risk parity funds (e.g. pension funds, hedge funds) can be analyzed by comparing their performance to passive risk parity portfolios (e.g. factor tilt or multi-factor portfolios), such as those constructed for this thesis.

Even though factor tilt indices provide a practically implementable way to harvest factor premia, the MSCI factor tilt indices are not representative of the performance of pure factors, and therefore, the results are should not be generalized to those. Further research could analyze the efficiency of market-neutral (i.e. pure) factor portfolios. Another concern is the fact that most research on factors is done on historical data, and this may cause issues with data-mining. Finally, as with other investments, past factor performance may not represent future performance.



## REFERENCES

- Alvarez, M. (2011). Risk Parity and Risk-Based Allocation. *Deutsche Bank Equity Quantitative Strategy* 13 October 2011.
- Ang, A. (2014). *Asset Management: A Systematic Approach to Factor Investing*. New York: Oxford University Press.
- Ang, A., Chen, J. & Xing, Y. (2006). Downside Risk. *Review of Financial Studies* 19(4), 1191–1239.
- Ang, A., Hodrick, R. J., Xing, Y. & Zhang, X. (2006). The Cross-section of Volatility and Expected Returns. *Journal of Finance* 61(1), 259–99.
- Arnott, R. D. & Bernstein, P. L. (2002). What Risk Premium is ‘Normal’? *Financial Analysts Journal* 58(2), 64–85.
- Asness, C. S., Frazzini, A. & Pedersen, L. H. (2014). Quality Minus Junk. *CFA Digest* 44(1). CFA Institute.
- Asness, C. S., Moskowitz T. J. & Pedersen, L. H. (2013). Value and Momentum Everywhere. *Journal of Finance* 68(3), 929–985.
- Asness, C., Frazzini, A. & Pedersen L. (2012). Leverage Aversion and Risk Parity. *Financial Analysts Journal* 68(1), 47–59. CFA Institute.
- Ben-Ami, D. (2016). Some Rough with the Smooth. *Investment & Pensions Europe*. Special Report: Factor Investing, April 2016.
- Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31(3), 307–327.
- Boudt K., Carl P. & Peterson B. (2013). Asset Allocation with Conditional Value-at-Risk Budgets. *Journal of Risk* 15(3), 39–68.
- Bruder, B. & Roncalli, T. (2012). Managing Risk Exposures Using the Risk Budgeting Approach. Lyxor Asset Management: Paris.
- Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. *Journal of Finance* 52(1), 57–82.
- Chen, N. F., Roll, R. R. & Ross, S. A. (1986) Economic Forces and the Stock Market. *Journal of Business* 59(3), 383–404.
- Choueifaty, Y. & Coignard, Y. (2008). Toward Maximum Diversification. *The Journal of Portfolio Management* 35(1), 40–51.
- Dudley, A. (2011). Risk Parity Primer. *Investment & Pensions Europe*. Special Report: Risk Parity, April 2011.

- Engle, R. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. *Journal of Business & Economic Statistics* 20(3), 339–350.
- Fama, E. F. & French, K. R. (1988). Dividend Yields and Expected Stock Returns. *Journal of Financial Economics* 22(1), 3–25.
- Fama, E. F. & French, K. R. (1995). Size and Book-to-Market Factors in Earnings and Returns. *Journal of Finance* 50(1), 131–155.
- Fama, E. F. & French, K. R. (1996). Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance* 51(1), 55–84.
- Fama, E. F. & French, K. R. (2010). Luck versus Skill in the Cross-Section of Mutual Fund Returns. *Journal of Finance* 65(5), 1915–1947.
- Fama, E. F. & French, K. R. (2016). Dissecting Anomalies with a Five-Factor Model. *Review of Financial Studies* 29(1), 69–103.
- Fama, E. F. & MacBeth, J. D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 81(3), 607–636.
- Fung, W. & Hsieh, D. (2004). Hedge Fund Benchmarks: A Risk-Based Approach. *Financial Analysts Journal* 60(5), 65–80.
- Goodwin, T. H. (1998). The Information Ratio. *Financial Analysts Journal* 54(4), 34–43.
- Gosh, S. (2011). Risk Parity and Portfolio Design. *Investment & Pensions Europe*. Special Report: Risk Parity, April 2011.
- Hudson, R. S. & Gregoriou, A. (2015). Calculating and Comparing Security Returns Is Harder Than You Think: A Comparison Between Logarithmic and Simple Returns. *International Review of Financial Analysis* 38(2), 151–162.
- Hurn, S., Martin, V., Phillips, P. & Yu, J. (2015). Financial Econometric Modelling. Working document, November 2015.
- Ilmanen, A. & Palazzolo C. (2014). Fewer Forecasts; More Diversification. *Investment & Pensions Europe*. Special Report: Risk and Portfolio Construction, June 2014.
- Jegadeesh, N. & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48(1), 65–91.
- Joenväärä, J., Kosowski, R. & Tolonen, P. (2016). Hedge Fund Performance: What Do We Know? Working paper, June 2016. (Accessed 5.4.2017) Available at: [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1989410](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1989410)

- Kahra, H. (2016). Portfolio Management (lecture). Risk Parity Portfolios. Oulu Business School. Oulu, Finland.
- Koedijk, K. G., Slager, A. M.H. & Stork, P. A. (2016). Investing in Systematic Factor Premiums. *European Financial Management* 22(2), 193–234.
- Koijen, R. S. J., Moskowitz, T. J., Pedersen, L. H. & Vrugt, E. B. (2013). Carry. NBER Working paper, August 2013.
- Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics* 47(1), 13–37.
- Lohre, H., Opfer, H. & Orszag, G. (2014). Diversifying Risk Parity. *Journal of Risk* 16(5), 53–79.
- Maillard S., Roncalli, T. & Teiletche, J. (2010). The Properties of Equally Weighted Risk Contributions Portfolios. *Journal of Portfolio Management* 36(4), 60–70.
- Mariathasan, J. (2011). Risk Parity: Nice Idea, Awkward Reality. *Investment & Pensions Europe*. Special Report: Risk Parity, April 2011.
- Markowitz, H. (1952). Portfolio selection. *Journal of Finance* 7(1), 77–91.
- Mesomeris, S., Wang, Y., Salvini, M. & Avettand-Fenoel, J.R. (2012). A New Asset Allocation Paradigm. *Deutsche Bank Quantitative Strategy Europe*, Thematic Report 5 July 2012.
- Meucci, A. (2009). Managing diversification. *Risk* 22, 74–79.
- Moore, C. (2016). At the Core of Smart Beta. *Investment & Pensions Europe*. Special Report: Factor Investing, April 2016.
- Moreolo, C. S. (2016). Calculated Risks. *Investment & Pensions Europe*. Special Report: Factor Investing, April 2016.
- Mossin, J. (1966). Equilibrium in a Capital Asset Market. *Econometrica* 34(4), 768–783.
- Pástor, L. & Stambaugh, R. F. (2003). Liquidity Risk and Expected Stock Returns. *Journal of Political Economy* 111(3), 642–685.
- Qian, E. (2005). Risk Parity Portfolios: Efficient Portfolios Through True Diversification. Research paper, PanAgora Asset Management, September 2005.
- Roncalli T. & Weisang G. (2016). Risk Parity Portfolios with Risk Factors. *Quantitative Finance* 16(3), 377–388.
- Roncalli, T. (2014). Introducing Expected Returns into Risk Parity Portfolios: A New Framework for Asset Allocation. Working paper, April 2014.

- Ross, S. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory* 13(3), 341–360.
- Sharpe, W. (1964). Capital Asset Prices: A Theory of Market Equilibrium. *Journal of Finance* 19(3), 425–442.
- Sharpe, W. (1994). The Sharpe Ratio. *Journal of Portfolio Management* 21(1), 49–58.
- Turner, D. (2014). From Solo to Tango. *Investment & Pensions Europe*. Special Report: Risk and Portfolio Construction, June 2014.

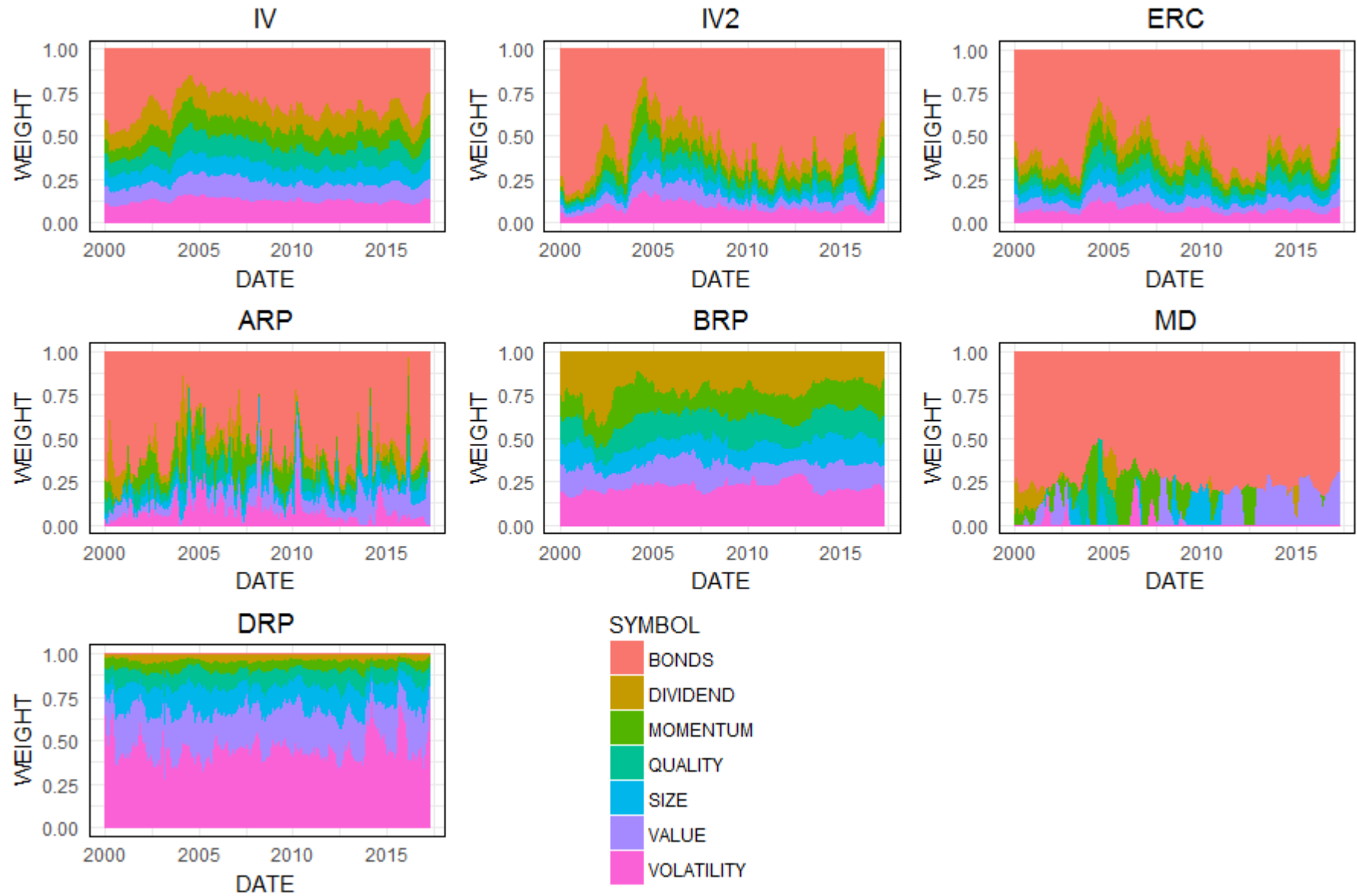
## APPENDICES

### 5.1 Appendix 1 Results of the robustness test using a rolling 12-month estimation window.

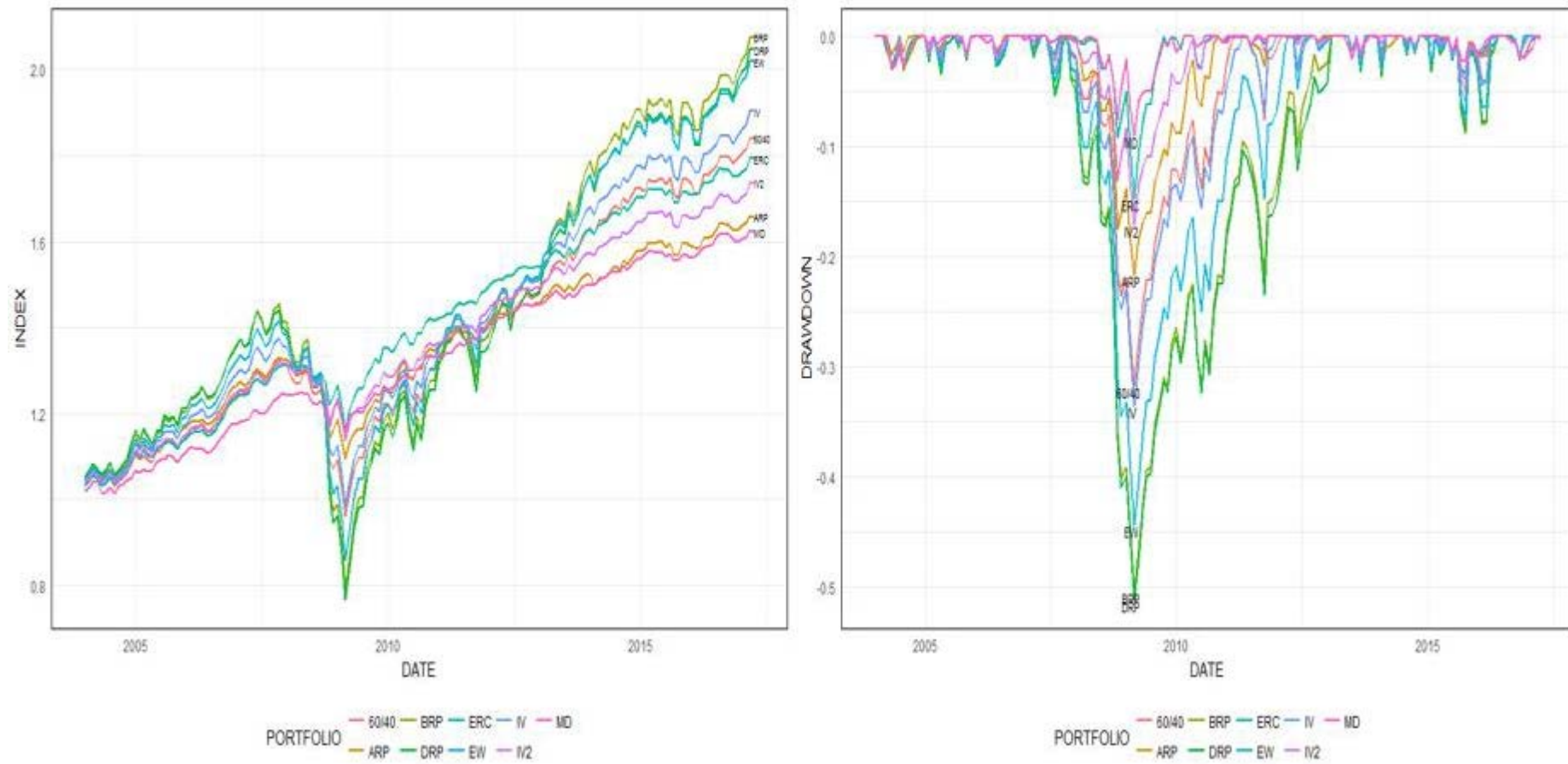
#### Results of the robustness test using a rolling 12-month estimation window.

	Risk Parity Portfolios							Benchmarks	
	IV	IV2	ERC	ARP	BRP	MD	DRP	EW	60/40
Return	6.77%	5.53%	5.94%	4.93%	8.07%	4.67%	7.84%	7.64%	6.31%
Excess return	5.56%	4.33%	4.74%	3.73%	6.87%	3.46%	6.64%	6.44%	5.10%
Volatility	8.47%	5.43%	5.16%	5.84%	12.95%	3.92%	13.09%	11.23%	8.00%
Sharpe	0.65	0.79	0.92	0.64	0.53	0.88	0.51	0.57	0.64
Information	−0.13	−0.20	−0.16	−0.28	0.31	−0.24	0.19	0.03	−0.20
Drawdown	−33.57%	−17.16%	−14.74%	−21.63%	−50.54%	−9.03%	−51.29%	−44.41%	−31.74%
Skewness	−1.12	−1.05	−0.94	−1.72	−1.07	−0.93	−1.10	−1.07	−1.07
Kurtosis	3.55	2.94	3.14	8.15	3.15	2.91	3.50	3.39	3.79
Normality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	−11.21%	−6.25%	−5.37%	−9.82%	−17.25%	−4.36%	−17.96%	−15.12%	−10.92%
Maximum	6.04%	4.43%	4.86%	4.15%	8.74%	3.57%	9.25%	7.89%	6.05%

This table presents the results of the robustness test where the portfolios are estimated using a rolling 12-month estimation window. All values are calculated from logarithmic returns. We have not considered transaction costs. The returns are the annual means and the volatilities are the annual standard deviations of the returns. The Sharpe ratios are calculated as the average annual return in excess of the risk-free rate divided by the annual standard deviation of the excess return. The Information ratios are calculated as the annual return in excess of the market divided by the standard deviation of the excess return. Drawdown measures the largest loss over the period as a percentage. Normality is the p-value of the Jarque-Bera test of normality with a null hypothesis of normality. Kurtosis is the excess kurtosis. The portfolios are constructed from the MSCI indices and the Citigroup U.S. Government Bonds Index. The market is represented by the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is December 1999 to March 2017.



Portfolio weights estimated with a rolling 12-month window, December 1999 to March 2017.



Performance of portfolios estimated with a rolling 12-month window, December 1999 to March 2017.

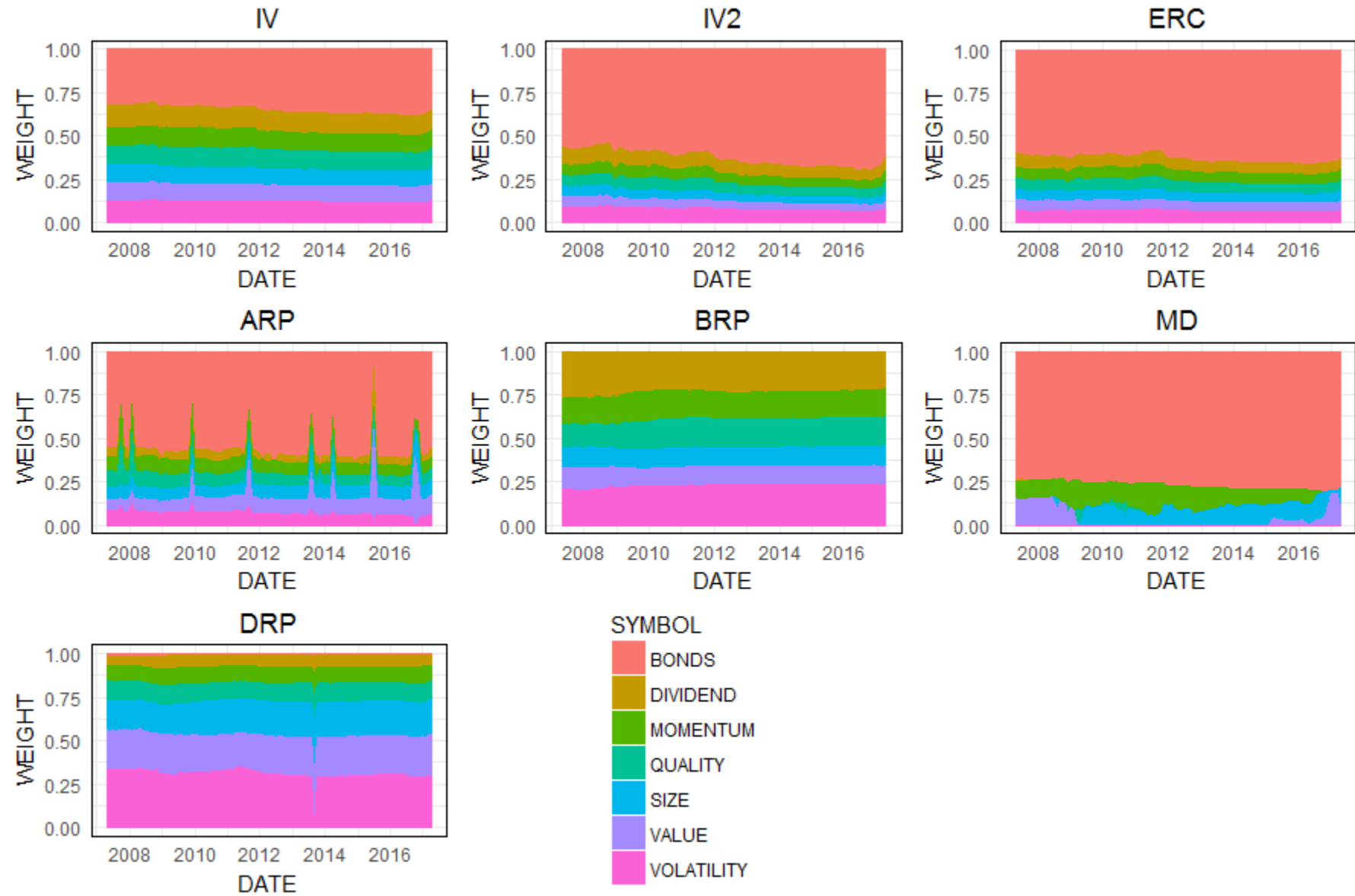
## 5.2 Appendix 2 Results of the robustness test using a rolling 100-month estimation window.

**Results of the robustness test using a rolling 100-month estimation window.**

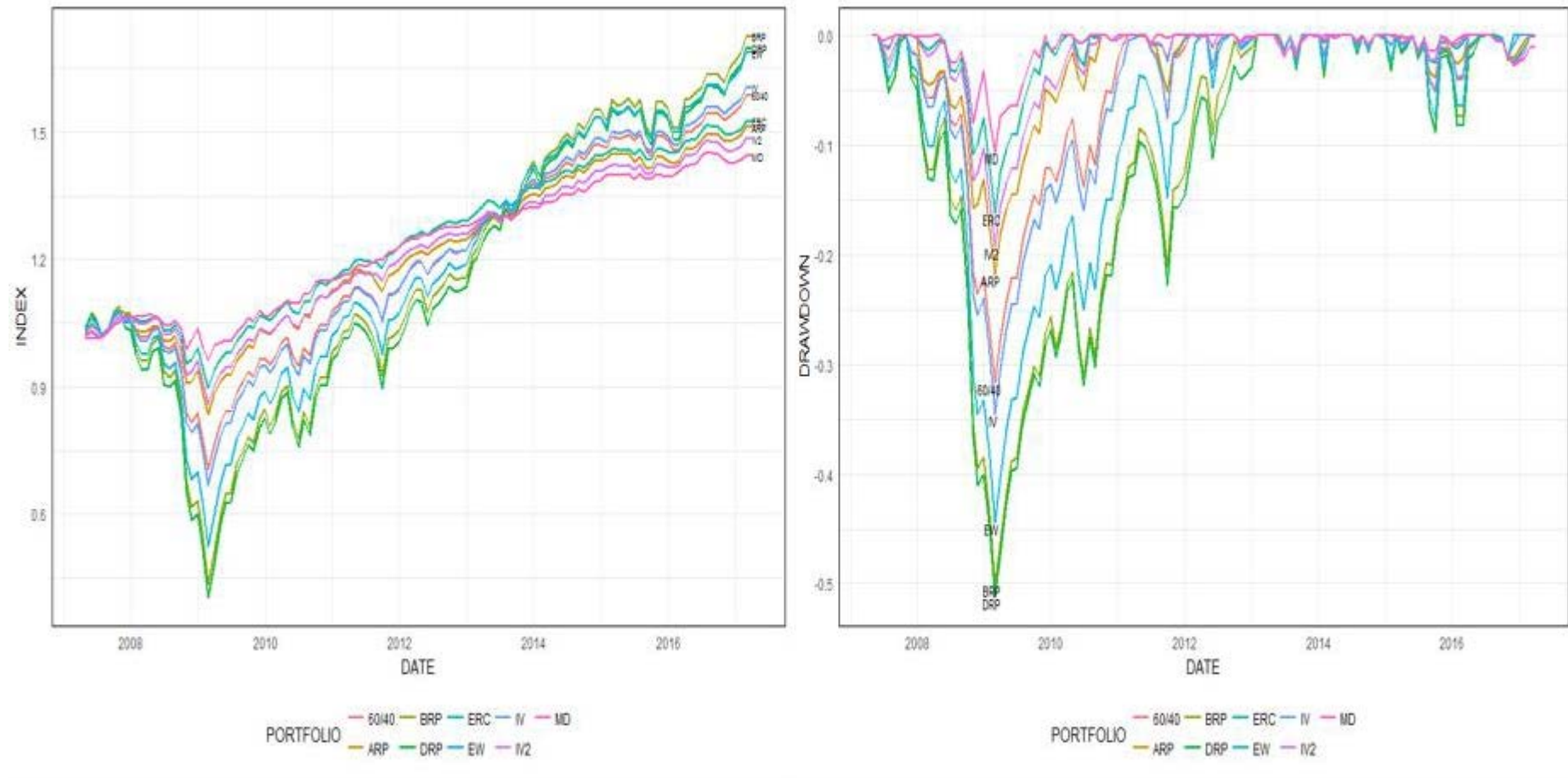
	Risk Parity Portfolios							Benchmarks	
	IV	IV2	ERC	ARP	BRP	MD	DRP	EW	60/40
Return	6.05%	4.83%	5.24%	5.12%	7.26%	4.43%	6.98%	6.86%	5.86%
Excess return	5.46%	4.24%	4.66%	4.53%	6.68%	3.85%	6.39%	6.28%	5.28%
Volatility	9.42%	5.71%	5.40%	6.36%	14.42%	4.15%	14.69%	12.52%	8.88%
Sharpe	0.58	0.74	0.86	0.71	0.46	0.93	0.43	0.50	0.59
Information	-0.09	-0.17	-0.13	-0.15	0.34	-0.17	0.23	0.06	-0.11
Drawdown	-34.51%	-19.30%	-16.20%	-21.77%	-50.06%	-10.56%	-51.32%	-44.41%	-31.74%
Skewness	-1.11	-1.42	-1.14	-1.04	-0.98	-1.17	-0.97	-0.98	-1.00
Kurtosis	3.24	5.12	3.81	3.09	2.24	4.18	2.34	2.43	2.87
Normality	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	-11.80%	-7.45%	-6.38%	-7.53%	-17.23%	-4.66%	-17.79%	-15.12%	-10.92%
Maximum	6.03%	4.57%	4.44%	4.63%	8.90%	3.58%	9.23%	7.89%	6.05%

This table presents the results of the robustness test where the portfolios are estimated using a rolling 100-month estimation window. All values are calculated from logarithmic returns. We have not considered transaction costs. The returns are the annual means and the volatilities are the annual standard deviations of the returns. The Sharpe ratios are calculated as the average annual return in excess of the risk-free rate divided by the annual standard deviation of the excess return. The Information ratios are calculated as the annual return in excess of the market divided by the standard deviation of the excess return. Drawdown measures the largest loss over the period as a percentage. Normality is the p-value of the Jarque-Bera test of normality with a null hypothesis of normality. Kurtosis is the excess kurtosis. The portfolios are constructed from the MSCI indices and the Citigroup U.S. Government Bonds Index. The market is represented by the MSCI USA equity index. The risk-free rate is the 3-month treasury bill secondary market rate from the FRED. The period is April 2007 to March 2017.





Portfolio weights estimated with a rolling 100-month window, April 2007 to March 2017.



Performance of portfolios estimated with a rolling 100-month window, April 2007 to March 2017.