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1 #####
2 ###Code for figure 4.2
3 ###Generating m paths of the Ornstein-Uhlenbeck process (OU)
4 ###on the time intervall [0,T]
5 ###SDE_SamplingOU.py
6 ###Python 2.7
7 #####
8
9 import numpy as np
10 import numpy.matlib
11 import matplotlib.pyplot as plt
12 from NumericalSDE import *
13
14 #####
15 ### Ornstein-Uhlenbeck process (OU)
16 ###  $dX_t = a(X_t)dt + b(X_t)dW_t$ 
17 ###  $X_0 = x_0$ 
18 ###  $a(x) = -\text{beta} \cdot x$ ,  $b(x) = \text{sigma}$ 
19 ### beta, sigma positive constants
20 ### True solution:
21 ###
22 #####
23 #Parameter
24 sigma = 1.5
25 beta = 1.0
26 #starting value x0
27 x0 = 1
28 #Parameters for the discretization
29 n = 2**8
30 t = timegrid(n)
31 #m discretized Wiener processes
32 m = 5
33 w = np.zeros((n+1,m))
34 for k in range(0,m):
35     w[:,k] = wiener(n)
36 #m sample paths
37 Xt = np.zeros((n+1,m))
38 stochIntApprox = np.zeros((n+1,m))
39 temp = np.zeros(n+1)
40 for s in range(0,m):
41     Xt[0,s] = x0
42     for k in range(0,n):
43         for j in range(0,k):
44             temp[j+1] = (w[j+1,s]-w[j,s])*np.exp((-beta)*((t[k+1])-t[j]))
45             stochIntApprox[k+1,s] = np.sum(temp)
46             temp = np.zeros(n)
47             Xt[k+1,s] = Xt[0,s]*np.exp((-beta)*(t[k+1])) + sigma*stochIntApprox[k+1,s]
48
49
50 #The code below is needed to test if the approximated stoch. integral is
51 #a good approximation or not (can be removed)
52 ##def a(x):
53 ##    return -beta*x
54 ##def b(x):
55 ##    return sigma
56 ##Yt = sde_euler(x0, a, b, w[:,1])
57 ##plt.scatter(t, Yt, 1, c='b')
58
59 #Plot
60 for sample_path in Xt.T:
61     plt.plot(t, sample_path,'r',linewidth=0.5)
62 plt.xlabel('t', fontsize=16)
63 plt.ylabel('x', fontsize=16)
64 plt.show()
65

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