Optimizing the trade-off between fuel consumption and travel time in an unsignalized autonomous intersection crossing

Andreas Hadjigeorgiou and Stelios Timotheou

***Abstract*— Connected and autonomous vehicles (CAVs) have the potential to disrupt road transportation. CAVs provide several attractive features, such as seamless connectivity and fine-grained control, which can be exploited to improve the efficiency of traffic networks. In this work, the problem of CAV coordination at an unsignalized intersection crossing is considered, aiming to select the CAV trajectories that minimize fuel consumption and/or travel time. Nonetheless, the minimization of travel time implies high fuel consumption and vice-versa. For this reason, this work considers the problem of simultaneously optimizing the fuel consumption-travel time trade-off for a set of CAVs that are expected to arrive at the intersection within a specific time-window. As the resulting problem is non-convex, we construct a Mixed-Integer Programming formulation that provides tight lower and upper bounds. We also develop a heuristic convex-concave procedure that yields fast, high-quality solutions. Simulation results validate the effectiveness of the proposed approaches and highlight the importance of optimizing the fuel consumption-travel time trade-off, as small compromises in travel time produce significant fuel savings.**

I. Introduction

Urbanization and the steady increase of vehicles are pushing transportation to its limits, resulting in congestion. This is a major problem as it leads to inefficient use of the transportation network causing drivers to spend more time travelling and increasing overall fuel consumption [1].

Among the major bottlenecks that lead to congestion are road intersections where different flows intersect. Traditionally, traffic lights are used to regulate flows that pass from an intersection to maximize some metric of interest (throughput, travel time, fuel consumption). Although a vast amount of literature exists utilizing different methodologies for traffic light control such as optimization [2], the intersection crossing is still inefficient as it results in stop-and-go behavior.

Advancements in information and communication technologies in different areas such as electronics, sensing, control, machine learning and communications is enabling new capabilities that can be exploited to significantly improve intersection crossing. The most promising is the technology of Connected and Autonomous Vehicles (CAVs) which offers full control of individual vehicles, as well as Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communications [3]. These powerful capabilities can be exploited to achieve *autonomous intersection crossing* where vehicles co-operate to pass an unsignalized intersection without stopping in order to minimize some objective function.

Recently, the body of literature around the coordination of CAVs at intersections and on-ramps is rapidly increasing for both centralized and decentralized approaches [4]. In centralized approaches the control input is decided for all vehicles by a single central controller. In decentralized approaches each autonomous vehicle decides the trajectory to follow based on information obtained from other vehicles and the infrastructure.

A popular centralized heuristic method uses the concept of reservations, where the intersection is divided into cells that are assigned to a specific vehicle for a certain amount of time [5]. This is a rule-based method which is simple to implement and computationally fast; nonetheless, it does not utilize rigorous mathematical tools to optimize performance and does not account for the vehicle dynamics. Other common centralized approaches utilize optimization and control where a certain objective is chosen to be minimized, such as throughput [6] and total travel time [7]. Although these approaches offer a higher degree of realism, they often result in combinatorial optimization problems which are computationally expensive to solve, so they can only be used to handle problems involving a small number of vehicles.

Decentralized methods often use heuristic control measures to minimize vehicle delay such as fuzzy logic [8]. Others employ model predictive control (MPC) [9] or optimal control [10] that explicitly consider vehicle dynamics to solve an optimization problem that optimizes a performance criterion, e.g. vehicle delay, throughput, fuel consumption and passenger discomfort. Although decentralized methods often result in computationally cheap schemes, they often yield suboptimal results as they optimize performance on a vehicle-to-vehicle basis.

This paper considers the problem of autonomous vehicle unsignalized intersection crossing from a centralized perspective. Contrary to other centralised approaches that mostly focus on throughput and total travel time minimization, this work focuses on the trade-off between fuel consumption and total travel time minimization for a specific set of vehicles that arrive at the intersection within a specific time-window. Controlling this *fuel consumption/travel time trade-off* is important because it allows the system operator to set different fuel consumption/travel time points of operation depending on the overall traffic conditions.

The contributions of this work are the following: *(i)* Con-

This work has been supported by the Research Promotion Foundation (Project: CULTURE/BR-NE/0517/14), the European Union's Horizon 2020 research and innovation programme under grant agreement No 739551 (KIOS CoE) and the Government of the Republic of Cyprus through the Directorate General for European Programmes, Coordination and Development.

The authors are with the Department of Electrical and Computer Engineering and the KIOS Research and Innovation Center of Excellence, University of Cyprus {hadjigeorgiou.g.andreas, timotheou.stelios}@ucy.ac.cy.

struction of a mathematical formulation that considers the fuel consumption/travel time trade-off optimization subject to the vehicle dynamics, as well as lateral and rear-end collisions. *(ii)* As the resulting problem is non-convex due to the presence of optimization variables in the integration limits, we device a novel decomposition of the problem into two subproblems. *(iii)* We develop two mathematical programming approaches for the solution of the first subproblem (nonconvex). First, we formulate a Mixed-Integer Program (MIP) for benchmarking purposes. Second, we develop an iterative heuristic convex-concave procedure that yields fast, high-quality solutions.

The remainder of this paper is organized as follows. Section II formulates the problem of CAV coordination at an intersection for optimization of the fuel consumption/travel time trade-off. Section III develops the two methods for solving the problem using mathematical programming techniques. Section IV presents and discusses the main simulation results. Concluding remarks are given in Section V.

II. Problem Formulation

In this work an intersection with two lanes per road is considered, as illustrated in Fig. 1. The area around the intersection is referred to as *Control Zone* (CZ) and extends L distance from the intersection boundary, while the center of the intersection is called *Merging Zone* (MZ). The length of the MZ is denoted by S and assumed to be $S < L$. The intersection is equipped with an *Intersection Controller* (IC) that is responsible for deriving and communicating the exact trajectory of each vehicle inside the control zone. Each CAV i that wishes to cross the intersection sends a message to the IC specifying its expected arrival time at the CZ, t_i^0, velocity, v_i^0 and direction of movement $d_i \in \mathcal{D}$. For simplicity it is assumed that vehicles do not turn so that the only directions of movement allowed are North-South (NS), South-North (SN), East-West (EW) and West-East (WE). Every T time units, the IC computes the CAV trajectories that minimize a certain objective in relation to the total fuel consumption and the total travel time for all available requests. For this optimization procedure, the IC follows a First-In-First-Out (FIFO) policy such that when vehicle i enters the CZ prior to vehicle j, i.e, $t_i^0 + t_i^m < t_j^0 + t_j^m$ and $t_i^0 + t_i^f < t_j^0 + t_j^f$, where t_i^m and t_i^f denote the required travel time of CAV i to arrive and exit the MZ. For notational convenience we consider that when $t_i^0 < t_j^0$ is also true that $i < j$. Also, we denote N as the total number of arrivals incurring within a time-window of length T, i.e, $i \in \mathcal{N} = \{1, \dots, N\}$.

Assumption 1: CAVs communicate their exact arrival time and velocity at the CZ without any errors or delays, T time units prior to their arrival.

CAVs follow second order dynamics such that,

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \quad x_i(t_i^0) = 0 \\ \dot{v}_i(t) &= u_i(t), \quad v_i(t_i^0) = v_i^0, \end{aligned} \tag{1}$$

where $x_i(t)$, $v_i(t)$ and $u_i(t)$ are the position, velocity and acceleration of CAV i at time t, with $u_i(t)$ being the control input. To ensure that the velocity and acceleration are within

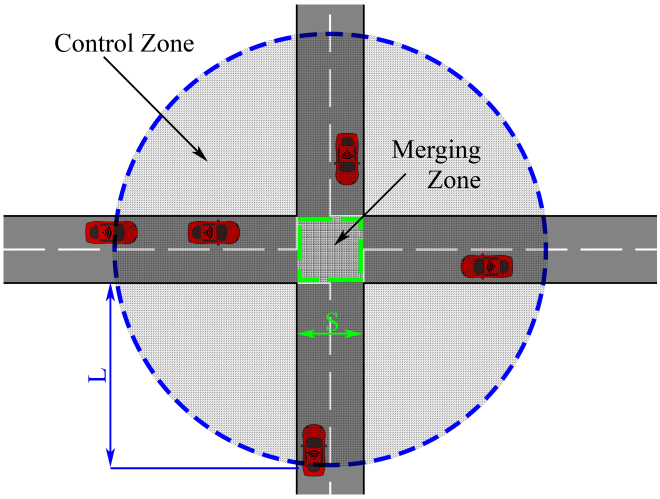

Fig. 1: Illustration of the intersection

physical or imposed limits (e.g. infrastructure requirements), the following constraints are imposed:

$$\begin{aligned} &0 \leq v_i(t) \leq v_i^{\max} \quad \text{and} \\ &u_i^{\min} \leq u_i(t) \leq u^{\max} \quad \forall t \in [t_i^0, t_i^0 + t_i^m]. \end{aligned} \tag{2}$$

For the safe crossing of all vehicles, lateral and rear-end collisions must be avoided at all times. Towards this direction, the following assumption is made.

Assumption 2: Each CAV $i \in \mathcal{N}$ arrives at the MZ at a predefined velocity $v_i^m = v^m$ and moves with constant velocity inside the MZ:

$$v_i(t) = v^m, \quad \forall t \in [t_i^0 + t_i^m, t_i^0 + t_i^f], \quad \forall i \in \mathcal{N}. \tag{3}$$

To ensure rear-end collision avoidance, all vehicles traveling in the same direction of movement are enforced to maintain a safe distance δ at all times such that

$$x_k(t) - x_i(t) \geq \delta, \quad \forall t \in [t_i^0, t_i^0 + t_i^f], \tag{4}$$

where k denotes the index of the vehicle in front of CAV i in the same direction of movement. Based on (3) and (4) the following constraint also holds:

$$t_k^0 + t_k^m + \frac{\delta}{v^m} \leq t_i^0 + t_i^m. \tag{5}$$

In order to avoid lateral collisions inside the MZ, the merging time of CAVs j and i, $i < j$ travelling in perpendicular directions (i.e. NS and EW) must adhere to the constraint

$$t_i^0 + t_i^m + \frac{S}{v^m} \leq t_j^0 + t_j^m, \tag{6}$$

which ensures that vehicle j enters the MZ only after vehicle i has exited the MZ.

Assumption 3: CAVs arriving at the CZ have none of the constraints (2), (4) violated at t_i^0.

In this work, the problem of selecting the control input, $u_i(t)$, and travel time, t_i^m, for each of N CAVs arriving at the CZ within window of T time-units is considered, in order to optimize the total fuel consumption subject to a desired total travel time and safety constraints.To optimize the travel time-fuel consumption trade-off, the approach taken is to consider the total fuel consumption as the objective and the total travel time as a constraint. The desired total travel time can be introduced considering a constraint of the form

$$\sum_{i=1}^{N} t_i^m \leq \gamma \sum_{i=1}^{N} t_i^{\min}, \tag{7}$$

where $\gamma \geq 1$ and $t_i^{\min}$ is the minimum feasible travel time CAV i can reach the intersection considering (5) and (6). The value $\gamma = 1$ minimizes the total travel time but maximizes the fuel consumption, while for increasing γ the total travel time of vehicles increases and the total fuel consumption decreases as vehicles choose from a larger family of trajectories. For the fuel consumption of CAV i, $J(u_i)$, we have that

$$J(u_i) = \int_{t_i^0}^{t_i^m + t_i^0} u_i^2(t)\, \mathrm{d}t, \tag{8}$$

which is the L^2 norm of the control input that minimizes the transient engine operation. Although this is not an accurate fuel consumption model, it has been used by several other studies, including [10], [11], as a simple yet powerful model, because minimizing acceleration is monotonically related to minimizing fuel consumption [12]. By considering all vehicles the following optimization problem can be formulated:

$$\begin{aligned} \underset{\mathbf{u}}{\text{minimize}} \quad & \sum_{i=1}^{N} \int_{t_i^0}^{t_i^m + t_i^0} u_i^2(t)\, \mathrm{d}t \\ \text{subject to:} \quad & (1)(2)(3)(4)(6)(7), \end{aligned} \tag{9}$$

where $\mathbf{u} = (u_1, ..., u_N)$.

III. Solution Approach

To tackle optimization problem (9), we first discretize the vehicle dynamics equations (1) using Euler's Forward method yielding

$$\begin{aligned} x_i(t + \Delta T) &= x_i(t) + \Delta T \;\; v_i(t), \quad x_i(t_i^0) = 0 \\ v_i(t + \Delta T) &= v_i(t) + \Delta T \;\; u_i(t), \quad v_i(t_i^0) = v_i^0, \end{aligned} \tag{10}$$

where ΔT is the time-step considered for the discretization. In this way, the following discretized version of (9) is obtained:

$$\begin{aligned} \underset{\mathbf{u}}{\text{minimize}} \; & \sum_{i=1}^{N} \sum_{j=0}^{H_i+1} u_i^2(t_i^0 + j\,\Delta T) \\ \text{subject to:} \; & (10)(2)(3)(4)(6)(7), \end{aligned} \tag{11}$$

where $H_i = \lceil t_i^m / \Delta T \rceil$. Similar to the continuous-time case, H_i is a variable and hence problem (11) is hard to solve. In order to deal with this problem, we develop a three-phase approximation procedure:

Phase 1: Solve the fuel minimization problem of individual vehicles for a range of fixed t_i^m to obtain a minimum fuel consumption profile as a function of t_i^m, $\tilde{J}_i(t_i^m)$.

Phase 2: Use $\tilde{J}_i(t_i^m)$ to solve a relaxed version of problem (11) with no rear-end collision constraints inside the CZ to derive close to optimal values for $\tilde{t}_i^m$.

Phase 3: Assuming constant H_i ($H_i = \lceil \tilde{t}_i^m / \Delta T \rceil$) solve one fuel minimization problem for each direction-of-movement with the consideration of rear-end collisions to obtain a feasible solution to problem (11).

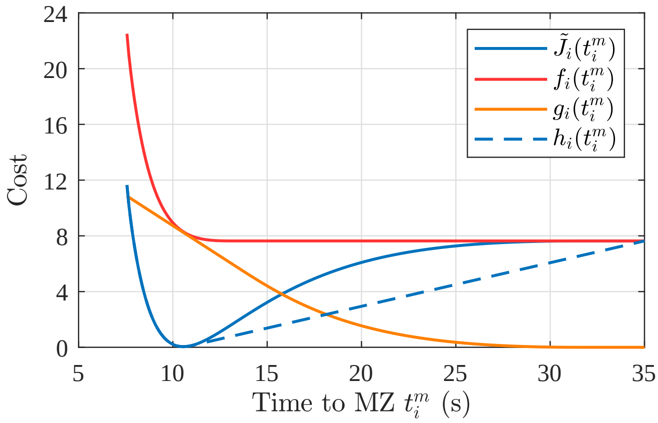

Fig. 2: Example of the cost function, $\tilde{J}_i(t_i^m)$, its decomposition into the difference of two convex functions $f_i(t)$ and $g_i(t)$, and the lower convex envelope, $h_i(t_i^m)$.

A. Phase 1

Phase 1 involves the solution of a series of optimization problems with fixed H_i in order to define a minimum fuel consumption profile , $\tilde{J}_i(t_i^m)$, as a function of t_i^m. Towards this direction, we define the problem

$$\begin{aligned} \tilde{J}_i(t_i^m) = \underset{u_i}{\text{minimize}} & \sum_{j=0}^{H_i+1} u_i^2(t_i^0 + j\,\Delta T) \\ \text{subject to:} \; & (2)(3)(10), \end{aligned} \tag{12}$$

which is a standard Quadratic Programming (QP) formulation. Solving (12) for all H_i such that $t_i^m \in \{t_i^c, t_i^c + \delta t, t_i^c + 2\delta t, \dots, t^{\max}\}$, where $t^{\max}$ is the maximum allowable time inside the CZ and t_i^c is the minimum feasible travel time without considering (5) and (6). Interpolating between the points, generates the function $\tilde{J}_i(t_i^m)$, $\forall t_i^m \in [t_i^c, t^{\max}]$. It is worth noting that $\tilde{J}_i(t_i^m)$ depends also on several other parameters such as u_i^0, v_m, L, S, δ. However, all except u_i^0 can be considered constant for a specific intersection. An example of a $\tilde{J}_i(t_i^m)$ realization can be seen in Fig. 2 which indicates that this function is non-convex. Nonetheless, $\tilde{J}_i(t_i^m)$ can be approximated by the difference of two convex functions f_i and g_i such that $\tilde{J}_i(t_i^m) \simeq f_i(t_i^m) - g_i(t_i^m)$. This approximation is important because it allows the development of high-quality optimization procedures for the solution of problems involving $\tilde{J}_i(t_i^m)$.

In our case, $\tilde{J}_i(t_i^m)$ is computed numerically and hence $f_i(t_i^m)$ and $g_i(t_i^m)$ cannot be derived in closed-form. For this reason, we consider that f_i and g_i are convex piecewise linear functions (PWL) of the form

$$f_i(t) = \sum_{\lambda=0}^{M_i-1} (a_\lambda t + b_\lambda) \mathbb{1}_{[t_{i,\lambda}^c, t_{i,\lambda+1}^c)}(t), \tag{13}$$

$$g_i(t) = \sum_{\lambda=0}^{M_i-1} (A_\lambda t + B_\lambda) \mathbb{1}_{[t_{i,\lambda}^c, t_{i,\lambda+1}^c)}(t), \tag{14}$$

where $M_i = \mu(\{t_i^c, t_i^c + \delta t, t_i^c + 2\delta t, \dots, t^{\max}\})$, $t_{i,\lambda}^c = t_i^c + \lambda \delta t$ and $\mathbb{1}_{[a,b)}(x)$ is an indicator function which is equal to 1 if $x \in [a,b)$ and 0 otherwise. To derive $f_i(t_i^m)$ and $g_i(t_i^m)$ we formulate the following Linear Programming (LP) problem

$$\min_{\mathbf{a},\mathbf{b},\mathbf{A},\mathbf{B}} \sum_{\lambda=0}^{M_i-1} |J_i(t_{i,\lambda}^c) - (f_i(t_{i,\lambda}^c) - g_i(t_{i,\lambda}^c))| \tag{15a}$$
$$\text{s.t. } a_\lambda(t_{i,\lambda-1}^c) + b_\lambda = a_{\lambda-1}(t_{i,\lambda-1}^c) + b_{\lambda-1}, \tag{15b}$$
$$A_\lambda(t_{i,\lambda-1}^c) + B_\lambda = A_{\lambda-1}(t_{i,\lambda-1}^c) + B_{\lambda-1}, \tag{15c}$$
$$a_{\lambda-1} \le a_\lambda,\ A_{\lambda-1} \le A_\lambda, \tag{15d}$$

where $\lambda = 2, ..., M_i - 1$ for (16b-16d). Constraints (15b) and (15c) ensure the continuity of $f_i(t_i^m)$ and $g_i(t_i^m)$ over successive PWL segments and (15d) their convexity. A realization of this decomposition is depicted in Fig. 2. Note that the approximation of $\tilde{J}_i(t_i^m)$ from $f_i(t_i^m)$ and $g_i(t_i^m)$ is improved as δt approaches zero.

B. Phase 2

Having obtained $\tilde{J}_i(t_i^m)$, $\forall i \in \mathcal{N}$, the next step is to solve a relaxed version of problem (11) to derive close-to-optimal values for t_i^m. Towards this direction we relax the rear-end collision constraints (4) to hold only in the MZ rather than both the CZ and MZ, yielding constraint (5). In this way, we eliminate dependence of the optimization problem from the trajectories $x_i(t)$, $t \in [t_i^0, t_i^0 + t_i^m]$, $i = 1, ..., N$, as well as the acceleration profiles $u_i(t)$. This relaxation results in the following optimization problem

$$\begin{aligned} &\underset{\mathbf{t}^m}{\text{minimize}} \sum_{i=1}^{N} \tilde{J}_i(t_i^m) \\ &\text{subject to: } (5)(6)(7), \end{aligned} \tag{16}$$

where $\mathbf{t}^m = (t_1^m, ..., t_N^m)$. The decomposition of $\tilde{J}_i(t_i^m)$, $i = 1, ..., N$ into the difference of two convex functions performed in Section III-A, allows the use of an optimization method to calculate $\mathbf{t}^m$, called the Convex-Concave Procedure (CCP) [13]. Given a non-convex objective function $\phi(x)$ expressed as the difference of two convex functions, $\phi_1(x)$ and $\phi_2(x)$, the CCP keeps all the information from the convex part $\phi_1(x)$ and linearizes the concave part $-\phi_2(x)$ in each iteration; hence, it retains more information about the objective function than other algorithms in each iteration. CCP has excellent convergence properties [13].

For the solution of problem (16) the CCP solves the following optimization problem for each iteration, τ:

$$\begin{aligned} &\underset{\mathbf{t}^m}{\text{minimize}} \sum_{i=1}^{N} f_i(t_i) - \hat{g}_i(t_i; t_i^m(\tau))) \\ &\text{subject to: } (5)(6)(7), \end{aligned} \tag{17}$$

where $\hat{g}_i(t_i; t_i^m(\tau)) = g_i(t_i^m(\tau)) + \nabla g_i(t_i^m(\tau))(t_i - t_i^m(\tau))$ and $t_i^m(\tau)$ is the time to the MZ calculated during the τ^{th} iteration. This is an LP problem since $\hat{g}_i$ is affine and f_i is a convex PWL function; hence, it can be solved to optimality very efficiently. The CCP used for the solution of the problem is depicted in Algorithm 1.

To initialize the procedure a good starting point $\mathbf{t}_{\text{init}}^m$, is required. To obtain such a point, we initially solve problem

Algorithm 1 Convex-Concave Procedure

Initilization
$\mathbf{t}^m(1) \leftarrow \mathbf{t}_{\text{init}}^m$
$\tau \leftarrow 1$
repeat
  for $i = 1 : N$ **do**
    Find λ such that $t_i^m(\tau) \in [t_{i,\lambda}^m, t_{i,\lambda+1}^m)$
    $\hat{g}_i(t) \leftarrow A_\lambda t + B_\lambda$
  end for
  $\mathbf{t}^m(\tau+1) \leftarrow$ Solve (17)
  $\tau := \tau + 1$
until $\sum_{i=1}^{N}((f_i(t_i^m(\tau)) - \hat{g}_i(t_i^m(\tau); t_i^m(\tau))) - (f_i(t_i^m(\tau+1)) - \hat{g}_i(t_i^m(\tau+1); t_i^m(\tau+1)))) \le \epsilon$

(16) using as objective the lower convex envelope[1] $h_i(t_i^m) = \text{conv}(\tilde{J}_i(t_i^m))$, depicted in Fig. 2.

C. Phase 3

Phase 3 aims to construct a close-to-optimal feasible solution to the original problem (11). Using the times to the MZ derived in Phase 2, the control input u_i can be chosen by solving (12) for each CAV. The trajectories of vehicles can be constructed using (10). However, the trajectories generated in this way, are not always safe, since constraint (4) was not explicitly considered. One way to tackle this problem, is using a Feasibility Enforcement Zone (FEZ) outside of CZ similar to [14]. A different approach is followed in this work. The following four optimization problems are formulated for each direction-of-movement z,

$$\begin{aligned} &\underset{\mathbf{u}}{\text{minimize}} \sum_{\{i|d_i=z\}} \sum_{j=0}^{H_i+1} u_i^2(t_i^0 + j\,\Delta T) \\ &\text{subject to: } \quad (10)(2)(3)(4), \end{aligned} \tag{18}$$

where $z \in \mathcal{D}$ indicates the direction of CAVs. This problem is a Quadratic Program that can be efficiently solved to derive feasible CAV trajectories; for this reason, this procedure will be referred to as CCPF.

To sum up, the implementation by the IC for a specific intersection can be done by solving (12) and (15) for all u_i^0 and storing them in memory. Then, every T time units the IC executes the following steps:

Step 1. If the intersection is empty it executes Algorithm 1. Otherwise, the IC executes Algorithm 1 with the additional constraint $t_1^0 + t_1^m \ge t_L + \delta/v^m$ or $t_1^0 + t_1^m \ge t_L + S/v^m$ in (17) depending on d_1, the direction of CAV 1, where t_L is the time needed for the last vehicle to arrive at the MZ during the previous time-window.

Step 2. If the intersection is empty the IC solves Problem (18) for each direction and transmits the control inputs to the vehicles. Otherwise, it solves Problem (18) with the additional constraint $x_{L_z}(t+T) - x_{1_z}(t) \ge \delta$, $\forall t \in [0, T]$ for all z, where x_{L_z} is the trajectory of the last vehicle in

[1]The lower convex envelope is given by the equation $h_i(t_i^m) = \sup\{g(t_i^m) \mid g(t_i^m) \le \tilde{J}_i(t_i^m)\ \forall t_i^m \in [t_i^{\min}, t_i^{\max}]\}$ where g is a convex function, calculated for each $\tilde{J}_i(t_i^m)$.

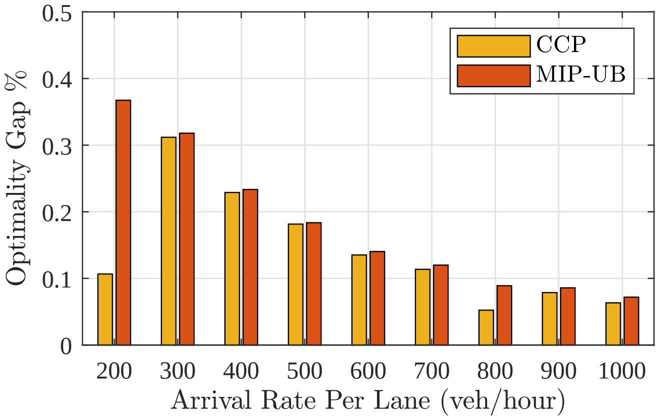

Fig. 3: Relative percentage difference of MIP-UB and CCP with MIP-LB for various arrival rates.

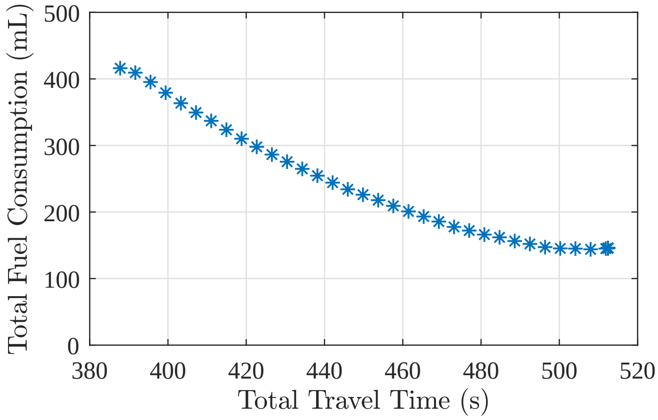

Fig. 4: Trade-off between (a) total travel time and total cost, (b) total travel time and total fuel consumption for an arrival rate of 500 $veh/hour$ per lane.

direction z, generated in the previous time-window and x_{1_z} is the trajectory of the first vehicle travelling in direction z in the current time-window. Then, the IC transmits the control inputs to the vehicles.

The ability of the IC to solve these problems in real-time emanates from the fact that all developed algorithms result in simple LP or QP formulations that can be solved to optimality very fast using standard solvers.

IV. Simulation Results and Discussion

To evaluate the performance of the proposed scheme, we executed several simulation scenarios using MATLAB and the Gurobi Mathematical Optimization Solver [15]. The parameters used are: $L = 100m$, $S = 6m$, $\delta = 3m$, $v^m = 10m/s$, $u^{\min} = -3m/s^2$, $u^{\max} = 2.25m/s^2$, $v^{\max} = 15m/s$, $\Delta T = \delta t = 0.1$. Arrival speeds were uniformly distributed between 8 to 12 m/s. CAVs were considered in batches of $N = 50$.

For the remaining of this paper, CCP will be referred to as the solution of Algorithm 1, with control inputs given by (12) and $\gamma = \infty$. In order to compare the CCP which is a heuristic to the optimal solution of (16), we derive tight PWL lower and upper bounds for each $\tilde{J}_i(t_i^m)$ for a specified number of linear segments. The breakpoints of the PWL segments were chosen to minimize the approximation error in an identical manner to [16]. Then, a Mixed-Integer Programming (MIP) algorithm that solves (9) is developed for benchmarking purposes. The objective function value of the CCP, MIP lower bound and MIP upper bound algorithms are defined as f_{CCP}, $f_{\text{MIP-LB}}$ and $f_{\text{MIP-UB}}$, respectively.

The simulation was run for various arrival rates, ranging between 200 to 1000 vehicles per hour per lane where f_{CCP}, $f_{\text{MIP-LB}}$ and $f_{\text{MIP-UB}}$ were calculated. A total of 100 linear segments were used to construct PWL lower and upper bounds. The performance of the developed algorithms is compared against the tight lower bounds obtained through MIP-LB, using the relative percentage optimality gap defined by

$$\text{Optimality Gap} = \frac{f_{\text{alg}} - f_{\text{MIP-LB}}}{f_{\text{MIP-LB}}} \times 100\%. \quad (19)$$

In the above equation, f_{alg} denotes either f_{CCP} or $f_{\text{MIP-UB}}$. The average results for 10 runs for each arrival rate are shown in Fig. 3. It can be seen that the optimality gap is very small, which indicates that the deviation of f_{CCP} from the optimal solution is on average less than 0.4%. In the worst case, the optimality gap is 0.7%. This confirms that the CCP produces close-to-optimal solutions, if not optimal. In addition to this, CCP was fast, converging after just two to three iterations.

Having illustrated the high-quality solutions produced by the CCP, next the trade-off between cost and travel time is explored through the developed solution for the considered problem. The case for $\gamma = 1$ minimizes the total travel time, and for this reason it is referred to as Travel Time Minimization (TTM). The cost-travel time trade-off for an arrival rate of 500 vehicles per hour per lane when γ is increased from 1 to 1.5 in steps of 0.01, was analyzed. It was observed that while the total travel time increased from 388s to 512s, the total cost (objective) was reduced by up to 98%, which indicated that for a moderate travel time increase one can obtain very significant cost saving using the simplistic energy model considered. To obtain a more realistic fuel consumption-travel time trade-off, we also considered the following empirical fuel consumption model

$$F_i = c_0 + c_1 v_i + c_2 v_i^2 + c_3 v_i^3 + u(c_4 + c_5 v_i + c_6 v_i^2), \quad (20)$$

where F_i is measured in millilitres per second (mL/s). The values used for the constants c_0 to c_6 are based on the torque-speed-efficiency map of a Nissan March K11 and given by: $c_0 = 0.1569$, $c_1 = 0.0245$, $c_2 = -7.415 \times 10^{-4}$, $c_3 = 5.975 \times 10^{-5}$, $c_4 = 0.07224$, $c_5 = 0.09681$ and $c_6 = 1.075 \times 10^{-3}$ [17]. Using the control input generated by solving (12), the velocity is calculated for each vehicle and used to derive the fuel consumption using the above model. The result can be seen in Fig 4. While, the reduction on fuel consumption is not as dramatic as using the cost function, the results indicate that an increase in travel time of 20% leads to a 50% reduction in fuel consumption, while further increase in travel time can lead to up to 65% fuel consumption reduction. It is important to highlight that these significant reductions in fuel consumption have been achieved through

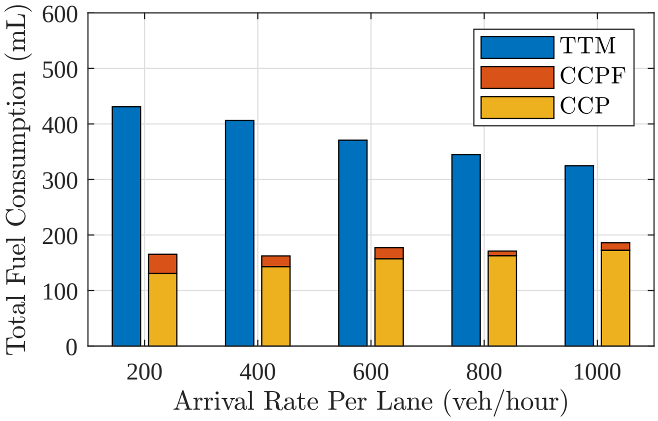

Fig. 5: TTM and CCP comparison for various arrival rates.

the use of different models for optimization (the simplistic model) and evaluation (the empirical model).

It is important to note that through the use of constraint (7), the IC is able to choose to minimize either the fuel consumption or the total travel time depending on the traffic conditions, or even use an intermediate state that combines the best of both worlds. Next, the effect of the arrival rate on the fuel consumption-travel time trade-off is examined. Fig. 5 illustrates total fuel consumption for the TTM approach and the CCP/CCPF using the realistic fuel consumption model for varying arrival rate between 200 and 1000 vehicles per hour per lane. The results are averaged over 10 runs. The total travel time of the CCP is higher than the TTM approach. This is expected as the emphasis in TTM is first to minimize the travel time and then minimize the fuel efficiency. However, the average increase in travel time experienced per vehicle is small, about 2.3 seconds with the maximum increase being 3.4 seconds. The increase in travel time translates into reduced fuel consumption. The CCP reduces the fuel consumption by 47%-70% compared to the TTM. However, the trajectories may not be feasible, so the CCPF is executed. On average the CCPF increases fuel consumption by 5%-25% compared to CCP, depending on the arrival rate, such that the fuel consumption improvement is between 43%-62% compared to the TTM approach.

V. Conclusion and Future work

This paper has investigated the coordination of CAVs passing through an unsignalized intersection in order to optimize the trade-off between travel time and fuel consumption. A centralized scheme was developed where the Intersection Controller simultaneously optimizes the trajectories of multiple vehicles that are expected to arrive at the Control Zone within a time-window. Simulation results showed the developed solution scheme can cut in half the fuel consumption with a moderate increase in travel time.

Ongoing work considers optimization and evaluation using advanced fuel consumption models, turning, as well as the effect of using CAV diversity on the travel time - fuel consumption trade-off. Finally, future research will also investigate the complications involved by considering noisy data, communication delays, unconstrained merging speeds, different priority models etc.

References

[1] European Commission, "European Semester Thematic Factsheet - Transport," Accessed: 2019-04-14. [Online]. Available: https://ec.europa.eu/info/sites/info/files/file_import/european-semester_thematic-factsheet_transport_en.pdf

[2] J. L. Fleck, C. G. Cassandras, and Y. Geng, "Adaptive quasi-dynamic traffic light control," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 3, pp. 830–842, May 2016.

[3] J. Guanetti, Y. Kim, and F. Borrelli, "Control of connected and automated vehicles: State of the art and future challenges," *Annual Reviews in Control*, vol. 45, pp. 18–40, 2018.

[4] J. Rios-Torres and A. A. Malikopoulos, "A survey on the coordination of connected and automated vehicles at intersections and merging at highway on-ramps," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 5, pp. 1066–1077, May 2017.

[5] K. Dresner and P. Stone, "A reservation-based multiagent system for intersection control," *IFAC Proceedings Volumes*, vol. 37, no. 8, pp. 136–141, July 2004.

[6] S. A. Fayazi and A. Vahidi, "Mixed-Integer Linear Programming for Optimal Scheduling of Autonomous Vehicle Intersection Crossing," *IEEE Transactions on Intelligent Vehicles*, vol. 3, no. 3, pp. 287–299, 2018.

[7] I. H. Zohdy, R. K. Kamalanathsharma, and H. Rakha, "Intersection management for autonomous vehicles using iCACC," in *Proceedings of the 15th International IEEE Conference on Intelligent Transportation Systems (ITSC'12), Anchorage, AK, USA, September 16-19*, 2012, pp. 1109–1114.

[8] E. Onieva, V. Milanés, J. Villagrá, J. Pérez, and J. Godoy, "Genetic optimization of a vehicle fuzzy decision system for intersections," *Expert Systems with Applications*, vol. 39, no. 18, pp. 13 148–13 157, December 2012.

[9] L. Makarem and D. Gillet, "Model predictive coordination of autonomous vehicles crossing intersections," in *Proceedings of the 16th International IEEE Conference on Intelligent Transportation Systems (ITSC'13), Hague, Netherlands, October 6-9*, 2013.

[10] A. A. Malikopoulos, C. G. Cassandras, and Y. J. Zhang, "A decentralized energy-optimal control framework for connected automated vehicles at signal-free intersections," *Automatica*, vol. 93, pp. 244–256, July 2018.

[11] Y. Zhang, A. A. Malikopoulos, and C. G. Cassandras, "Decentralized optimal control for connected automated vehicles at intersections including left and right turns," in *Proceedings of the 56th Annual IEEE Conference on Decision and Control (CDC'17), Melbourne, Australia, December 12-15*. IEEE, 2017, pp. 4428–4433.

[12] J. Rios-Torres, A. Malikopoulos, and P. Pisu, "Online optimal control of connected vehicles for efficient traffic flow at merging roads," in *Proceedings of the 18th International IEEE Conference on Intelligent Transportation Systems (ITSC'15), Las Palmas, Gran Canaria, Spain, September 15-18*, 2015.

[13] T. Lipp and S. Boyd, "Variations and extension of the convex–concave procedure," *Optimization and Engineering*, vol. 17, no. 2, pp. 263–287, November 2015.

[14] Y. Zhang, C. G. Cassandras, and A. A. Malikopoulos, "Optimal control of connected automated vehicles at urban traffic intersections: A feasibility enforcement analysis," in *Proceedings of the 2017 American Control Conference (ACC), Seattle, WA , USA, May 24-26*, 2017.

[15] Gurobi Optimization Inc., "Gurobi Optimizer Reference Manual," 2016. [Online]. Available: http://www.gurobi.com

[16] S. Timotheou, "Asset-task assignment algorithms in the presence of execution uncertainty," *The Computer Journal*, vol. 54, no. 9, pp. 1514–1525, September 2010.

[17] M. A. S. Kamal, M. Mukai, J. Murata, and T. Kawabe, "Model predictive control of vehicles on urban roads for improved fuel economy," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 3, pp. 831–841, May 2013.