Cooperative Method of Traffic Signal Optimization and Speed Control of Connected Vehicles at Isolated Intersections

Biao Xu[®], *Member, IEEE*, Xuegang Jeff Ban[®], Yougang Bian, Wan Li, Jianqiang Wang[®], Shengbo Eben Li[®], *Senior Member, IEEE*, and Keqiang Li

Abstract—Signalized intersections play an important role in transportation efficiency and vehicle fuel economy in urban areas. This paper proposes a cooperative method of traffic signal control and vehicle speed optimization for connected automated vehicles, which optimizes the traffic signal timing and vehicles' speed trajectories at the same time. The method consists of two levels, i.e., roadside traffic signal optimization and onboard vehicle speed control. The former calculates the optimal traffic signal timing and vehicles' arrival time to minimize the total travel time of all vehicles; the latter optimizes the engine power and brake force to minimize the fuel consumption of individual vehicles. The enumeration method and the pseudospectral method are applied in roadside and onboard optimization, respectively. Simulation studies are conducted to compare the proposed method with benchmark methods. The results show significant improvement of transportation efficiency and fuel economy by the cooperation method.

Index Terms—Traffic signal optimization, vehicle speed control, connected automated vehicle, V2I cooperation, intelligent transportation systems, travel time, fuel consumption.

I. INTRODUCTION

RBAN traffic is critical to businesses, people's daily life, and local economies, which also causes heavy congestion and related energy use and emission issues. The US Energy Information Administration reported that, of the total petroleum consumed in the US in 2016, vehicles accounted for

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B. Xu, Y. Bian, J. Wang, S. E. Li, and K. Li are with the Department of Automotive Engineering, Tsinghua University, Beijing 100084, China, and also with the Collaborative Innovation Center of Electric Vehicles in Beijing, Beijing 100084, China (e-mail: b-xu13@mails.tsinghua.edu.cn; byg14@mails.tsinghua.edu.cn; wjqlws@tsinghua.edu.cn; lishbo@tsinghua.edu.cn; likq@tsinghua.edu.cn).

X. J. Ban and W. Li are with the Department of Civil and Environmental Engineering, University of Washington, Seattle, WA 98105 USA (e-mail: banx@uw.edu; wan5@uw.edu).

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a staggering portion of about 50% [1]. Furthermore, Texas Transportation Institute reported that, in 2014, traffic jams wasted over 3.1 billion gallons of fuel and 6.9 billion hours in urban areas in the US [2]. Vehicle fuel economy and transportation efficiency are affected by many factors such as road capacity and infrastructure design. Among these factors, signalized intersections in urban areas play an important role, which could block traffic flow and cause vehicle deceleration and idling. This further aggravates traffic jams and decreases the vehicle fuel economy.

Researchers have made great efforts to improve transportation efficiency and vehicle fuel economy at signalized intersections. In the transportation field, traffic signal control/optimization has been an extensively studied area for various types of signal control such as fixed-time and actuated signal control [3]. More advanced methods, such as adaptive signal control, have also been proposed and deployed (e.g., OPAC [4], RHODES [5], SCOOT [6], and Ren et al. [7]). In the vehicle control field, vehicle engine control, startup control and speed control have been studied to improve the transportation efficiency and vehicle fuel economy. Chen et al. [8] investigated the control strategy of engine start/stop system that can shut off the engine automatically during idling to decrease fuel consumption. Wang et al. [9] proposed a startup assistance system at signalized intersections to help vehicles start up with less delay for improving the transportation efficiency. Li et al. [10] demonstrated an eco-departure system that aims to optimize the speed profiles of the vehicle departure operations at signalized intersections and to decrease the fuel consumption during this

Recent technology developments on connected automated vehicles (CAVs) have shown great potential to significantly improve the mobility, fuel economy, and safety of urban traffic [10], [11]. CAVs, especially the vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) communications and vehicle automation, also provide new opportunities for traffic signal control and vehicle control at signalized intersections.

First, with V2I and V2V communications, traffic signal controllers can acquire the more exact positions and motion information of approaching vehicles in real time, which can be used for more effective traffic signal control. Lee *et al.* [12]

obtained vehicles' states from connected vehicles to estimate the travel times that were used for arriving intersection control, which proved to improve the total delay and average speed of vehicles compared to the actuated signal control. Priemer and Bernhard [13] designed a decentralized adaptive traffic signal control method that estimated the queue length and traffic flow using the V2I data. The simulation results showed that the proposed method performed better than TRANSYT-7F signal control when the penetration exceeded 20%. Goodall et al. [14] used simulation methods to predict the queue length and delay after obtaining vehicle position and speed information via V2I, which was then used to optimize traffic signal timing. This method was able to improve the traffic mobility compared to coordinated actuated signal control in different levels of saturation rates according to the simulation results. Zhao et al. [15] developed a V2I-based signal timing optimization method by considering individual vehicles' fuel consumption characteristics of which the simulation results suggested the potential to improve vehicle fuel economy around the intersection. Feng et al. [16] presented a bi-level adaptive signal control algorithm using the connected vehicle data to minimize the vehicle delay and queue length, and the simulation results showed that it reduced total delay significantly under high penetration rates compared to the actuated control. Actually, the ability to acquire the exact vehicle motion information promotes estimation accuracy of the queue length, vehicle travel time, and vehicle fuel consumption for traffic signal control, which results in better signal control performance.

Secondly, an approaching vehicle can also obtain traffic signal phases/timing and traffic conditions by V2I/V2V communications in real time, through which the speed trajectory of the vehicle can be optimized and controlled to reduce fuel consumption or other desirable objectives. Such individual vehicle control can be more easily deployed on CAVs since the recommended vehicle speed trajectory may be implemented as part of the automation algorithm. Asadi and Vahidi [17] proposed a predictive cruise control method at signalized intersections, which aimed to make vehicles cruise at the pre-set speed and arrive at the green light timely with the minimal use of braking. Jin et al. [18] suggested a power-based optimal longitudinal control for ICE (Internal combustion engine) vehicles to optimize the speed profiles considering the brake specific fuel consumption map, the traffic signal, and the road grade. Wu et al. [19] made effort to optimize the speed profiles for electric vehicles on signalized arterials. Xu et al. [20] used the branch and bound algorithm to optimize vehicle speed profiles in adjacent signalized intersections. He et al. [21] introduced a multi-stage optimal control method that optimizes the vehicle speed considering the traffic signal and the queue length at intersections. HomChauduri et al. [22] developed a speed optimization method for a group of connected vehicles using decentralized model predictive control, which was proved to be effective for vehicle platoon going through signalized intersections. In Europe, eCoMove project [23] built a prototype system for energy efficiency and mobility of vehicles which was able to provide driving assistance with the human-machine interface to avoid vehicle stops at traffic signals.

In summary, CAV-based traffic signal control has great potential to improve the transportation efficiency and fuel economy for all vehicles in the system (defined as the *macro* traffic level in this paper), while CAV-based vehicle speed optimization can improve efficiency and reduce fuel use at the individual vehicle level (defined as the *micro* vehicle level). It is expected that combining the control at the macro and micro levels can help further improve transportation efficiency and decrease fuel consumption. For this, Li *et al.* [24] developed a joint optimization method for traffic signal timing and vehicle speeds, which was shown to decrease the travel time in different traffic demands. However, they did not consider the fuel consumption for the approaching vehicles. Additionally, the vehicle speed optimization is rule-based, which may not lead to the optimal speed trajectories.

Besides traffic signal control and vehicle control, there are other CAV-based traffic intersection management methods. Dresner and Stone [25] designed a reservation-based method to coordinate the motion of CAVs at unsignalized intersections. Lee and Park [26] proposed a so-called Cooperative Vehicle Intersection Control method to manipulate individual CAVs' maneuvers to avoid vehicle collision at unsignalized intersections. Tachet et al. [28] proposed a capacity-optimal Slot-based Intersection management system for CAVs which allocates the vehicles' access time with the consideration of safety constraints. Yang et al. [28] proposed a method to optimize the departure sequence and trajectories of the vehicles at an intersection which had potential to minimize the traffic delay for a mix-traffic environment with conventional, connected, and automated vehicles. Most of these methods, by eliminating the traffic signal completely, can only work when CAVs reach the 100% penetration, and no pedestrian or bicyclist is present at the intersection. This, however, will not happen in the near future. Additionally, they did not separate the conflicting traffic flow, which needs high robustness requirement for vehicle control to ensure traffic safety at intersections.

In this paper, we propose a cooperative method of traffic signal optimization and vehicle speed control which can simultaneously optimize traffic signal timing and vehicle speed trajectories. The purpose is to improve transportation efficiency and decrease CAV fuel consumption. The cooperation method consists of traffic optimization and vehicle optimal control. Traffic optimization calculates the optimal traffic signal timing and vehicles' arrival times cycle by cycle, according to the vehicles' initial speed and position information. Here we apply the best practices in signal design, i.e., the dualring diagram for signal design [3]. Such method has been proven to be effective in ensuring safety (e.g., separating conflicting movements and providing proper clearance times), while at the same time maximizing the efficiency for vehicles passing signalized intersections. The vehicle optimal control is applied to each individual vehicle to obtain the optimal trajectories based on the optimal arrival time calculated in the traffic optimization to minimize the vehicle fuel consumption. Here we assume that all vehicles are CAV-equipped so that the optimized speed trajectory can be calculated and executed by the vehicles automatically without human involvement.

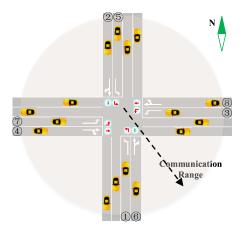


Fig. 1. Studied Scenario

The main contributions of this paper are as follows:

- (1) We develop a cooperative method for the simultaneous optimization of traffic signal timing (macro level) and vehicle control (micro level), by considering two objectives: transportation efficiency and vehicle fuel consumption.
- (2) We consider transportation efficiency at the macro signal timing control level and fuel economy at the micro vehicle control level. This also implies that the primary goal of the proposed method is to ensure the efficiency of all vehicles, while at the same time to minimize vehicle fuel consumption. Note that these two goals are interrelated: better efficiency will usually lead to less fuel consumption. Such consideration also helps to decompose the method into two interactive components, which makes the cooperative method easier to construct and solve.

This paper is structured as follows. In Section II, we define the problem studied in the paper. Section III presents the methodology of traffic signal optimization and vehicles speed control. Section IV shows the enumeration method for solving the traffic optimization problem and the pseudospectral method for solving the vehicle optimal control problem. The simulation and results are presented in Section V. Finally, Section VI gives some concluding remarks.

II. PROBLEM STATEMENT

We consider a typical 4-leg, 2-lane (in each direction) signalized intersection (see Fig. 1). The two lanes in each incoming approach of the intersection are a left-turning lane and a through/right-turning lane, respectively. Here, we assume that vehicles for different destinations are already on the corresponding lanes so that we do not need to consider the lane changing behavior. In other words, vehicle lane changing has been done before they enter the study area (when they approach the intersection) or after they leave the study area. As a result, vehicles from the same approaching direction will also follow FIFO (first-in-first-out) and no vehicle overtaking will occur. The study area is defined by the communication range of V2I as shown in Fig. 1, which is usually a few hundred meters in radius.

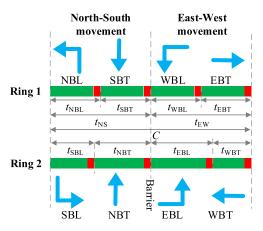


Fig. 2. Signal phase sequence.

There are thus eight approaching traffic movements in the studied intersection, after we combine the through and right turn movements for the same direction. They are northbound left (NBL), northbound through (NBT), west-bound left (WBL), westbound through (WBT), southbound left (SBL), southbound through (SBT), eastbound left (EBL) and eastbound through (EBT) movements. The purpose of the traffic signal control is to coordinate those movements. For a vehicle, the movement it belongs to is denoted as L when $L \in \mathbb{F} = \{\text{NBL}, \text{NBT}, \text{WBL}, \text{WBT}, \text{SBL}, \text{SBT}, \text{EBL}, \text{EBT}\}$. We number the vehicles in movement L with $1 \sim N_L$ according to the vehicles' distance to the stop line where N_L represents the total vehicle number of traffic movement L. As a result, a vehicle can be represented with a pair (L, i) where i is the serial number of the vehicle and $i \leq N_L$.

As aforementioned, we apply the dual-ring approach for signal design [29] with predetermined fixed cycle length *C* (to readily extend the proposed model to consider the coordination of multiple signals in future research); see Fig. 2. The phase sequence of Ring 1 is NBL – SBT – WBL – EBT and the phase sequence of Ring 2 is SBL – NBT – EBL – WBT. Each phase can be divided into effective green interval and clearance time (i.e., yellow and all-red times). Clearance time is also predetermined and fixed using standard procedures [3]. Notice that since a particular phase may be skipped, the actual phase sequence after optimization may be different as that shown in Fig. 2.

We assume that vehicles can pass the stop lines during the effective green intervals of the traffic signal, once they appear in the study area. Note that this does NOT mean that vehicles must pass the intersection during the same cycle they arrive at the stop lines. They may pass the intersection in the next few signal cycles, which essentially indicates the occurrence of *oversaturation* due to heavy traffic demands. The design of the signal clearance time will make sure that the vehicles passing the stop line at the end of the effective green time can be cleared during the clearance time, which is standard signal design practice [3].

In Fig. 2, we demonstrate the schematic diagram of the dual-ring signal control design. The green lines represent the effective green intervals of the traffic signal phases and

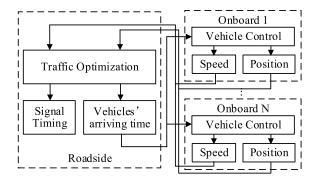


Fig. 3. Overview of the cooperation of traffic signal and vehicles.

the red lines represent the clearance time R. $t_{\rm NBL}$, $t_{\rm SBT}$, $t_{\rm WBL}$, $t_{\rm EBT}$, $t_{\rm SBL}$, $t_{\rm NBT}$, $t_{\rm EBL}$ and $t_{\rm WBT}$ denote the phase time of NBL, SBT, WBL, EBT, SBL, NBT, EBL, and WBT movement, respectively. $t_{\rm NS}$ and $t_{\rm EW}$ are the phase time of the northbound/southbound and eastbound/westbound movements, respectively. C is the (fixed) cycle length.

In practice, left turn phases or phases of the minor streets may be skipped. Hence, we use binary variables b_{NBL} , b_{SBT} , b_{WBL} , b_{EBT} , b_{SBL} , b_{NBT} , b_{EBL} , and b_{WBT} to denote if the corresponding phases are skipped. Specifically, if $b_L = 0$ ($L \in \mathbb{F}$), then the phase for the movement L is skipped and the phase time $t_L = 0$. When $b_L = 1$ ($L \in \mathbb{F}$), the phase for the movement L is reserved and the phase time t_L is greater than the minimum green time. Briefly, $\forall L \in \mathbb{F}$, we have

$$(1 - b_L) t_L = 0, (1)$$

$$b_L \in \{0, 1\}.$$
 (2)

As discussed before, we assume that all vehicles are equipped with positioning and V2V/V2I devices so that they can send location and movement information to the signal controller when they enter the communication range of the intersection. Additionally, all approaching vehicles are assumed to be automated vehicles so that vehicles can control their speed, strictly follow the optimized speed profile, and pass the intersection automatically. That is, we assume 100% penetration of CAV equipped vehicles in this research.

III. METHODOLOGY

A. Overview of Cooperation

The proposed cooperative method consists of two major components: traffic optimization and vehicle control. As shown in Fig. 3, the traffic optimization resides at the traffic controller. It acquires the speed and position information from approaching vehicles, calculates the optimal traffic signal timing, and plans the vehicles' arrival times with the aim to decrease the total travel time of all vehicles at the macro level. The optimized vehicles' arrival times are then sent to each corresponding vehicle. The onboard vehicle control decides the optimal speed trajectory of the vehicle to optimize the engine power and brake force by considering the constraint of the arrival time to minimize the fuel consumption in the whole trip of the vehicle at the micro vehicle level.

It is worth noting that the traffic optimization and the vehicle control are conducted with the rolling horizon procedure. For the traffic optimization, it optimizes the traffic signal timing and vehicles' arrival time at the end of each signal cycle, which means its control period is one signal cycle. The calculated optimal signal timing is adopted by the traffic signal for the next cycle. For the vehicle control, the control period is much shorter, e.g., 0.2s.

B. Traffic Optimization

1) Traffic Model: As all vehicles are assumed to be automated vehicles, we use some basic rules to build the traffic model in this paper. Specifically, when vehicles are approaching the intersection, they must (i) arrive at the stop line within the green interval of the corresponding phase; and (ii) keep a safe time headway to avoid collisions. Mathematically, if the vehicle (L,i) meets the first condition above, we have

$$\tau_L^{(i)} \in g_L, \quad \forall i \le N_L, L \in \mathbb{F},$$
 (3)

where $\tau_L^{(i)}$ is the arrival time of the vehicle (L, i), which is the vehicle's running time from the initial position to the stop line, and g_L is the union set of the traffic signal green interval for the traffic movement L. Therefore Equation (3) means the vehicle's arrival time falls into the green intervals' union set.

Given the phase time of NBL, SBT, WBL, EBT, SBL, NBT, EBL and WBT as $t_{\rm NBL}$, $t_{\rm SBT}$, $t_{\rm WBL}$, $t_{\rm EBT}$, $t_{\rm SBL}$, $t_{\rm NBT}$, $t_{\rm EBL}$ and $t_{\rm WBT}$, the effective green interval at the beginning of the cycle can be written as:

$$g_{\text{NBL}} = \bigcup_{k=0}^{K-1} [kC, kC + (t_{\text{NBL}} - R) b_{\text{NBL}}]$$
 (4)

$$g_{\text{SBT}} = \bigcup_{k=0}^{K-1} \left[kC + b_{\text{NBL}} t_{\text{NBL}}, kC + b_{\text{NBL}} t_{\text{NBL}} + b_{\text{SBT}} \left(t_{\text{SBT}} - R \right) \right]$$
 (5)

$$g_{WBL} = \bigcup_{k=0}^{K-1} \left[kC + b_{SBT}t_{SBT} + b_{NBL}t_{NBL}, kC + b_{SBT}t_{SBT} + b_{NBL}t_{NBL} + b_{WBL}\left(t_{WBL} - R\right) \right]$$
(6)

$$g_{EBT} = \bigcup_{k=0}^{K-1} [kC + C - b_{EBT}t_{EBT}, kC + C - b_{EBT}R]$$
 (7)

$$g_{SBL} = \bigcup_{k=0}^{K-1} [kC, kC + (t_{SBL} - R) b_{SBL}]$$
 (8)

$$g_{NBT} = \bigcup_{k=0}^{K-1} \left[kC + b_{SBL}t_{SBL}, kC + b_{SBL}t_{SBL} + b_{NBT} \left(t_{NBT} - R \right) \right]$$

$$(9)$$

$$g_{EBL} = \bigcup_{k=0}^{K-1} \left[kC + b_{NBT} t_{NBT} + b_{SBL} t_{SBL}, kC + b_{NBT} t_{NBT} + b_{SBL} t_{SBL} + b_{EBL} \left(t_{EBL} - R \right) \right]$$
(10)

$$g_{WBT} = \bigcup_{k=0}^{K-1} [kC + C - b_{WBT}t_{WBT}, kC + C - b_{WBT}R]$$
(11)

where R refers to the clearance time, k means the cycle number, and K denotes the total number of cycles considered by the method and is pre-set. Here, we take (4) as an example to explain the equations. As shown in (4), $g_{\rm NBL}$ is a union set of several green intervals in K cycles, of which kC is the start time of the green interval in the $k^{\rm th}$ cycle and

 $kC+(t_{\text{NBL}}-R)b_{\text{NBL}}$ is the end time of the green interval in the k^{th} cycle. It means that the vehicles may pass the intersection at any green interval of the K cycles.

In addition, the allocated time of different traffic phases must meet the requirement of the dual-ring constraints:

$$b_{\text{SBT}}t_{\text{SBT}} + b_{\text{NBL}}t_{\text{NBL}} = b_{\text{NBT}}t_{\text{NBT}} + b_{\text{SBL}}t_{\text{SBL}} = t_{\text{NS}}, \quad (12)$$

$$b_{\text{EBT}}t_{\text{EBT}} + b_{\text{WBL}}t_{\text{WBL}} = b_{\text{WBT}}t_{\text{WBT}} + b_{\text{EBL}}t_{\text{EBL}} = t_{\text{EW}}, (13)$$

$$t_{\rm NS} + t_{\rm EW} = C, \tag{14}$$

$$t_{L} - R \in [g_{\min}, g_{\max}] \cup \{0\}, L \in \mathbb{F}. \tag{15}$$

In (15), g_{\min} means the minimum green time, and g_{\max} means the maximum green time. Notice that a phase L may be skipped, in which case t_L is zero.

Since the signal timing is modeled via the dual-ring diagram in Fig. 2, constraints (1) - (15) ensure that movement conflicts will not occur while compatible movements may occur concurrently if needed.

Additionally, to ensure safety, vehicle (L, i) must keep a safe time headway to the front vehicle (L, i - 1):

$$\tau_L^{(i)} - \tau_L^{(i-1)} \ge \text{THW}_{\min}, \quad \forall 2 \le i \le N_L, i \in N^+, L \in \mathbb{F},$$
(16)

where THW_{min} denotes the minimum safe time headway which we can set as 2s [30].

Remark: In Traffic Optimization, headway constraint (16) can only ensure the overall collision avoidance of two consecutive vehicles, in particular at the stop line. Vehicle collision may still occur before they get to the stop line. This will be further considered in Vehicle Optimal Control with a safety distance constraint imposed to ensure collision avoidance of the trajectories of two consecutive vehicles.

2) Vehicle Kinematic Model: A vehicle is a complex nonlinear dynamic system with numerous nonlinear components such as the engine and the transmission. In this paper, we build a simple vehicle kinematic model to describe the vehicle movement in the traffic optimization model since the model needs to consider all approaching vehicles. In the vehicle control model, a more detailed vehicle model will be developed to more accurately control the speed of a single vehicle.

It is assumed that the vehicle tends to accelerate to the desired speed and keep uniform motion when running through the intersection. Based on this assumption, the vehicle model can be written as:

$$\dot{d} = -v$$
, $\dot{v} = a$, $a = \begin{cases} a(t), & \text{if } v \neq v_t \\ 0, & \text{if } v = v_t, \end{cases}$

where t, d, v, and a are the time and distance to the intersection, and the velocity and the acceleration of the vehicle respectively. Denote v_t the target speed; in this paper, we assume the maximum target speed is the road speed limit.

Given the speed limit v_{max} and the maximum comfortable acceleration a_{max} , we can calculate the minimum arrival time $\tau_{L,\text{min}}^{(i)}$ of the vehicle (L, i) with the distance $d_L^{(i)}$ and the velocity $v_L^{(i)}$. When a vehicle's distance is far enough so that the vehicle can speed up to the speed limit before arriving at

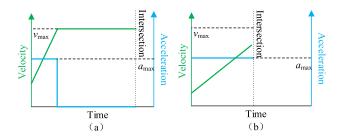


Fig. 4. Two cases for vehicle arriving with minimal time.

the intersection (see Fig. 4(a)), the minimum arrival time is

$$\tau_{L,\min}^{(i)} = \frac{2a_{\max}d_L^{(i)} + \left(v_{\max} - v_L^{(i)}\right)^2}{2a_{\max}v_{\max}}, \quad \text{if } d_L^{(i)} \ge \frac{v_{\max}^2 - v_L^{(i)2}}{2a_{\max}}.$$
(17)

On the other hand, when the distance is not far enough (see Fig. 4(b)), the minimum arrival time is

$$\tau_{L,\min}^{(i)} = \frac{-v_L^{(i)} + \sqrt{v_L^{(i)2} + a_{\max} d_L^{(i)}}}{2a_{\max}}, \quad \text{if } d_L^{(i)} < \frac{v_{\max}^2 - v_L^{(i)2}}{2a_{\max}}.$$
(18)

It is obvious that the vehicle's arrival time cannot be less than its minimum arrival time (see constraints (19)). These minimal arrival time constraints consider the vehicle kinematic limitation, and ensure the optimized arrival time satisfies the vehicle kinematic constraints.

$$\tau_L^{(i)} \ge \tau_{L,\min}^{(i)}, \quad \forall i \le N_L, L \in \mathbb{F}.$$
 (19)

3) Cost Function: In traffic signal control, minimizing delay is often used as the objective. In this paper, we use the total travel time (from the initial position to the stop line) of all the approaching vehicles as the cost function for traffic signal optimization:

$$f\left(\tau_L^{(i)}\right) = \sum_{L \in \mathbb{F}} \sum_{i=1}^{N_L} \tau_L^{(i)}.$$

4) Optimization Model: The traffic optimization aims to optimize the phase time of the traffic signal and the arrival time of all the approaching vehicles. Based on the traffic model, the vehicle model, and the cost function, the traffic optimization model is as follows:

$$\left\{t_{\text{SBT}}^{*}, t_{\text{NBL}}^{*}, t_{\text{NBT}}^{*}, t_{\text{SBL}}^{*}, t_{\text{EBT}}^{*}, t_{\text{WBL}}^{*}, t_{\text{WBT}}^{*}, t_{\text{EBL}}^{*}, \tau_{\text{L}}^{(i)*}\right\}$$

$$= \arg\min \sum_{L \in \mathbb{F}} \sum_{i=1}^{N_{L}} \tau_{L}^{(i)},$$
subject to: constraints: (1)~(19),

where $t_{\rm SBT}^*$, $t_{\rm NBL}^*$, $t_{\rm NBT}^*$, $t_{\rm SBL}^*$, $t_{\rm EBT}^*$, $t_{\rm WBL}^*$, $t_{\rm WBL}^*$, $t_{\rm EBL}^*$, and $\tau_L^{(i)*}$ are the optimal signal timing and the vehicles' optimal travel time

Note that when the total cycle number K in the optimization model is large enough, we can always find the feasible arrival time for vehicles in the optimization model under any circumstances. Hence, the optimization model is always feasible.

Remark: The traffic optimization is conducted at the end of each traffic signal cycle. The vehicle speed and position are regarded as input to the optimization model. Furthermore, we conduct the optimization in a "receding horizon" way. We first consider multiple cycles (K) in one optimization and a vehicle may pass the intersection within the K cycles. The optimized signal timing is only applied for the next cycle after each optimization. This method essentially is a receding horizon optimization technique, which enhances both the global optimality and the model precision.

C. Vehicle Optimal Control

1) Vehicle Dynamic Model: In vehicle control, we use a longitudinal dynamic model as the vehicle model [17],

$$\dot{s} = -v,$$

$$\dot{v} = \frac{1}{m} \left(\frac{P}{v} - F_b - \frac{1}{2} C_D A \rho_a v^2 - mgf \cos \alpha - mg \sin \alpha \right),$$
(22)

where P is the engine power, F_b means the braking force, m is the vehicle's mass, C_D is the drag coefficient, A is the vehicle frontal area, ρ_a is the air density, f is the rolling resistance coefficient of the tires, g is the gravitational acceleration, α is the road slope, ν is the vehicle speed, and s is the vehicle displacement. Notice that (21) & (22) constitute a more detailed vehicle model compared with the vehicle model in traffic optimization.

We can adjust the vehicle speed by controlling either the engine power P or the braking force F_b with the uncontrolled one being zero. We then select the Virginia Tech Comprehensive Power-Based Fuel Consumption Model (VT-CPFM) [31] as the vehicle fuel consumption model, which is a power-based model to calculate the fuel consumption. It is defined as follows:

$$F_C = \begin{cases} \alpha_0 + \alpha_1 P + \alpha_2 P^2, & \text{if } P > 0\\ \alpha_0, & \text{if } P \le 0, \end{cases}$$
 (23)

where F_c is the fuel consumption rate, and α_0 , α_1 , and α_2 are the coefficients of the fuel consumption model.

2) Safety Constraint: When a vehicle is approaching the intersection, it must keep a safe distance to the front vehicle to avoid front collision:

$$s(t) - s_n(t) \ge d_{\text{safe}},\tag{24}$$

where s_p is the distance of the preceding vehicle to the stop line and d_{safe} is the safe following distance. Here (24) ensures the safe distance at any time, i.e., between the trajectories of the two vehicles.

3) Terminal Constraints: Some terminal constraints including initial state constraints and the terminal state constraints should be met.

Firstly, the initial state is determinate:

$$s(0) = d_0, (25)$$

$$v(0) = v_0,$$
 (26)

where v_0 is the initial velocity of the vehicle and d_0 is the initial distance to the stop line.

The vehicles must follow the time schedules planned by *Traffic Optimization*, so the terminal time of *Vehicle Optimal Control* refers to arrival time in *Traffic Optimization*. Therefore.

$$s\left(t_{f}\right) = 0,\tag{27}$$

where $t_{\rm f}$ is the fixed terminal time which is optimized in *Traffic Optimization* as $\tau_L^{(i)}$, i.e. $t_{\rm f} = \tau_L^{(i)}$.

4) Cost Function: Primarily, the fuel consumption of the whole trip is considered in the cost function in individual vehicle control, while ensuring the arrival time constraint from the traffic optimization model. Besides, the terminal velocity of the vehicle (which reflects the vehicle kinetic energy at the terminal time) has great influence on the fuel consumption after the vehicle passes the intersection. This is because when a vehicle passes an intersection with a low speed, it has to accelerate to a higher speed (maybe the speed of free traffic flow) after going through the intersection. During the acceleration, its engine power increases sharply, which results in lots of fuel consumption. Since the terminal speed of the vehicle has great influence on the future fuel consumption, we considered the cost function with two parts, i.e., the vehicle fuel consumption of the total trip from the initial position to the intersection and the future fuel consumption after leaving the intersection (represented by the terminal speed). Hence, the cost function [32] is:

$$k_1 \int_0^{t_f} F_C(t) dt - k_2 v (t_f)^2,$$
 (28)

where k_1 and k_2 are the weight coefficients of the fuel consumption and vehicle kinetic energy.

5) Vehicle Optimization Model: An optimal control problem is built to optimize the vehicle engine power P and the brake force F_b . Combining the vehicle dynamics, the safety constraint, the terminal constraints and the cost function, we construct the vehicle optimal control model as:

$$\begin{aligned} \left\{P^*, F_b^*\right\} \\ &= \arg\min k_1 \int_0^{t_f} F_C\left(t\right) dt - k_2 v \left(t_f\right)^2, \\ &\text{subject to: vehicle dynamics: (21) and (22),} \\ &\text{fuel consumption model: (23),} \\ &\text{safety constraint: (24),} \\ &\text{terminal constraints: (25), (26), and (27).} \end{aligned}$$

As a summary of this section: by considering the total vehicle arrival time in the traffic optimization model and the fuel consumption in the individual vehicle control model, the proposed cooperative method (i) simultaneously considers traffic efficiency and fuel use of the intersection; and (ii) considers efficiency with higher priority, i.e., the primary goal is to ensure efficiency of all vehicles, while at the same time to minimize vehicle fuel consumption. Such consideration also decomposes the efficiency and energy objectives, which makes the cooperative method easier to construct and solve.

IV. SOLUTION METHODS

A. Solution Method for Traffic Optimization

The proposed traffic optimization model is not a standard nonlinear programming (NLP) problem because of the discrete interval constraints in $(4)\sim(11)$. Thus, it is difficult to solve the optimization model by standard NLP solvers. Here, we adopt an enumeration method to achieve the minimal total travel time of the approaching vehicles. The detailed procedures of the enumeration method are shown below:

- (i) We first discretize the traffic signal timing [t_{NBL} , t_{SBT} , twbl, tebt, tsbl, tnbt, tebl, twbt] considering the constraints (1), (2), (12), (13), (14), and (15), which leads to N signal timing schemes. Here, N is the number of traffic signal timing schemes after we conduct the discretization. We give an example to show how to discretize the traffic signal timing. Assume C = 60s, $g_{min} = 8s$, $g_{max} = 24s$, R = 2s, and the discrete step of 2s. We first discretize t_{NBL} as [0, 10, 12, 14, 16, 18, 20, 22, 24, 26]. Secondly, for given t_{NBL} (say 14), we can discretize t_{SBT} as [0, 10, 12, 14, 16, 18, 20, 22, 24, 26]. Thirdly, for given t_{SBT} (say 20), we discretize t_{SBL} and t_{NBT} as [(10, 24), (12, 22), (14, 20), (16, 18), (18, 16), (20, 14), (22, 12), (24, 10)] so that they add up to 34s. Then, we discretize t_{WBL} and t_{EBT} as [(10, 16), (12, 14), (14, 12), (16, 10)]. Finally, we discretize $t_{\rm EBL}$ and $t_{\rm WBT}$ as [(10, 16), (12, 14), (14, 12), (16, 10)] so that they add up to 26s. After the discrete procedures described above, we get 1092 (N) signal timing schemes.
- (ii) Next, we will calculate the total travel time for each signal timing scheme with the constraints of $(3)\sim(11)$ and $(16)\sim(19)$.
- (iii) Finally, we select the optimal signal timing scheme with the minimal total travel time.

Notice that the discrete interval of the traffic signal timing should not be too small. Otherwise, the number of traffic signal timing schemes N will be too large, which is computationally demanding. In this sense, the enumeration method is a brute force method that may lead to a huge computation load, especially when the traffic signal cycle length is long or the discrete step of signal timing is small. However, if the discrete step of the traffic signal timing is chosen properly, the enumeration method is able to calculate the approximate global optimum of the traffic signal timing with fairly efficient computation. In this paper, we set the discrete step to 2s because: (1) the smaller discrete step leads to a huger calculation load; (2) 2s is a common discrete step in traffic optimization which can also help us find a suboptimal solution. In a word, the discrete step to 2s is a balance of the computation load and the solution precision.

Taking a typical case with C=80s, $g_{\min}=8$ s, $g_{\max}=30$ s, and the discrete step of 2s as an example, we conduct the calculation for 500 times, of which the results show that the average and maximum CPU times are 0.28s and 0.45s.

B. Solution Method for Vehicle Control

The proposed vehicle control method is an optimal control (OPC) problem with nonlinear objective function and

kinetic equations, which is challenging to solve. Since the 1950s, several theoretical developments, such as the variational method and maximum principle, have been introduced to solve such OPCs. These methods are generally demanding computationally and highly dependent on the initial values. They also tend to have a small radius of convergence, which may result in suboptimal or infeasible solutions, especially for complex and high-dimensional OPCs. To solve high-dimensional and complex nonlinear OPCs, the shooting method, the dynamic inversion method, and the pseudospectral method have been proposed [33]. Among these methods, the pseudospectral method is of higher computational efficiency and accuracy [34]. In this study, we use the Legendre pseudospectral method to solve the proposed OPC in (29). The method uses Legendre interpolating polynomial to approximate the state and control variables.

The steps of Legendre pseudospectral method are shown as step 1 \sim step 6 [34]. The main idea of Legendre pseudospectral method is to transform a continuous optimal control problem into a discrete combinational optimization problem using a global interpolating polynomial. Step 1 and Step 2 try to find the discrete time, i.e., the collocation points for the differential state equations and the integration objective function using Legendre polynomial. Step 3 \sim Step 5 carry out the discretization of states and control variables in state equations and the integration objective function. Additionally, in Step 6, we eventually obtain the discrete NLP.

Step 1 (Time Domain Transformation):. We transform the time domain $t \in [0, t_f]$ to τ in the standard time interval [-1, 1] as (30).

$$\tau = \frac{2t}{t_f} - 1, \, \tau \in [-1, 1]. \tag{30}$$

Step 2 (Collocations Calculation): We use the Newton-Raphson method to iterate and finally find the collocation points. Here, collocation points denote the discrete points for the state and control variables. The initial iteration value of the collocation points $\tau_{j,0}$ is selected as the roots of the N-order Chebyshev-Gauss-Lobatto polynomial.

$$\tau_{j,0} = \cos\left(\frac{\pi j}{N}\right), \quad j = 0, 1, \dots, N.$$
(31)

For each collocation point τ_j , we use Legendre polynomial to iteratively calculate the final collocations as (32),

$$\tau_{j,i+1} = \tau_{j,i} - \frac{\tau_{j,i} P_N(\tau_{j,i}) - P_{N-1}(\tau_{j,i})}{N P_N(\tau_{i,i})}$$
(32)

where

$$P_1\left(\tau_{i,i}\right) = 1\tag{33}$$

$$P_2\left(\tau_{j,i}\right) = \tau_{j,i},\tag{34}$$

$$P_{n+1}(\tau_{j,i}) = \frac{(2n+1)\tau_{j,i}P_n(\tau_{j,i}) - nP_{n-1}(\tau_{j,i})}{n+1}.$$
 (35)

When the iteration errors are small enough, we can finish the iteration and find the final N collocation points τ_j $(j = 0, 1, 2, \dots, N)$.

TABLE I ASC SIMULATION PARAMETERS

Distance of Detectors [m]	20
Minimum Green Time [s]	7
Maximum Green Time [s]	50
Unit Extension Time [s]	3
All-red Time [s]	2

Step 3 (States and Control Variables Discretization): After we determine the collocation points, we can use the Lagrange interpolation as $(36)\sim(38)$ to approximate the state and the control variables using the value of the state and the control variables at the collocation points.

$$x(\tau) \approx X(\tau) = \sum_{i=0}^{N} L_i(\tau) X_i, \qquad (36)$$
$$u(\tau) \approx U(\tau) = \sum_{i=0}^{N} L_i(\tau) U_i, \qquad (37)$$

$$\mathbf{u}\left(\tau\right) \approx U\left(\tau\right) = \sum_{i=0}^{N} L_{i}\left(\tau\right) U_{i},\tag{37}$$

$$L_{i}(\tau) = \prod_{j=0, j \neq i}^{N} \frac{\tau - \tau_{j}}{\tau_{i} - \tau_{j}}.$$
(38)

In this paper, $\mathbf{x} = [s, v]^{\mathrm{T}}$, $\mathbf{u} = [P, F_{\mathrm{b}}]^{\mathrm{T}}$, $X_i = [s_i, v_i]^{\mathrm{T}}$, and $U_i = [P_i, F_{bi}]^T$, where s_i, v_i, P_i , and F_{bi} are the displacement, the speed, the engine power and the brake force respectively at the collocation points.

Step 4 (State Space Equation Transformation): We transform the differential form of the state space equations into the algebraic form with the values of the state and the control variables at the collocation points, as (39).

$$\dot{\boldsymbol{x}}\left(\tau_{j}\right) \approx \sum_{i=0}^{N} \dot{L}_{i}\left(\tau_{j}\right) X_{i} = \sum_{i=0}^{N} \boldsymbol{D}_{ji} X_{i}$$

$$\left(j = 0, 1, \dots N\right), \quad (39)$$

where D_{ii} denotes the element of the differential matrix D

$$D_{ji} = \begin{cases} \frac{P_{N}(\tau_{j})}{P_{N}(\tau_{i})(\tau_{j} - \tau_{i})}, & \text{if } l \neq j \\ -N(N+1)/4, & \text{if } l = j = 0 \\ N(N+1)/4, & \text{if } l = j = N \\ 0, & \text{otherwise.} \end{cases}$$
(40)

Then the state space equation can be transformed as the following constraints.

$$\sum_{i=0}^{N} \mathbf{D}_{ji} \begin{bmatrix} s_i \\ v_i \end{bmatrix} - \frac{t_f}{2} \begin{bmatrix} \frac{-v_j}{m} - F_{bj} - \frac{1}{2} C_D A \rho_a v_j^2 - \\ \frac{1}{m} \left(\frac{P_j}{v_j} - F_{bj} - \frac{1}{2} C_D A \rho_a v_j^2 - \right) \end{bmatrix} = 0,$$
(41)

where $i = 0, 1, \dots, N$.

Step 5 (Objective Function Transformation): We transform the objective function using the Gauss-Lobatto quadrature rule to enhance the accuracy of transforming the integral:

$$J = k_1 \frac{t_f}{2} \sum_{i=1}^{N} \omega_j F_C(P_j, \tau_j) - k_2 v_N^2, \tag{42}$$

where w_i denotes the integration weight, defined as:

$$\omega_j = \frac{2}{N(N+1)P_N(\tau_i)^2}.$$
 (43)

TABLE II VEHICLE PARAMETERS

Mass	Drag Coefficient	Vehicle Frontal Area
1500kg	0.4	1.5m^2
Air Density	Rolling Resistance Coefficient	Maximum Power
1.2kg/m ³	0.015	110kw
Fuel Consumption Model		
α_1	α_2	a_3
0.59	0.057	0.00014

Step 6 (NLP Transformation): Using the above steps, we can transform the proposed OPC to NLP problem as follows,

$$\left\{P_{j}^{*}, F_{bj}^{*}\right\}$$

$$= \arg\min k_{1} \frac{t_{f}}{2} \sum_{j=1}^{N} \omega_{j} F_{C}\left(P_{j}, \tau_{j}\right) - k_{2} v_{N}^{2},$$

$$\text{subject to: } \sum_{i=0}^{N} D_{ji} \begin{bmatrix} s_{i} \\ v_{i} \end{bmatrix}$$

$$-\frac{t_{f}}{2} \left[\frac{1}{m} \left(\frac{P_{j}}{v_{j}} - F_{bj} - \frac{1}{2} C_{D} A \rho_{a} v_{j}^{2} - \right)\right] = 0,$$

$$s_{j} - s_{fj} \geq d_{0},$$

$$s_{0} = 0,$$

$$v_{0} = 0,$$

$$s_{N} = d.$$
(44)

We conducted the Legendre pseudospectral method for 500 times in cases with the collocation number of 20, the random initial speed from 5m/s to 15m/s, the random distance from 500m to 800m, and random arrival time, of which the results show that the average CPU time is 0.16s and the maximum CPU time is 0.3s.

The NLP problem in (44) is a nonlinear problem with a non-convex objective and non-convex constraints. Generally, this kind of problems is challenging to solve. Fortunately, our vehicle control problem here has a relatively small dimension (in hundreds of variables and constraints), which can be solved by standard NLP solvers such as SNOPT [35]. These solvers always result in a local solution, which is acceptable for the vehicle control model here.

V. SIMULATION AND RESULTS

A. Simulation Setup

To verify the proposed cooperation method, we conducted simulation with VISSIM and MATLAB. First, we implemented the actuated signal control [3] (ASC) method in VISSIM and considered it as one of the baseline algorithms. Some parameters of the ASC simulation can be seen in TABLE I. Meanwhile, the vehicles' inputs and trajectories were exported via the COM port of VISSIM. Secondly, we used Equation (23) to calculate the fuel consumption of the vehicles simulated in ASC with the recorded trajectories data. TABLE II shows some vehicle parameters used in the fuel consumption calculation which we obtained from a vehicle model of CARSIM [36]. Thirdly, we conducted simulations

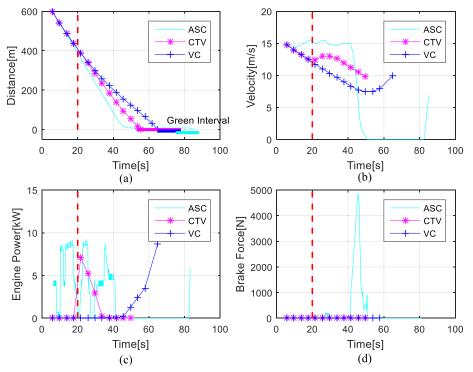


Fig. 5. Vehicle state and control variables of ASC, VC, and CTV. (a) Displacement. (b) Velocity. (c) Engine power. (d) Brake force.

of the Cooperative method of Traffic signal optimization and Vehicle control (CTV) using the same vehicle input data from VISSIM. Then, we used the Vehicle Control algorithm (VC) with fixed traffic signal timing as another baseline algorithm. In this algorithm, vehicles will be firstly assigned the optimal arrival times based on the fixed timing of traffic signal. Their trajectories will be optimized by the vehicle control model in Section III. The same vehicle model as TABLE II was used in CTV simulation and VC simulation.

B. Case Study

We first show the results of one case study with the traffic demand of 300 vehicle/(hour-lane) in the northbound/ southbound flow and the eastbound/ westbound flow. The communication ranges of CTV and VC are both 600m. The cycle length of CTV and VC is 60s.

Fig. 5 shows the trajectories, speed profiles, engine power curves, and brake force curves of one randomly selected vehicle from NBL with the CTV, VC and ASC control.

Fig. 5(a) shows the trajectories. The bold horizontal lines indicate the effective green intervals of different algorithms. It can be seen that the effective green interval of CTV is earlier than those of ASC and VC. This leads to the shorter travel time compared with the ASC and CV algorithms.

Fig. 5(b) shows the speed profiles. We can see that the vehicle with ASC runs with higher speed during $5s \sim 40s$, and has to brake and to keep idling after 40s. The vehicles with CTV and VC slow down ahead and then go through the intersection without any stop and idling. Furthermore, the vehicle under VC reaches a lower speed because of the longer travel time compared with that under the CTV.

Fig. 5(c) shows engine power curves. The vehicle speed with ASC fluctuates because of the car following model

used in VISSIM, which causes 0.9g more fuel consumption according to our computation. Furthermore, the average engine power of ASC during 5s \sim 40s is around 4.5kW which is higher than those of CTV and VC.

Fig. 5(d) shows the brake force curves. We can see that the ASC vehicle brakes at about 40s, while the CTV vehicle and the VC vehicle pass the intersection without any braking.

It is worth noticing that the traffic optimization of CTV was conducted in 20s (shown as the vertical bold dashed lines in Fig. 5), after which the vehicle travel times were re-optimized. It caused the saltation of the engine power curve and the speed profile of the CTV vehicle in Fig. 5.

Since vehicle fuel consumption depends on the specific driving pattern (i.e., the trajectory or speed profile) of the vehicle, we expect that the three algorithms will lead to very different fuel consumption on the selected vehicle. The simulation results confirm this. In particular, the vehicle consumes less fuel under CTV because it slows down ahead to avoid braking and idling at the intersection compared with the ASC. Besides, the vehicle under CTV passes the intersection using shorter time because its green interval starts earlier compared with the VC. Specifically, the vehicle fuel consumption under CTV, VC, and ASC for the whole trip are 32.8g, 38.9, and 57.8g, respectively. Furthermore, the vehicle travel times of CTV, VC, and ASC for the whole trip are 49.7s, 59.2s, and 79.5s, respectively.

C. Impact of Cycle Length and Communication Range

The cycle length and the communication range are both critical parameters for the proposed cooperative method. To find out the algorithm performance with different cycle lengths and communication ranges, we conducted more simulations.

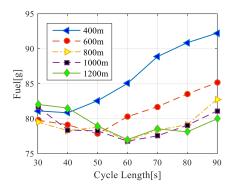


Fig. 6. Fuel consumption.

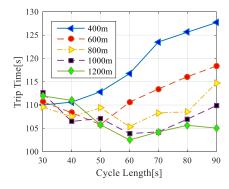


Fig. 7. Travel time.

The traffic demand of the northbound/ southbound movements is 300 vehicle/(hour·lane) and that of the eastbound/westbound movements is 400 vehicle/ (hour·lane). The cycle length varies from 30s to 90s and the communication range varies from 400m to 1200m. The fuel consumption and travel time in the whole trip were calculated.

Fig. 6 shows the average fuel consumption of CTV in the scenarios with different cycle length and communication range conditions. Fig 7 shows the average travel time. It can be seen from these two figures that a wider range of communication leads to a better performance of fuel economy and transportation efficiency in general. However, for a given cycle length, the fuel consumption and travel time decrease with the increase of the communication range more sharply when the range is small and less so after the range reaches 600m. For example, when the communication range is extended from 400m to 600m with the cycle length of 60s, the fuel consumption drops from 85g to about 80g, while fuel consumption keeps nearly unchangeable (about 76g) after the communication range reaches over 800m. It means that we do not need to extend the communication range to acquire better performance when the communication range is over 800m. Actually, in the situation studies, when the communication range reaches 800m, the vehicles and traffic signal have enough time and space to adjust their speed and timing, respectively. In this case, increasing the communication range further does little to improve the performance of the proposed algorithm.

From Fig. 6 and Fig. 7, we can see that the communication range correlates with the cycle length. The best cycle length for communication range of 400m, 600m, 800m, 1000m, and 1200m are about 40s, 50s, 60s, 60s, and 60s, respectively.

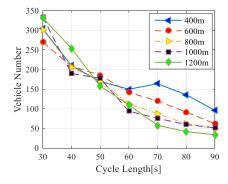


Fig. 8. Number of oversaturated vehicles.

Fig. 8 shows the number of oversaturated vehicles in the scenarios with different cycle length and communication range conditions. We can see from the figure that longer cycle length leads to fewer oversaturated vehicles. It is because the longer cycle length means longer green intervals that result in more vehicles passing the intersection within a cycle length.

When the communication range is small, the travel time within the communication range is short. In this case, shorter cycle length can lead to more frequent adjustment of the traffic signal timing, which results in less fuel consumption and smaller travel time. In reverse, when the communication range is large, the travel time within the communication range becomes long. There is enough time to adjust the signal timing. In this case, the short cycle length results in large loss time of the traffic signal, which causes degraded performance. Based on the results and analysis above, we can conclude that the wider the communication range, the longer the optimal traffic signal cycle length.

D. Impact of Traffic Demand

To further test the proposed cooperative method in different traffic conditions, we conducted simulations with different levels of traffic demands.

We set three groups of simulation cases:

Group A: the traffic demands of each entrance lane are the same. It consists of four cases with the traffic demands of 100, 200, 300, and 400 vehicles/hour in each•lane, respectively.

Group B: the traffic demands of south/north-bound entrance lanes are different from those of east/west-bound entrance lanes. It consists of four cases with the south/north-bound traffic demands of 50, 150, 250, and 350 vehicles/hour•in each lane, and the east/west-bound traffic demands of 150, 250, 350, and 450 vehicles/hour•in each lane, respectively.

Group C: the traffic demands of each entrance lane are equal, and first increasing and then decreasing with time.

The communication range of the three groups of simulation cases is set as 600m and the cycle length is 60s. The time of each simulation in Group A and B is three hours, and the total simulation time of Group C is six hours. We calculated the fuel consumption, travel time for the entire trip of 600m, and the ratio of stopped vehicles. Here, the speed threshold for stop definition is set as 3 m/s, i.e., when the speed of a vehicle is lower than 3 m/s, it is categorized as the stopped vehicle. The performances of CTV, CV, and ASC in each group of simulation cases are shown in Fig. 9~Fig. 11.

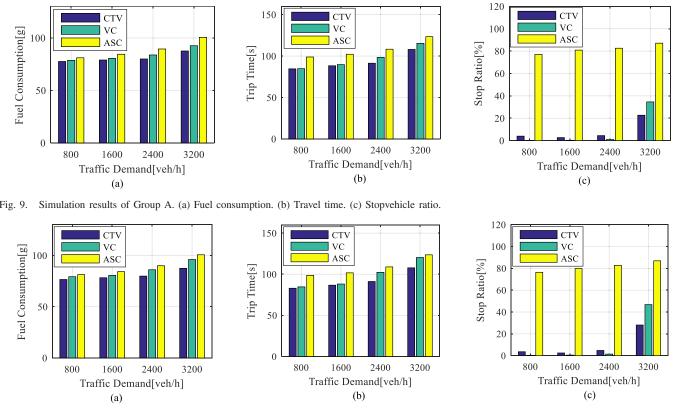


Fig. 10. Simulation results of Group B. (a) Fuel consumption. (b) Travel time. (c) Stopvehicle ratio.

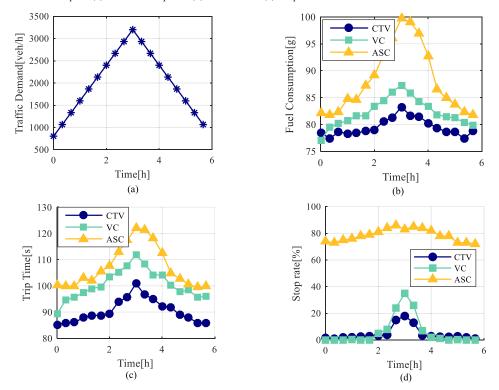


Fig. 11. Simulation results of Group C. (a) Traffic demand. (b)Average fuel consumption. (c)Average travel time. (d) Stop rate.

From the Fig 9 \sim 10, we can see that vehicles with all three algorithms consume less fuel and spent shorter time for the trips in low traffic demands than those in heavy traffic demands. Additionally, vehicles with the CTV algorithm always consumed less fuel and spent less time during the

trips compared with VC and ASC. Additionally, the CTV and VC algorithms can significantly decrease the number of vehicle stops, which results in short travel time and less fuel consumption. Furthermore, as demands become heavier, the improvements of VC/CTV over ASC decreases.

Fig 11(a) shows the total traffic demand variation with time in the simulation of Group C. From the simulation results in Fig 11(b), Fig 11(c) and Fig 11(d), we can also draw conclusions that the proposed cooperative method can improve the traffic efficiency and vehicle economy in the environment of varying traffic demand.

Note that the vehicles in ASC cannot predict the traffic signal phasing and timing while the vehicles in CTV and VC can receive this information from V2X communication. Therefore, the vehicles in CTV and VC can adjust their speed to avoid meeting the red light and stopping. Consequently, the stop rates of CTV and VC in the simulation is significantly smaller than that of ASC.

VI. CONCLUSION AND FUTURE WORK

In this paper, we proposed a cooperative method for traffic signal optimization and vehicle speed control to improve the transportation efficiency and vehicle fuel economy both in macro traffic level and micro vehicle level. First, we built a mathematic model of cooperative optimization between traffic signal and vehicle speed, which consists of traffic optimization and vehicle optimal control. In the roadside traffic optimization, the traffic signal timing and vehicles' arrival time were optimized cycle by cycle with the aim to reduce the total vehicle travel times. Then the vehicles' optimal arrival times were sent to corresponding vehicles. In onboard vehicle optimal control, the engine power, brake force, speed profiles and trajectories of an approaching vehicle were optimized, considering the constraints of vehicle dynamics, arrival time, and car following safety with the aim to reduce fuel consumption of the vehicle. The enumeration method and the pseudospectral method were used to solve the traffic optimization problem and vehicle optimal control problem, respectively. Finally, we conducted numerical experiments in simulations to test the proposed cooperative method, and compared the performance of the method with two benchmark algorithms with different traffic demands, cycle lengths, and V2I communication ranges.

The simulation results showed that the cooperative method can effectively adjust the traffic signal timing according to the real-time traffic condition and simultaneously produce optimal vehicle trajectory/speed profiles, which results in the improved transportation efficiency and vehicle fuel economy. In addition, the cycle length and communication range have significant effects on the performance of the algorithm. Specifically, a wider communication range works better with a longer cycle length, and the algorithm performs the best with the cycle length of 60s and communication range of about 800m in the studied scenarios. Furthermore, the simulations with different traffic demands demonstrated that the proposed algorithm improves the transportation efficiency and fuel economy in the stable and varying traffic volumes.

Note that the proposed cooperative method is a general idea for the intersection management of CAVs and it can be applied in not only the 4-leg and 2-lane intersections but also other kinds of intersections. To extend this method to other intersections, we only need to re-formulate the green phase constraints ($4\sim11$) and the signal timing constraints ($12\sim14$).

The proposed method can be extended in a few important directions. First, the method assumes 100% penetration of CAVs. Before the complete deployment of CAVs, the CAVs and human-driven vehicles may co-exist for a long time. In future work, the human-driven vehicles must be considered in the cooperative optimization method, and the algorithm performance with different CAV penetration should be studied. Secondly, the enumeration method for solving the traffic optimization model is brute force, and may be computationally demanding. Future research should focus on developing more rigorous methods to solve the problem. Thirdly, the current method assumes a rather restrictive intersection geometry and lane group scenarios (i.e, two lanes for each approaching direction), which should be relaxed in future studies. Last but not least, coordination of multiple intersections has the potential to further improve the transportation efficiency and vehicle fuel economy, which is worth studying in future research.

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Biao Xu (M'18) received the B.E. degree from Tsinghua University, Beijing, China, in 2013, where he is currently pursuing the Ph.D. degree with the Department of Automotive Engineering. From 2016 to 2017, he was a Visiting Scholar with University of Washington, Seattle, WA, USA.

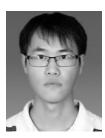
His active research interests include connected and automated vehicles, vehicle control, and V2I cooperation. He was a recipient of the Best Paper Award from the 14th Intelligent Transportation Systems Asia-Pacific Forum in 2015 and the Best Paper

Award from the 2017 IEEE Intelligent Vehicle Symposium.



Xuegang Jeff Ban received the B.E. and M.S. degrees in automotive engineering from Tsinghua University, Beijing, China, in 1997 and 2000, respectively, and the M.S. degree in computer sciences and the Ph.D. degree in transportation engineering from University of Wisconsin–Madison, Madison, WI, USA, in 2003 and 2005, respectively. He is currently an Associate Professor with the Department of Civil and Environmental Engineering, University of Washington, Seattle, WA, USA.

He has authored over 100 papers in peer-reviewed journals and conferences. His research interests include transportation network modeling and simulation, urban traffic system modeling and operations, intelligent transportation systems, and connected and automated vehicles. He was a recipient of the 2011 CAREER Award from the U.S. National Science Foundation, the New Faculty Award from the Council of University Transportation Centers and the American Road and Transportation Builders Association in 2012, and the Best Paper Award in the 2017 IEEE Intelligent Vehicle Symposium. He serves as an Associate Editor for *Journal of Intelligent Transportation Systems* and *Networks and Spatial Economics*, and serves on the Editorial Board of *Transportation Research*, Parts B and C.



Yougang Bian received the B.E. degree from Tsinghua University, Beijing, China, in 2014, where he is currently pursuing the Ph.D. degree with the Department of Automotive Engineering. His research interests include distributed control of dynamical systems and its applications to connected and automated vehicles.

Mr. Bian received the Excellent Undergraduate Dissertation Award at Tsinghua University in 2014 and the Best Paper Award at the 2017 IEEE Intelligent Vehicles Symposium.



Wan Li received the B.S. degree in traffic engineering from Sun Yat-sen University, Guangzhou, China, in 2012 and the M.S. degree in civil engineering from Louisiana State University, Baton Rouge, LA, USA, in 2014. She is currently pursuing the Ph.D. degree in civil engineering with University of Washington, Seattle, WA, USA.

Her current research interests include connected/autonomous vehicles and transportation big data analytics.



Jianqiang Wang received the B. Tech. and M.S. degrees from Jilin University of Technology, Changchun, China, in 1994 and 1997, respectively, and the Ph.D. degree from Jilin University, Changchun, in 2002. He is currently a Professor with the Department of Automotive Engineering, Tsinghua University, Beijing, China.

He has authored or coauthored over 40 journal papers and is the co-holder of 30 patent applications. He was involved in over 10 sponsored projects. His active research interests include intelligent vehicles,

driving assistance systems, and driver behavior. He was a recipient of the Best Paper Award in the 2014 IEEE Intelligent Vehicle Symposium, the Best Paper Award in the 14th ITS Asia Pacific Forum, the Best Paper Award in the 2017 IEEE Intelligent Vehicle Symposium, the Distinguished Young Scientists of NSF China in 2016, and the New Century Excellent Talents in 2008.



Shengbo Eben Li (SM'16) received the M.S. and Ph.D. degrees from Tsinghua University in 2006 and 2009, respectively. He was with Stanford University, The University of Michigan, and University of California at Berkeley. He is currently an Associate Professor with the Department of Automotive Engineering, Tsinghua University.

He has authored over 100 peer-reviewed journal/conference papers and the co-inventor of over 20 Chinese patents. His active research interests include intelligent and connected vehicles,

learning-based driver assistance, distributed control and optimal estimation, and electrified powertrain management. He serves as a TPC Member of the IEEE Intelligent Vehicle Symposium and an ISC Member of FAST-zero 2017, Japan. He was a recipient of the Best Paper Award in the 2014 IEEE Intelligent Transportation System Symposium, the Best Paper Award in the 14th ITS Asia Pacific Forum, the National Award for Technological Invention, China, in 2013, the Excellent Young Scholar of NSF China in 2016, and the Young Professorship of the Changjiang Scholar Program in 2016. He serves as an Associated Editor of IEEE Intelligent Transportation Systems Magazine and IEEE Transactions on Intelligent Transportation Systems Magazine



Keqiang Li received the B.Tech. degree from Tsinghua University, Beijing, China, in 1985, and the M.S. and Ph.D. degrees from Chongqing University, Chongqing, China, in 1988 and 1995, respectively. He is currently a Professor in automotive engineering with Tsinghua University.

He has authored over 90 papers and is a co-inventor on 12 patents in China and Japan. His research interests include vehicle dynamics, control for driver-assistance systems, and hybrid electrical vehicles. He is a Senior Member of the Society

of Automotive Engineers of China. He was a recipient of the Changjiang Scholar Program Professor Award and some awards from public agencies and academic institutions of China. He is on the Editorial Boards of *International Journal of Intelligent Transportation Systems Research* and *International Journal of Vehicle Autonomous Systems*.