

iii - Ankit Singh - 23BCS12931 246

Q-1) Given three integers n, a, b , return the n th magical number.

A positive integer is magical if it is divisible by either a or b .

$1 \leq n \leq 10^9$

$1 \leq a \leq 10^5$

→ Approach:

→ Brute Force: $O(n)$ We start from 1 and then check if it is divisible by a or b and have a count to count frequency.
→ Not optimal for large range.

→ Efficient Approach

$O(\log n)$ (Binary Search) + (Inclusion-exclusion)

$$\rightarrow l = \min(a, b)$$

$$\rightarrow r = n * \min(a, b)$$

→ First find a mid then check count by:

$$\left\lfloor \frac{mid}{a} \right\rfloor + \left\lfloor \frac{mid}{b} \right\rfloor + \left\lfloor \frac{mid}{lcm(a, b)} \right\rfloor$$

→ If count < n then move to right

else if count $\geq n$ move to left

→ Code: int Magical(int min, int max) {

$$long l = \min(a, b);$$

$$long r = n * \min(a, b);$$

long long ans;

while ($l \leq r$) {

$$long mid = l + (r - 1) / 2;$$

$$long long count = \text{floor}(\frac{mid}{a} + \frac{mid}{b} - \frac{mid}{lcm(a, b)})$$

If (count < n) {

$$l = mid + 1; } }$$

else if (count $\geq n$) {

$$ans = mid;$$

$$mid = r - 1; } }$$

return ans; } }

int GCD (int a, int b){
if (a == b == 0) return a;
return (a % b);}

int lcm (int a, int b){

return 1LL * a * b / GCD(a, b);}