

Foundation Exam-Summer 2021-Section 2A

1. a) Worst case: $O(nq)$

This would be the runtime since the worst case scenario would be if the boundaries were 0 to $n-1$. This would lead to the contiguous subsequence sum to have a runtime of $O(n)$. Since there are q questions, worst case runtime is $O(nq)$.
↑ variable

b) best case: $O(q)$

The best-case scenario would be if the low and high boundaries are the same index (0 to 0). The contiguous subsequence sum would have a constant number of operations (for-loop executed once), so the runtime is $O(1)$. Since there are q questions $O(1) \times q = O(q)$, which is the best-case runtime.

2. n users $\rightarrow O(\log(n))$
- 10^4 users is 10 milliseconds
- assume $O(\log_{10}(n))$

$$10 \text{ milliseconds} = \text{constant} \times \log_{10}(10^4)$$

$$10 \text{ milliseconds} = \text{constant} \times 4$$

$$\text{constant} = 2.5 \text{ milliseconds}$$

$$20 \text{ milliseconds} = \text{constant} \times \log_{10}(\text{users})$$

$$20 \text{ ms} = 2.5 \text{ ms} \times \log_{10}(\text{users})$$

$$8 \text{ ms} = \log_{10}(\text{users})$$

$$10^8 = \text{users}$$

We can support 10^8 users while taking no more than 20 seconds per query.

(Question 3 on back)

$$3 \quad 2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

~~$$2^5 - 1$$~~

$$i=3 \quad 2^{i+1} - 1$$

$$\sum_{i=0}^n \sum_{j=0}^i 2^j = \sum_{i=0}^n (2^{i+1} - 1) = 2 \sum_{i=0}^n 2^i - n$$

$$= 2(2^{n+1} - 1) - n$$

$$= 2^{n+2} - n - 2$$

$$\boxed{\sum_{i=0}^n \sum_{j=0}^i 2^j = 2^{n+2} - n - 2}$$

due to
constraint
of time
I have
"skipped"
some parts
of work
but they are
obvious
nonetheless