Physical Quantities

Physical Quantities

The quantities which can be measured by an instrument & which describes the laws of physical world

Types of physical Quantities

Physical Quantities

- Fundamental Physical Quantity
- Derived Physical Quantity

Fundamental Quantity: Physical Quantities which does not depend upon other quantity.

S.N	Physical Quantity	Units	Symbol
1.	Length	Meter	m
2.	Mass	kilogram	kg
3.	Time	Second	s
4.	Temperature	kelvin	K
5.	Electric Current	Ampere	A
6.	Luminous Intensity	Candela	cd
7.	Amount of substance	Mole	mol

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Derived Quantity: Physical quantity which are made up of by Fundamental Quantity.

(i) Area → m^2 → length × length
(ii) Volume → m^3 → length × length × length
(iii) Velocity → m/s → $\frac{\text{length}}{\text{time}}$
(iv) Force → ~~m~~ kg m/s^2 → mass × acceleration

Dimensions: The Dimension are the power to which the fundamental units are raised in order to express the derived unity of quantity.

(i) Length → [L]
(ii) Mass → [M]
(iii) Time → [T]
(iv) Temperature → [K] or [θ]
(v) Electric Current → [A] or [I]
(vi) Luminous Intensity → [C]
(vii) Amount of substance → [mol]

∴ If two physical quantities have same units their dimension is also same.

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Dimension of some Physical Quantity

(I) Area → m^2 → $[M^0 L^2 T^0]$
(II) Volume → m^3 → $[M^0 L^3 T^0]$
(III) Density → kg/m^3 → $[M^1 L^{-3} T^0]$
(IV) Force → $kg\ m/s^2$ → $[M^1 L^1 T^{-2}]$
(V) Acceleration → m/s^2 → $[M^0 L^1 T^{-2}]$

Principle of Homogeneity

The Dimension of each term of an equation must be same.

$4s + 2s = 6s$ ✓
$4s + 2kg =$ ✗

Application of Dimensional Analysis

(CRC^3)

Relation   Correctness   Constant   Conversion

(i) To check the correctness of given physical equation.

e.g

$$v^2 = u^2 + 2as$$

Dimension of $[v^2] = [M^0 L^1 T^{-1}]^2 = [M^0 L^2 T^{-2}]$

Dimension of $[v^2] = [M^0L^1T^{-1}]^2 = [M^0L^2T^{-2}]$
Dimension of $[2as] = [M^0L^1T^{-2}] \times [L] = [M^0L^2T^{-2}]$

Since, the Dimension of each term is correct, we can say given equation is correct.

(11) It helps to find the value of any constant in the given physical equation.

e.g In, the equation we can find the value of G where symbols have their usual meaning.

$$F = \frac{G m_1 m_2}{r^2}$$

$$\text{or, } G = \frac{Fr^2}{m_1 m_2}$$

$$\text{or, } [G] = \frac{[MLT^{-2}] \times [L]^2}{[M] \times [M]}$$

$$\text{or, } [G] = \frac{[ML^3T^{-2}]}{[M^2]}$$

$$\text{or, } [G] = [M^{-2}L^3T^{-2}]$$

(iii) It helps us to convert system of measurnment.

Let v_1 and v_2 are two system of measurnment an n_1 and n_2 are numeric values associated with those respectively.

$$n_1 v_1 = n_2 v_2$$

Let, $M_1, L_1 T_1$ are the fundamentals untis for v_1 system and $M_2 L_2 T_2$ are fundamental units for v_2 System the above equation becomes.

$$n_1 \times [M_1^a \; L_1^b \; T_1^c] = n_2 [M_2^a \; L_2^b \; T_2^c]$$

$$n_2 = n_1 \times \left[\frac{M_1^a}{M_2^a}\right] \left[\frac{L_1^b}{L_2^b}\right] \left[\frac{T_1^c}{T_2^c}\right]$$

$$n_2 = n_1 \times \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

e.g.

conversion of 10J in erg

(IV) To find the relation between differrent physical quantities.

e.g

If the force in circular path depends on the m, v & r where the symbol have their usual meaning, then find the relation between them.

Since, the force depends om m, v and r

$F \propto m^a$ .... (I)

$F \propto v^b$ .... (II)

$F \propto r^c$ .... (III)

Combining (I) and (II) and (III)

$F \propto m^a v^b r^c$

$F = K\, m^a v^b r^c$ .... (IV)

where k is propotinality constant.

In, terms of Dimension,

$[MLT^{-2}] = [M]^a \times [LT^{-1}]^b \times [L]^c$

$(MLT^{-2}) = [M]^a \times [L^{b+c}T^{-b}]$

Equating the power of given dim.

we get,

$a = 1$

$b = 2$

$b + c = 1$ $[\because b = 2]$

or, $2 + c = 1$

or, $c = -1$

Using these values in eqn (iv)

$$F = K\, m^1 \times v^2 \times r^{-1}$$

or, $F = \frac{K m v^2}{r}$

Experimentally $K = 1$ unit

$$F = \frac{mv^2}{r}$$

This is required relation between given physical quantities

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Limitation (Drawbacks) of Dimensions.

=) It fails to determine the dimension of dimensionless constant.

=) It doesn't give us the dimension of trignometry, exponentional and algebra like function.

=) It doesn't tell us about the vector or scalar nature of physical quantity.

=) Some physical equation are expressed in ratio and are not able to express in dimension.

Accuracy:

The closeness between calculated value and the value is known as accuracy. Accurate value always coincide with true or standarized value.

e.g If we calculate a mass in lab for three times and found to be 520gm in all three experiments and the true mass of that body is also 520gm then our calculated data is accurate own.

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Precision:

Precision is repetation of data to the particular point is known as precision. For eg. if we calculate a masst af any body and found to be precise data. The precise data may or may not be accurate.

Significant figure.

The meaningful digits in a Number are known as significant figure. With increase in significant figure, the data becomes more significant.

The rules to determine the significant figure:

(i) All non-zero digits are significant e.g 123 or 1.23 have 3 significant.

(ii) All the trapped zeros are significant e.g 1005 2005 have 4 significant figure.

(iii) All the zeros right of the decimal are significant. e.g 0.002 or, 0.1000 both have 3 significant figures.

(iv) All the trailing zero after Number are non-significant. For eg 2000 have only one significant figure

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(v) If the number can be expressed in the power of 10, then it is insignificant.
5×10^{23} or 49×10^{-1} have only two significant.

SHORT ANSWER QUESTION

1.a State principle of Homogenity of dimension
Principle of Homogenity of dimension states that, "In an equation, the dimension of each term should be equal."
i.e

$$a+b=c$$

where, the dimension of a, b, c are same

1.(b) The time dependent physical Quantity P is given by $P=P_0e^{-\alpha t^2}$, where α is constant and t is time. Find the Dimension of α.

$$P=P_0e^{-\alpha t^2}$$

we know, $-\alpha t^2$ has no dimension because it is exponential value

$$-\alpha t^2=[M^0L^0T^0]$$
$$-\alpha[T]^2=[M^0L^0T^0]$$
$$-\alpha=[M^0L^0T^{-2}]$$
$$\therefore \alpha=[M^0L^0T^{-2}]$$

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(c) In, equation $F = at^2$, F is force a is constant t is time, find dimension of a

$$F = at^2$$
$$[MLT^{-2}] = a \times [T]^2$$
$$a = \frac{[MLT^{-2}]}{[T^2]}$$
$$a = [MLT^{-4}]$$

2(a) A student writes $\sqrt{\frac{R}{2GM}}$ for escape velocity check the correctness of the formula using Dimension analysis.

$$\text{Escape Velocity} = \sqrt{\frac{R}{2GM}}$$

$$[LT^{-1}] = \sqrt{\frac{[L]}{2[M^{-1}L^3T^{-2}] \times [M]}}$$

$$[LT^{-1}] = \sqrt{\frac{[L]}{[L^3T^{-2}]}}$$

$$[LT^{-1}] \neq \sqrt{[L^{-2}T^2]}$$

$\therefore$ The formula is incorrect

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2.(b) A simple pendulum of length l and mass m be oscilliating with time period T under gravity. show dimensionally $T = 2\pi\sqrt{\frac{l}{g}}$

Let's Assume

$T \propto l^a$ .... (I)

$T \propto g^b$ .... (II)

combining (I) and (II)

$T \propto l^a g^b$

or, $T = k l^a g^b$.(III) [∵ k is proponality constant]

Now,

$[T] = [L]^a \times [LT^{-2}]^b$

$[T] = [L^a] \times [L^b T^{-2b}]$

$[T] = [L^{a+b} T^{-2b}]$

Eauiting corresponding powers

$a+b = 0$ ... (IV)

$-2b = 1$

$b = -1/2$

Putting value of b in (IV)

$a - 1/2 = 0$

$a = 1/2$

Again Putting values of a, b at (III)

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$$T = K\, l^{1/2}\, g^{-1/2}$$

$$T = K \frac{l^{1/2}}{g^{1/2}}$$

$$T = K\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$ [$\because$ $2\pi = K$, Experimentally

$\therefore$ This is the required equation.

3.(a) What do you mean by relative Error

The relative error is defined as the ratio of the absolute error of the measurnment to the actual measurnment.

(b) Find the percentage Error, 2.226 ± 0.25

$$\text{Percentage Error} = \frac{0.25}{2.226} \times 100$$

$$= 11.06\%.$$

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4. (a) Taking Force, length and Time to be the fundamental quantities find the dimension of (a) density (b) Pressure (c) momentum (d) energy

(a) Density

$D = \frac{m}{V}$

$= \frac{[M]}{[L^3]}$

$= [ML^{-3}]$

(b) Pressure

$P = \frac{F}{A}$

$P = \frac{[MLT^{-2}]}{[L^2]}$

$P = [ML^{-1}T^{-2}]$

(c) Momentum

$M = m \times v$

$= [M\cancel{LT^{-2}}] \times [LT^{-1}]$

$= [MLT^{-1}]$

Energy

$P.E = mgh$

$= [M][LT^{-2}][L]$

$= [ML^2T^{-1}]$

(b) The position x of a particle at time is given by

$$x = \frac{v_0}{d}(1 - e^{at})$$

we know

$at = [M^0L^0T^0]$

$a = [M^0L^0T^{-1}]$

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5.(b) Add 7.21, 12.141 and 0.0028 and then write to an appropriate number of significant figures.

Significant figures's sum
$= 7.21 + 12.141 + .0028$
$= 19.35$

LONG ANSWER QUESTIONS

1.(a) A body of mass m be moving with constant speed v in circle of radius r under centripetal Force F. show dimensionally $F = \frac{mv^2}{r}$.

let's assume

$F \propto m^a$ ---- (I)

$F \propto v^b$ ---- (II)

$F \propto r^c$ ---- (III)

Combining (I), (II) and (III)

$F \propto m^a v^b r^c$

$F = K m^a v^b r^c$ ---- (IV)

[$\because$ K is constant]

Now,

$$[MLT^{-2}] = [M]^a \times [LT^{-1}]^b \times [L]^c$$

$$[MLT^{-2}] = [M^a] \times [L^b T^{-b}] \times [L^c]$$

$$[MLT^{-2}] = [M^a L^{b+c} T^{-b}]$$

Equating the corresponding sides

$a = 1$

$b = 2$

$b + c = 1$

$2 + c = 1$

$c = -1$

Again putting the value of a, b, c in (IV)

$$F = K m^1 v^2 \times r^{-1}$$

$$F = K \frac{mv^2}{r}$$

we know, $K = 1$, Experimentally

$$F = \frac{mv^2}{r} \times 1$$

$$\therefore \quad F = \frac{mv^2}{r}$$

Hence required formula is $F = \frac{mv^2}{r}$

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(b) The distance covered by a Particle in time t is given by.

$$x = a + b + ct^2 + dt^3$$

Given,

$x = [L]$

$a = [L]$ [∵ Principle of Homogenity]

$b = [L]$ [∵ Principle of Homogenity]

$ct^2 = [L]$

$c = \frac{[L]}{[T^2]}$

$c = [LT^{-2}]$

$dt^3 = [L]$

$d = \frac{[L]}{[T^3]}$

$d = [LT^{-3}]$

(c) Calculate the dimensional formula of from the relation in 1 (a)

$$F = \frac{mv^2}{r}$$

$$F = \frac{[M] \times [LT^{-1}]^2}{[L]}$$

$$F = \frac{[M.L^2T^{-2}]}{[L]}$$

$$\therefore F = [MLT^{-2}]$$

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2(a) What do you mean by error in measurnment Is significant figure related with error?

The difference between real value of physical Quantity andmeasured value of Physical quantity is Error

If there are more number of significant figure than there is less error and vice versa.

2.(b) A student measures a distance of about 75 cm with a meter stick. Estimate the percentage error in this measurnment

Measurnment = 75 cm

Error = 0.1 cm

$$\% \text{ Error} = \frac{0.1}{75} \times 100$$

$$= 0.13\%$$

2.(c) How many significant figures in the measurnment 75.0 cm

=) 2 significant figures.

(I) A body is moving through air at high speed v experiments a retarding force F give by $F = kA\rho v^x$, where A is the surface area of a body, ρ is density of air and k is dimensionless constant. Deduce the value of x.

$$F = kA\rho v^x$$

we know.

$$[MLT^{-2}] = [L^2] \times [ML^{-3}] \times [LT^{-1}]^x$$

$$[MLT^{-2}] = [ML^{-1}] \times [L^x T^{-x}]$$

$$[MLT^{-2}] = [ML^{-1+x} T^{-x}]$$

Equiting corresponding sides.

$$-x = -2$$

$$x = 2$$

(II) The velocity $(v) = at + \frac{b}{t+c}$ where a, b, and c are constant. Find the dimension of a, b and c

Given

a, b, c are constant

The Dimension of $c = [T]$ {∵ Principle of Homogenity}

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we know

$$\frac{b}{t+c} = v$$

Taking Dimensions

$$\frac{b}{[T]} = [LT^{-1}]$$

$$b = [LT^{-1+1}]$$

$$b = [L]$$

Again

$$at = [LT^{-1}]$$

$$a = \frac{[LT^{-1}]}{[T]}$$

$$a = [LT^{-2}]$$

$\therefore$ $a = [LT^{-2}]$ $b = [L]$ $c = [T]$

3. The velocity ~~of~~ (v) of a particle depending on time t is given

$$v = a + bt + \frac{c}{d+t}$$

Dimension of $a = [LT^{-1}]$

Dimension of $bt = [LT^{-1}]$

or, $b = \frac{[LT^{-1}]}{[T]}$

or, $b = [LT^{-2}]$

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Dimension of $D = [T]$ {∵ Principle of Homogenity}

Dimension of $\frac{c}{d+t} = [LT^{-1}]$

or, $\frac{c}{[T]+[T]} = [LT^{-1}]$

or $\frac{c}{[T]} = [LT^{-1}]$

or $c = [LT^{0}]$

or, $c = [L]$

9. The Density of Earth is given by $\rho = kg^{x} r^{y} G^{z}$. Obtain the values of x, y, z

Let's assume

$\rho = kg^{x} r^{y} G^{z}$

Taking the Dimension on both sides

$[ML^{-3}] = [LT^{-2}]^{x} \times [L]^{y} \times [M^{-1}L^{3}T^{-2}]^{z}$

$[ML^{-3}] = [L^{x}T^{-2x}] \times [L^{y}] \times [M^{-z}L^{3z}T^{-2z}]$

$[ML^{-3}] = [M^{-z}L^{x+y+3z}T^{-2x-2z}]$

Equating corresponding components

$-z = 1$ $\quad$ $-2x - 2z = 0$ $\quad$ $2x = -2$

$z = 1$ $\quad$ $2x + 2z = 0$ $\quad$ $x = -1$

$2x + 2 = 0$

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$x+y+3z=-3$
$-1+y+3=-3$
$-1+y=-6$
$y=-6+1$
$y=-5$

5. The velocity of a particle change with time according to the relation $v=xt+yt^2+z$. Find the dimensions of x, y and z if v is in ms^{-1} and t in s.

The Dimension of z = $[LT^{-1}]$
The Dimension of xt = $[LT^{-1}]$
$x = [LT^{-2}]$
The Dimension of yt^2 = $[LT^{-1}]$
$y = [LT^{-3}]$

∴ The Dimension of x, y, z are $[LT^{-2}]$ $[LT^{-3}]$ $[LT^{-1}]$

6. If acceleration due to gravity g, the speed of light c and pressure p are taken as fundamental quantities, then find the dimension of length

Let's assume
$l \propto g^a$ ...(I)
$l \propto c^b$ ...(II)
$l \propto p^x$ ...(III)

combining (I) and (II) (III)

$l \propto g^a c^b p^x$

$l = K g^a c^b p^x$

Taking dimension on both side

$[L] = [LT^{-2}]^a \times [LT^{-1}]^b \times [ML^{-1}T^{-2}]^x$

$[L] = L^a T^{-2a} \times L^b T^{-b} \times M^x L^{-x} T^{-2x}$

$[L] = [L^{a+b-x} M^x T^{-2a-b-2x}]$

Equating corresponding values

$x = 0$

$a+b-x = 1$

$a+b = 1 \cdots$ (IV)

$-2a-b-2x = 0$

$-2a-b = 0$

$2a+b = 0 \cdots$ (V)

Subtracting (IV) from (V)

$2a+b = 0$

$\underline{a+b = 1}$

$a = -1$

Again Putting into (IV)

$-1+b = 1$

$b = 2$

So, The required dimension of l is

$g^{-1} c^2 p^0$

$= c^2 g^{-1} p^0$

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7. Find the value of 15 ~~Joule~~ Joule in a system which has 10cm, 100gm and 20s as fundamental units

Given,

$n_1 = 15$ joule — $n_2 = ?$

$m_1 = 1\,kg$ — $M_2 = 100\,gm$

$L_1 = 1\,m$ — $L_2 = 10\,cm$

$T_1 = 1\,s$ — $T_2 = 20\,s$

we know that

$$n_2 = n_1 \times \left[\frac{M_1}{M_2}\right]^a \times \left[\frac{L_1}{L_2}\right]^b \times \left[\frac{T_1}{T_2}\right]^c$$

$$n_2 = 15 \times \left[\frac{1\,kg}{100\,gm}\right]^a \times \left[\frac{1\,m}{10\,cm}\right]^b \times \left[\frac{1}{20\,s}\right]^c$$

we know, the Dimension of Joule is $[ML^2T^{-2}]$ so, $a = 1$, $b = 2$, $c = -2$

$$n_2 = 15 \times \left[\frac{1000}{100}\right]^1 \times \left[\frac{100}{10}\right]^2 \times \left[\frac{1}{20}\right]^{-2}$$

$n_2 = 15 \times 10 \times 10 \times 10 \times 400$

$n_2 = 6000000$

$n_2 = 6 \times 10^6$ new units.

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9\. The length of each side of a cube is 7.203. What is the total surface area and the volume of the cube to appropriate significant fiure.

Length of side = 7.203 m

Total surface area = $6a^2$

$= 6 \times (7.203)^2$

$= 311.3 \, m^2$

Volume = a^3

$= (7.203)^3$

$= 346.39 \, m^3$

8\. The length and breadth of a field are measured as: l (120 ± 2) m and b = (100 ± 5) m respectively. What is the area of field ?