Linear Optimization: Exercises

***Question 1** Put the following problem in standard form:

$$\begin{aligned} \max F &= 3X + 4Y - 5Z \\ 2X - 3Y &\leq 5 \\ Z - 3Y &\geq -3 \\ Z + 3Y &= 3 \\ X \geq 0, Y &\geq 0. \end{aligned}$$

***Question 2** Give the **tableau** corresponding to the following problem:

X3	=	160	-	8 X1	-	4 X2
X4	=	120	-	4 X1	-	6 X2
X5	=	34	-	1 X1		
X6	=	14			-	1 X2
Z	=	0	+	10 X1	+	15 X2

***Question 3** Solve the following maximization problem

B			X1	X2	X3	X4	X5	X6	X7	X8
X3	160	=	8	4	1	0	0	0	0	0
X4	120	=	4	6	0	1	0	0	0	0
X7	34	=	1	0	0	0	-1	0	1	0
X8	14	=	0	1	0	0	0	-1	0	1
	Z+48	=	1	1	0	0	-1	-1	0	0

***Question 4**

A farmer has a piece of farm land, $L = 30$ km^2, to be planted with either wheat or barley or some combination of the two. The farmer has a limited amount of fertilizer, $F = 400$ kg, and pesticide, $P = 100$ kg. Every square kilometer of wheat requires $F_w = 25$ kg of fertilizer and $P_w = 6$ kg of pesticide, while every square kilometer of barley requires $F_b = 15$ kg of fertilizer and $P_b = 10$ kg of pesticide. Let $S_w = 5000$ be the selling price of wheat per square kilometer, and $S_b = 3000$ be the selling price of barley. The farmer want to maximize total selling price of his wheat and barley.

Give the **standard form** of this optimization problem. Then, perform the **first step of the simplex algorithm**.

Question 5 : Give the standard form of the following problem. You can use several steps:

1. model the problem
2. simplify (remove some variables)
3. put into standard form.

A refinery buy crude oil (of two different quality) and produce and sell several products (gasoline, kerosen, fuel oil and residue) (see first table). The second table gives the amount of each product produced from one barrel of crude oil, the production cost and the maximal production for each product.

	Product	€/barrel
purchase	crude oil 1	24
	crude oil 2	15
sale	Gasoline	36
	Kerosene	24
	Fuel Oil	21
	Residue	10

	Yield (%)		maximal production
	crude oil 1	crude oil 2	(barrel/day)
Gazoline	80	44	24000
Kerosene	5	10	2000
Fuel Oil	10	36	6000
Residue	5	10	
Production cost (€/barrel)	0.5	1	

***Question 6** Solve the problem:

$$\begin{aligned} \max_{X1,X2,X3} Z = 3X1 + 4X2 - 5X3 \\ X1 + X2 + X3 \geq 4 \\ X1 - X3 \leq 1 \\ X2 + 2X3 \leq 2 \\ X1, X2, X3 \geq 0. \end{aligned}$$

***Question 7** Solve the problem:

$$\begin{aligned} \max_{X1,X2} Z = 3X1 - X2 \\ -X1 + X2 \geq -1 \\ -2X1 + X2 \leq 1 \\ 2X1 - X2 \leq 5 \\ X1, X2 \geq 0. \end{aligned}$$

***Question 8** Solve the problem:

B			X1	X2	X3	X4	X5	X6	X7
X5	0	=	1/4	-60	-1/25	9	1	0	0
X6	0	=	1/2	-90	-1/50	3	0	1	0
X7	1	=	0	0	1	0	0	0	1
	Z-0	=	3/4	-150	1/50	-6	0	0	0

Exercise 2

For each problem below, you need to perform **one step** of the simplex algorithm. This means that:

- You will first eventually need to **transform the problem** and / or **introduce slack variables** (for instance, if you need to perform a phase 1), in this case you must explain what you do and why.
- Then do **one step** of the simplex algorithm. Explain which is the **entering / leaving variable** and why. You also have to give the tableau **before and after** this step.
- Finally, give the **new feasible solution** and explain what will happen next (e.g., this is the optimal solution, there is no optimal solution, ...).

Question 9 : The problem is given by the following tableau (the objective is to maximize Z):

B			X1	X2	X3	X4
X3	3	=	-1	3	1	0
X4	4	=	2	-2	0	1
	Z-2	=	1	-1	0	0

Question 10 : The problem is given by the following tableau (the objective is to maximize Z):

B			X1	X2	X3	X4	X5	X6
X4	2	=	-1	2	-1	1	0	0
X5	3	=	1	-1	-1	0	1	0
X6	3	=	0	-1	3	0	0	1
	Z-3	=	1	2	-1	0	0	0

Question 11 : Solve the following problem:

$$\begin{aligned} \max_{X1,X2,X3} \; & 3X1 - X2 + 5X3 \\ & X1 - X3 \leq 3 \\ & X2 + X3 - 5 \geq 0 \\ & X1, X2, X3 \geq 0. \end{aligned}$$

Question 12 : The problem is given by the following tableau (the objective is to maximize Z):

B			X1	X2	X3	X4
X3	3	=	-1	-2	1	0
X4	4	=	-2	2	0	1
	Z-2	=	2	1	0	0

Question 13 : The problem is given by the following tableau (the objective is to maximize Z):

B			X1	X2	X3	X4	X5	X6
X4	2	=	2	2	-1	1	0	0
X5	4	=	1	-1	-1	0	1	0
X6	3	=	0	-1	3	0	0	1
	Z-3	=	1	2	-1	0	0	0

Question 14 : The problem is given by the following tableau (the objective is to maximize Z):

B			X1	X2	X3	X4
X2	3	=	-1	1	1	0
X4	6	=	8	0	-2	1
	Z-3	=	-7	0	1	0