Time allocated: 3h - No documents allowed. Scoring will depend on the cleanliness of the copy and the clarity of the explanations. TAKE CARE: any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

1 Truth table ($\backsimeq$ 2 points)

1. Using the truth table method, prove that the premises: $p \Rightarrow q$ and $r \Rightarrow p \vee q$, logically entail $r \Rightarrow q$

2. Using the truth table method, prove that $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ is valid

2 Validity, unsatisfiability, contingency ($\backsimeq$ 3 points)

Using resolution reasoning, say whether each sentence below is valid, unsatisfiable or contingent:

1. $(\neg q \vee (\neg(p \vee \neg p) \wedge r) \Rightarrow s) \Rightarrow (s \vee q)$

2. $(a \Rightarrow b) \wedge (\neg a \Rightarrow (b \vee c)) \wedge (\neg c \Rightarrow \neg b) \wedge ((b \wedge c) \Rightarrow \neg a) \wedge (c \Rightarrow a)$

3 Problem modeling and solving ($\backsimeq$ 5 points)

Brown, Jones, and Smith are suspected of a crime. In front of a jury they testify as follows:

Brown: "Jones is guilty and Smith is innocent."

Jones: "If Brown is guilty then so is Smith."

Smith: "I'm innocent and at least one of the others is guilty."

1) Model this universe using Propositional Logic, that is, provide three judicious proposition constants and convert those three testimonies to three proposition sentences.
2) Write a truth table for the three testimonies.
3) Use the above truth table to answer the following questions (explain your answer):

(a) Are the three testimonies satisfiable?

(b) The testimony of one of the suspects logically entails that of another. Say which one entails which one?

(c) Assuming that everybody is innocent, who committed perjury?

(d) Assuming that all testimonies are true, who is innocent and who is guilty?

(e) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

4 Problem modeling and solving ($\backsimeq$ 5 points)

Here are some informations about a simple world:

When Mary isn't sick, she sings, she dances with John, and Harry is jealous. When Lucy is sick and wants to run outside, John is afraid. When Mary is not happy, she cannot eat. When Mary is dancing with John or Harry, Lucy is sick. When John or Harry is jealous, Lucy is sick. Mary isn't sick. When John is afraid or Harry is jealous, Mary is not happy. When Mary sings, Lucy wants to run outside.

Model this universe using propositional logic, and then provide a resolution proof of: *Harry is jealous and Mary cannot eat.*

5   Unification ($\backsimeq$ 2 points)

For each pair of logical sentences below, say whether they are unifiable or not. In case they are unifiable give their most general unifier, in case they are not unifiable, explain why.

1. p(A,a) and p(b,B)
2. r(f(Y),Y,b) and r(f(X), f(X), Z)
3. s(a,X,c) and s(Y,d,Z)
4. t(a,f(X,Y),b) and t(Z,f(a),W)
5. u(X,X,b) and v(a,a,Y)
6. w(a,b,c,c,b,a) and w(X,Y,Z,Z,Y,X)
7. x(a,f(X,g(a,Y),Z,W),b,T) and x(X,f(a,T,c,b),Y,g(X,W))
8. y(a,f(X,g(a,Y),Z,W),b,T) and y(X,f(a,T,c,b),Y,g(Y,W))

6   Validity, unsatisfiability, contingency ($\backsimeq$ 4 points)

Using resolution reasoning, say whether each sentence below is valid, unsatisfiable or contingent

1. $\forall X.\forall Y.((p(X) \land p(Y)) \Rightarrow q(X,Y)) \Rightarrow \forall X.(p(X) \Rightarrow \exists Y.q(X,Y))$

2. $(\exists X.q(X) \land (\forall X.(p(X) \Rightarrow \neg q(X)))) \Rightarrow \exists X.\neg p(X)$

7   Resolution reasoning ($\backsimeq$ 3 points)

Given the following first-order sentences:
   a) $\forall X.(p(X) \Rightarrow \exists Y.q(Y))$
   b) $\neg\exists X.(q(X) \land \exists Y.\neg w(Y))$
   c) $\forall X.((p(X) \land w(X)) \Rightarrow s(X))$
   d) $p(mary)$
Using resolution reasoning, show that: s(mary)

8   Problem modeling and solving ($\backsimeq$ 6 points)

Here are some informations about a simple world:
1) If someone is young and plays the guitar then that person is happy.
2) If someone is old and plays the violin then that person is happy.
3) If someone plays the drums then he's happy.
4) Mary is young, owns a guitar, has learned the guitar and loves Paul.
5) John is old and he knows how to dance.
6) If a person owns an instrument and has learned to play it then that person plays that instrument.
7) If a person is a genius and a musical instrument is a string instrument, then that person plays that instrument.
8) If a person has built a musical instrument then that person plays that instrument.
9) Lindsey has built a violin.
10) The guitar and violin are string instruments.
11) George is a genius.
12) For any person p1 and p2,
   a) If p1 is happy and loves p2, then p1 is a happy lover of p2.
   b) If p1 knows how to dance and p2 is happy then p1 is a happy dancer with p2.
   c) If p1 is a happy dancer with p2 then p1 dances with p2.
   d) If p1 is a happy lover of p2 then p1 dances with p2.
   e) If p1 has built and instrument, then p1 dances with this instrument.

Using resolution reasoning in first order logic, answer the question: *"Who dances with whom/what?"*