Master MLDM/DSC/CPS2 - First year
Introduction to Artificial Intelligence
Exam on Propositional and First Order Logics
October 28, 2021

Time allocated: 3h
No documents authorized. Your copy must be written in English. Grading will depend on the cleanliness of your copy and the clarity of your explanations.
TAKE CARE: any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

1 Truth table (2 points)

The Sheffer stroke, written "$\uparrow$", denotes a logical operation that is equivalent to the negation of the conjunction operation. Hence, $\Phi \uparrow \Psi$ is logically equivalent to $\neg(\Phi \wedge \Psi)$. **Using the truth table method**, show that:

1. $\neg p$ is logically equivalent to $p \uparrow p$
2. $p \Rightarrow q$ is logically equivalent to $p \uparrow (q \uparrow q)$
3. $p \Rightarrow q$ is logically equivalent to $p \uparrow (p \uparrow q)$
4. $p \wedge q$ is logically equivalent to $(p \uparrow q) \uparrow (p \uparrow q)$
5. $p \vee q$ is logically equivalent to $(p \uparrow p) \uparrow (q \uparrow q)$

2 Validity, unsatisfiability, contingency (5 points)

Using resolution reasoning in proposition logic or first order logic, and the methodology we saw during the course, say whether each sentence below is valid, unsatisfiable or contingent:

1. $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$
2. $(((p \vee \neg q) \wedge (r \vee q)) \Rightarrow (p \vee r))$
3. $\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
4. $\forall X.p(X) \Rightarrow \exists X.p(X)$
5. $\forall X.\forall Y.((lt(X,Y) \Rightarrow \neg lt(Y,X)) \vee \exists Z.\forall X.(lt(X,Y) \Rightarrow \forall X.lt(Z,X)))$

3 Resolution principle (3 points)

For each pair of clauses below, say whether the resolution principle can be applied. If yes give the resolvent, if no, explain why.

1. $\{r(X), q(X,a,f(X)\}$ and $\{\neg q(b,Y,f(b),s(Y)\}$
2. $\{\neg p(X,f(Y),k), \neg p(g(a,b),f(c),W), \neg p(W)\}$ and $\{p(A,f(B),C), p(A,B)\}$
3. $\{p(X,a,Y),s(Y)\}$ and $\{r(X), \neg p(b,X,Z)\}$
4. $\{p(a,f(X,g(Y),Z),b,T), r(X,Y), p(X,f(a,T,c),Y,g(Y))\}$ and $\{\neg p(A,f(C,D,E),b,F), s(A,F)\}$
5. $\{p(A), \neg q(b)\}$ and $\{\neg p(a), q(B)\}$
6. $\{r(A), p(A,B), s(B), p(a,C), t(C), p(A,b)\}$ and $\{\neg p(X,b), \neg p(a,Y), r(X), w(Y)\}$

4 Problem modeling and solving I (5 points)

Below is information about a simple world.

If Mary has taken the course on database and the course on algorithms then she masters the fundamental units and can continue in computer science. If John has taken the course on algorithms or the course on maths then he can continue in maths. If Mary can continue in computer science then she can have a good job and earn money. Mary has taken the courses on algorithms and databases. John has taken the course on maths. If John can continue in maths then he is happy and wants to dance with Mary. If Mary can have a good job or sees John is happy then she wants to dance with John. If John wants to dance with Mary and Mary wants to dance with John, then life is beautiful.

Model this world using **propositional logic**, and then provide a resolution proof of: *Life is beautiful*.

Take care: Modelling this using first-order logic would lead to a score of 0/5 for this exercise.

5 Problem modeling and solving II (5 points)

Below is information about a simple world, for any person `P1`, `P2` and `P3`.

- If a person `P1` is richer than another person `P2`, and they live in the same place then `P1` pays more taxes than `P2`.
- If a person `P1` is richer than another person `P2`, and `P1` lives in town and `P2` lives in the countryside, then `P1` is smarter than `P2`.
- If a person `P1` is richer than a person `P3` that is richer than a person `P2`, then `P1` is richer than `P2` (the relationship "richer than" is transitive).
- The relationship "younger than" is also transitive.
- If a person `P1` is smarter than a person `P2`, or if `P1` is younger than `P2` and `P1` is a student and `P2` is a teacher, then `P1` is happier than `P2`.
- If `P1` pays more taxes than `P2` then `P1` is jealous of `P2`.
- If `P1` is happier than `P2` or `P1` is jealous of `P2`, then `P1` has feelings.
- Bess and Dana live in the same place.
- John is a student.
- John is younger than Mary.
- Suzy is a teacher.
- Cody lives in town.
- Cody is richer than Dana.
- Dana lives in the countryside.
- Bess is richer than Cody.
- Mary is younger than Suzy.

Model this world using **first order logic** and then, using resolution reasoning in first order logic, **find the three persons that have feelings**.