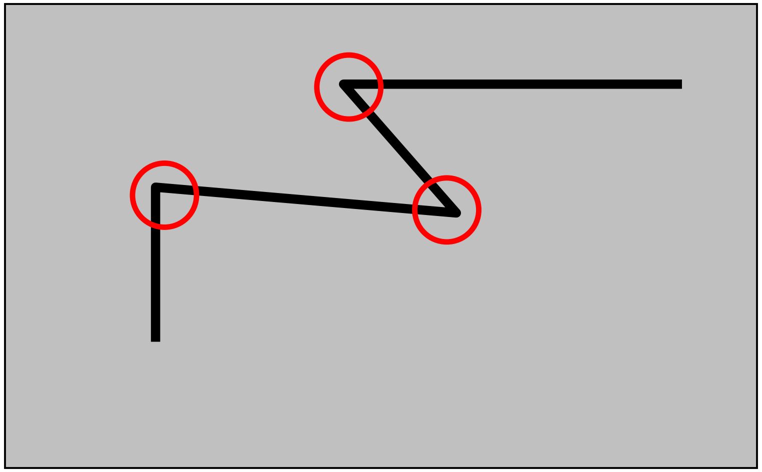
Notes on the Harris Detector

from Rick Szeliski's lecture notes, CSE576, Spring 05

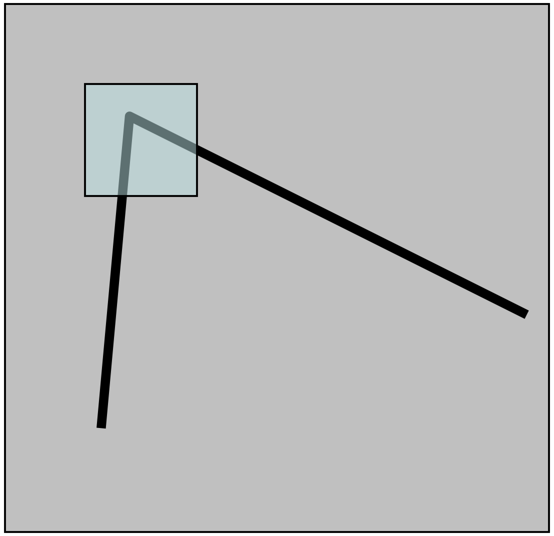
Harris corner detector

- C.Harris, M.Stephens. “A Combined Corner and Edge Detector”. 1988

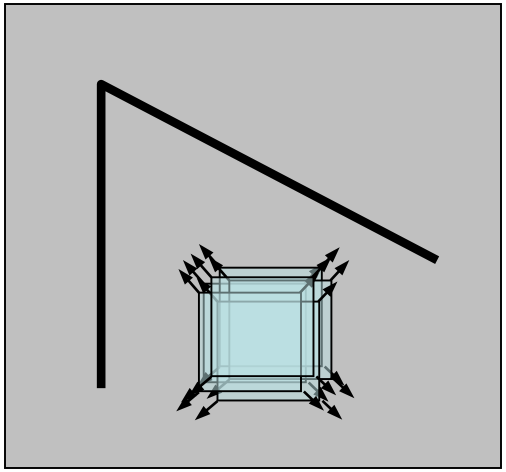


The Basic Idea

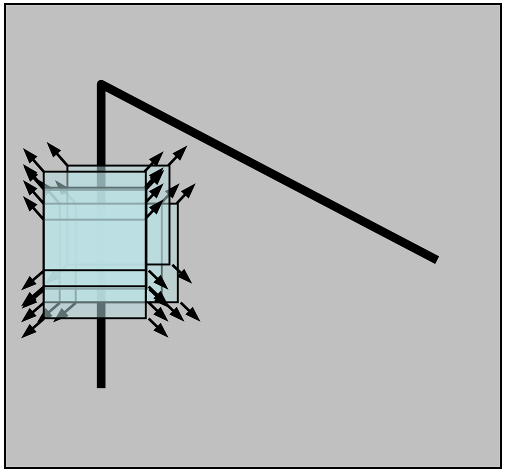
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



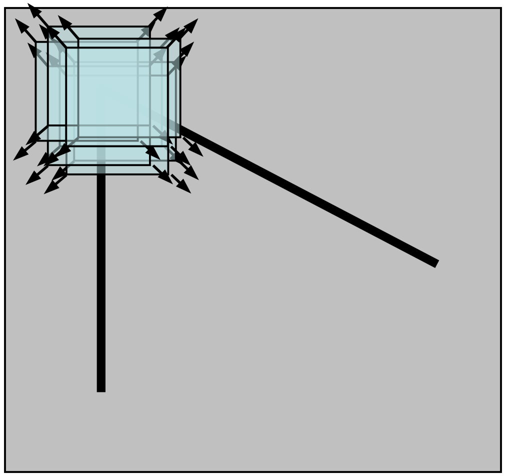
Harris Detector: Basic Idea



"flat" region:
no change in
all directions



"edge":
no change along
the edge direction



"corner":
significant change
in all directions

Harris Detector: Mathematics

Change of intensity for the shift $[u,v]$:

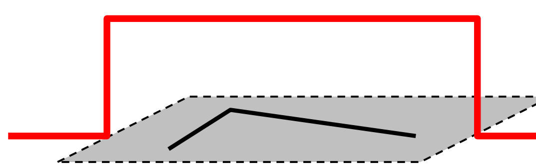
$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

Window function

Shifted intensity

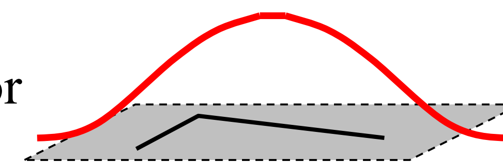
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Harris Detector: Mathematics

For small shifts $[u,v]$ we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

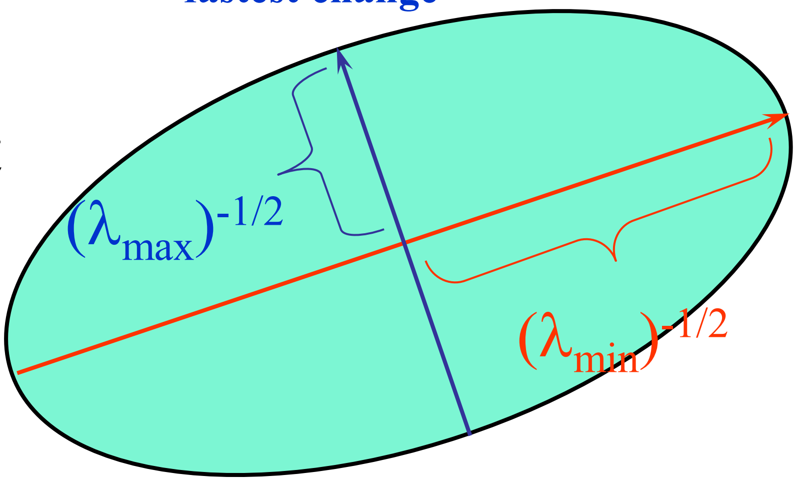
Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

λ_1, λ_2 – eigenvalues of M

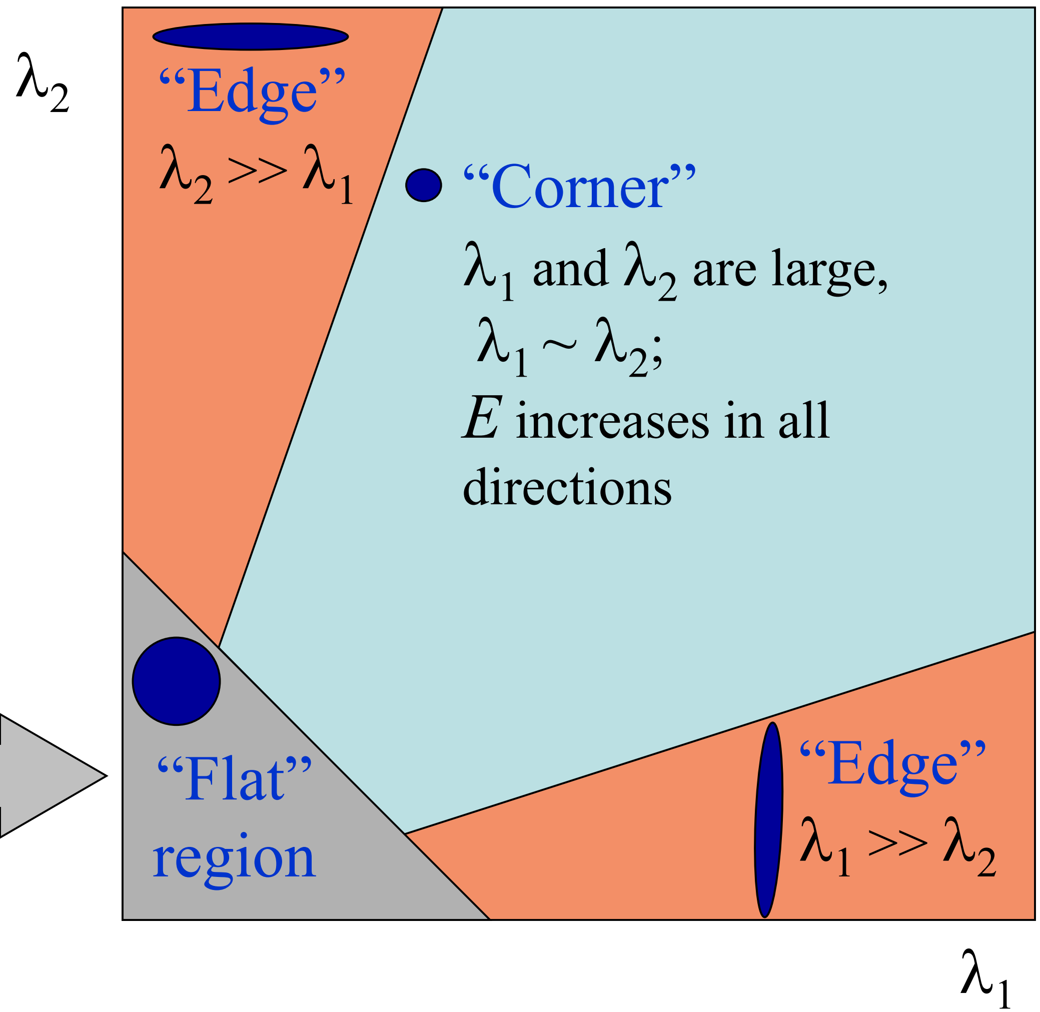
Ellipse $E(u,v)$ = const



Harris Detector: Mathematics

Classification of image points using eigenvalues of *M*:

λ_1 and λ_2 are small;
E is almost constant in all directions



Harris Detector: Mathematics

Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(k – empirical constant, k = 0.04-0.06)

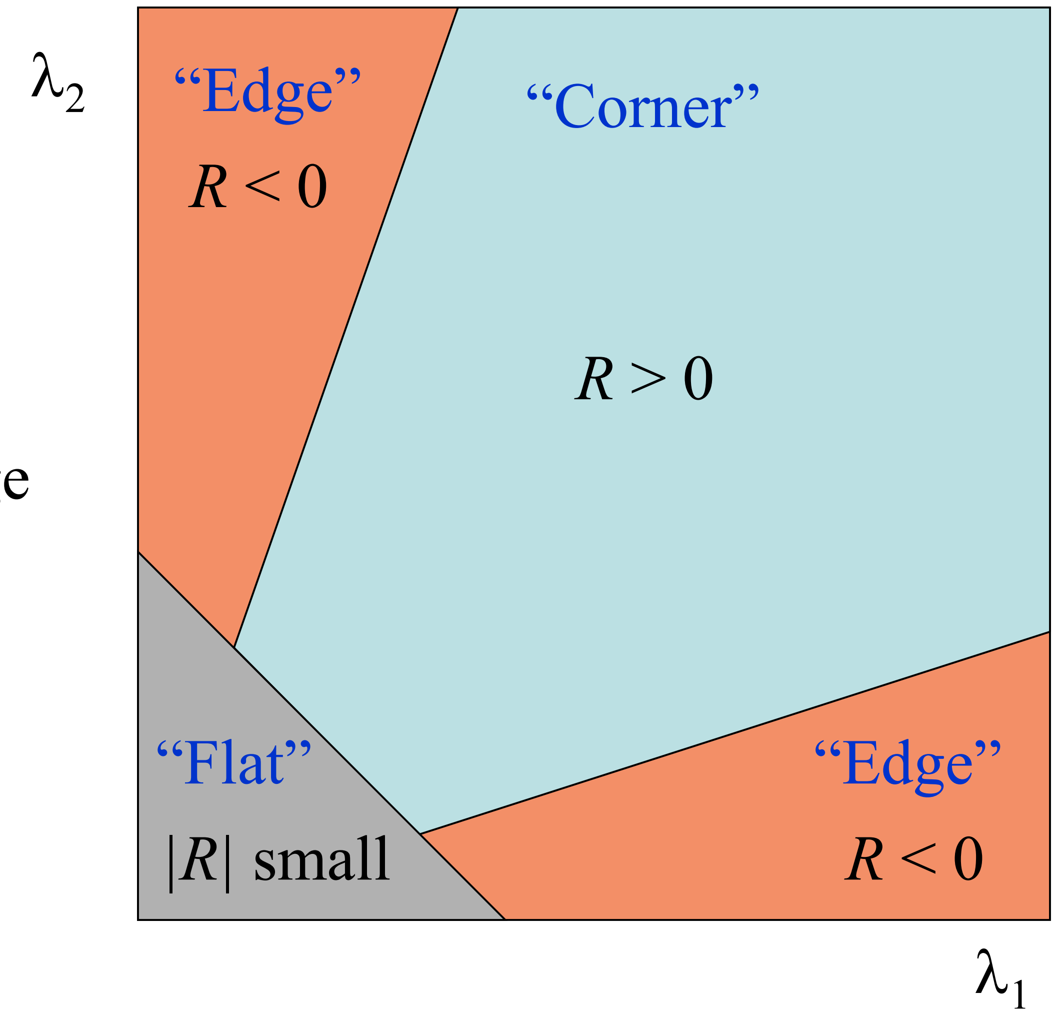
Harris Detector: Mathematics

• R depends only on eigenvalues of M

• R is large for a corner

• R is negative with large magnitude for an edge

• $|R|$ is small for a flat region



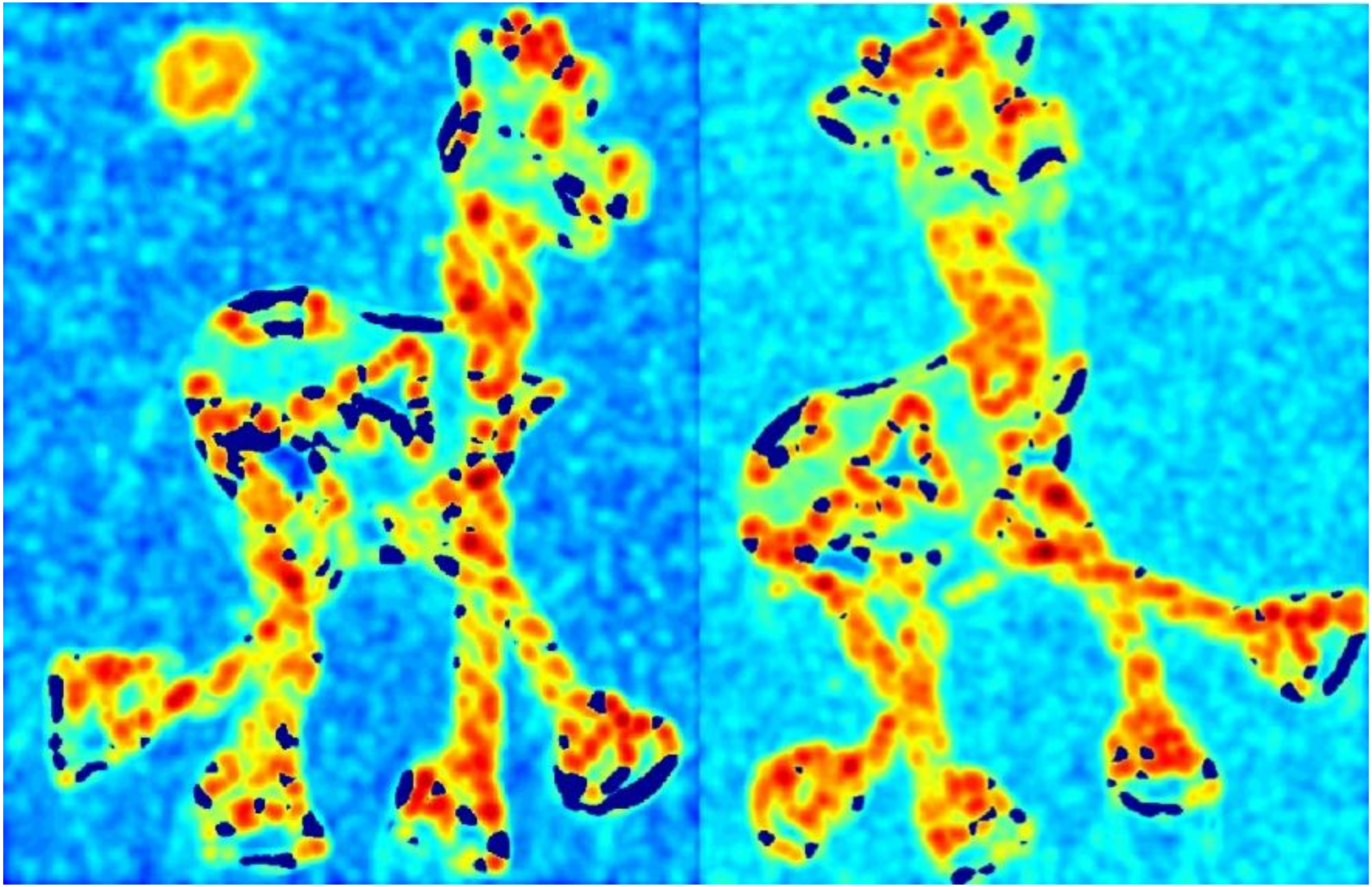
Harris Detector

- The Algorithm:
  - Find points with large corner response function R (R > threshold)
  - Take the points of local maxima of R

Harris Detector: Workflow

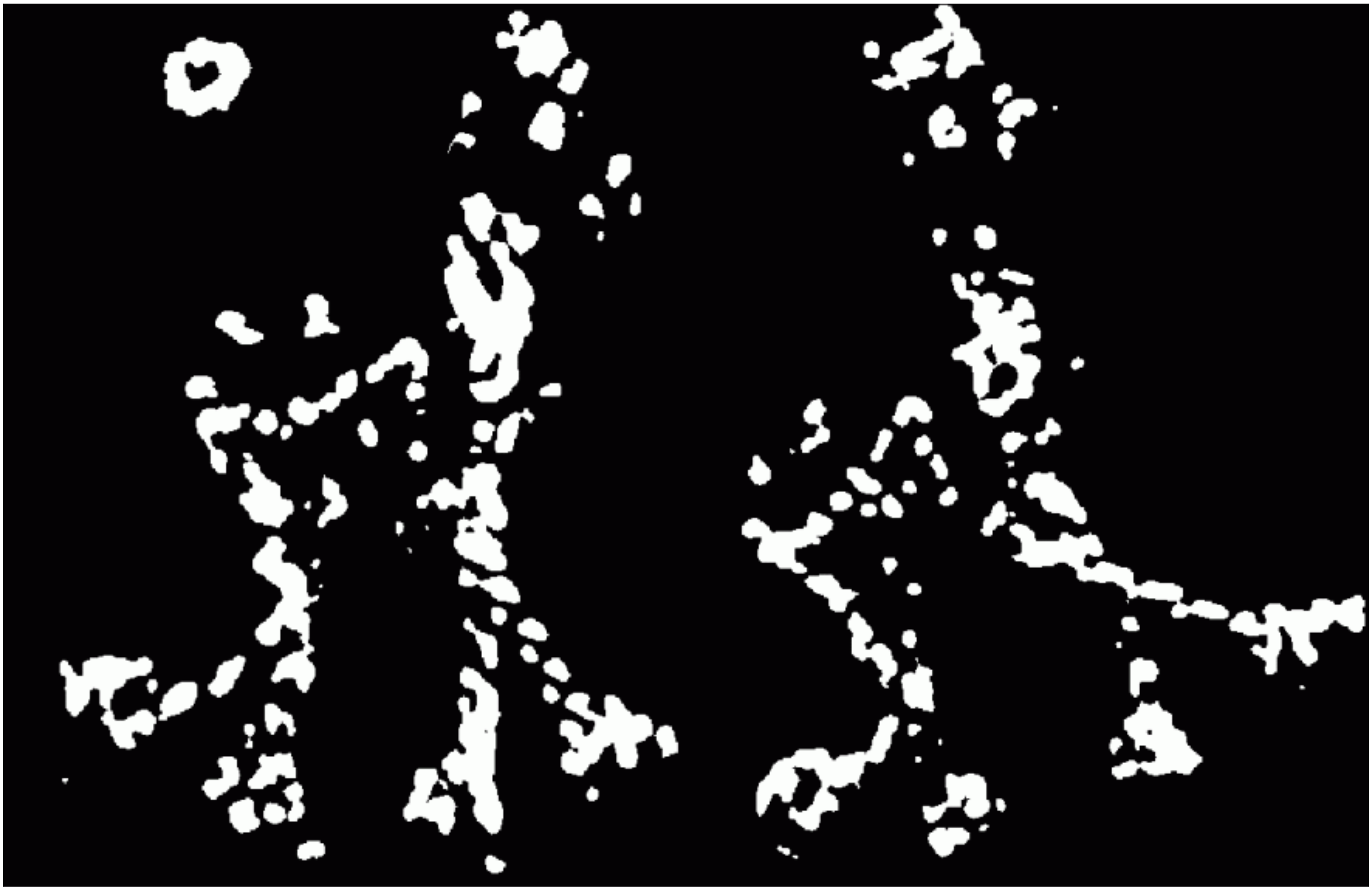
Harris Detector: Workflow

Compute corner response R



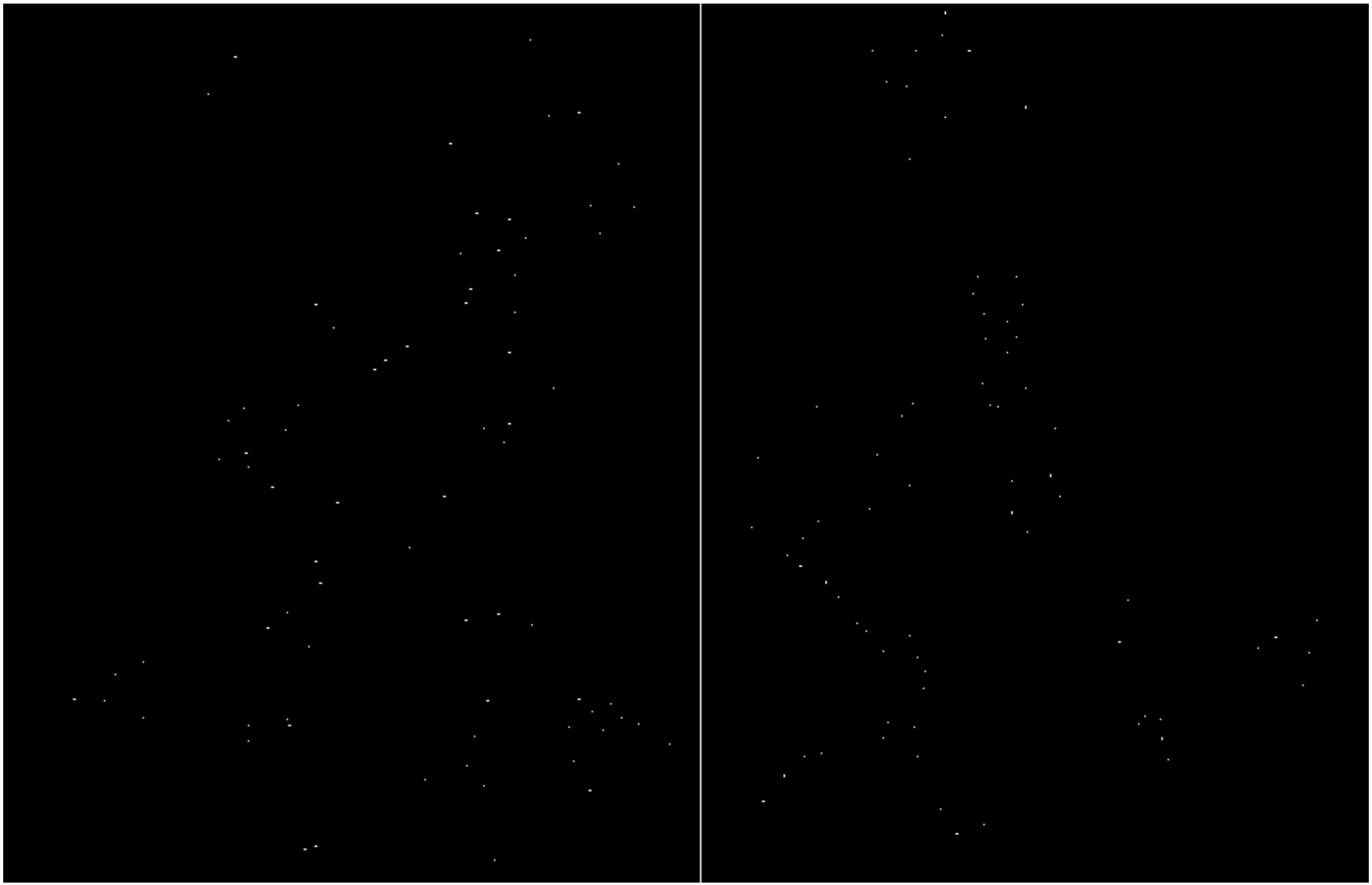
Harris Detector: Workflow

Find points with large corner response: $R >$ threshold



Harris Detector: Workflow

Take only the points of local maxima of R



Harris Detector: Workflow

Harris Detector: Summary

- Average intensity change in direction $[u,v]$ can be expressed as a bilinear form:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

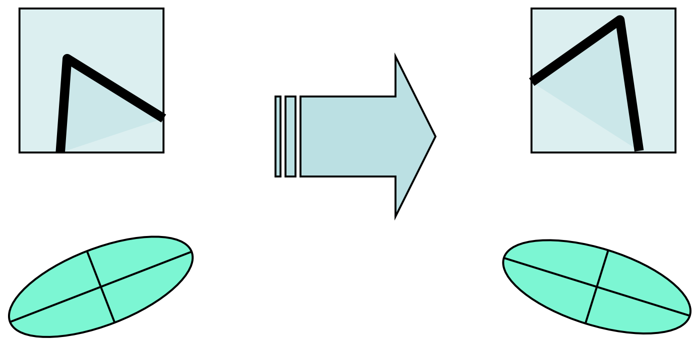
- Describe a point in terms of eigenvalues of M: *measure of co*

$$R = \lambda_1 \lambda_2 - k\left(\lambda_1 + \lambda_2\right)^2$$

- A good (corner) point should have a *large intensity change* in *all directions*, i.e. R should be large positive

Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

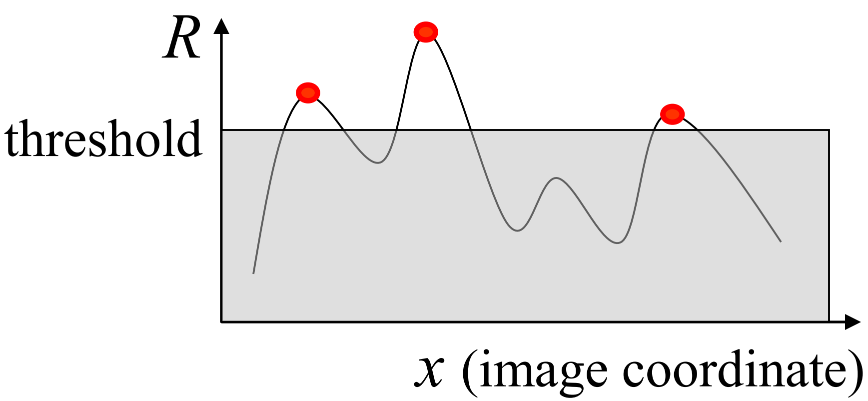
Corner response R is invariant to image rotation

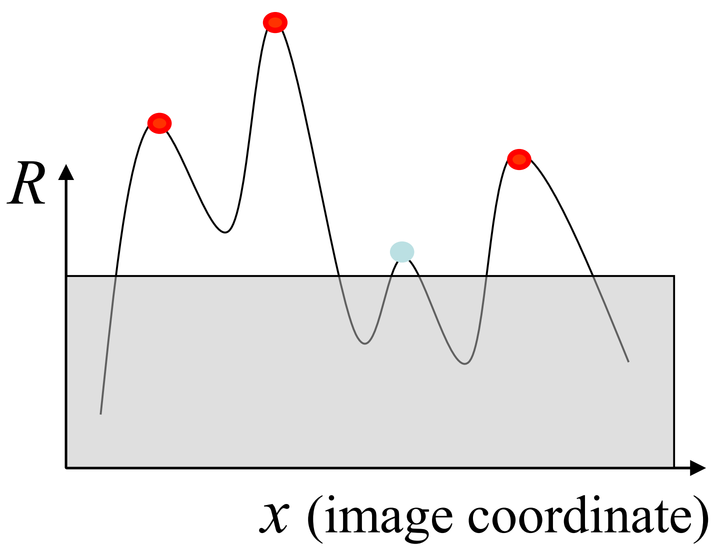
Harris Detector: Some Properties

- Partial invariance to *affine intensity* change

  ✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

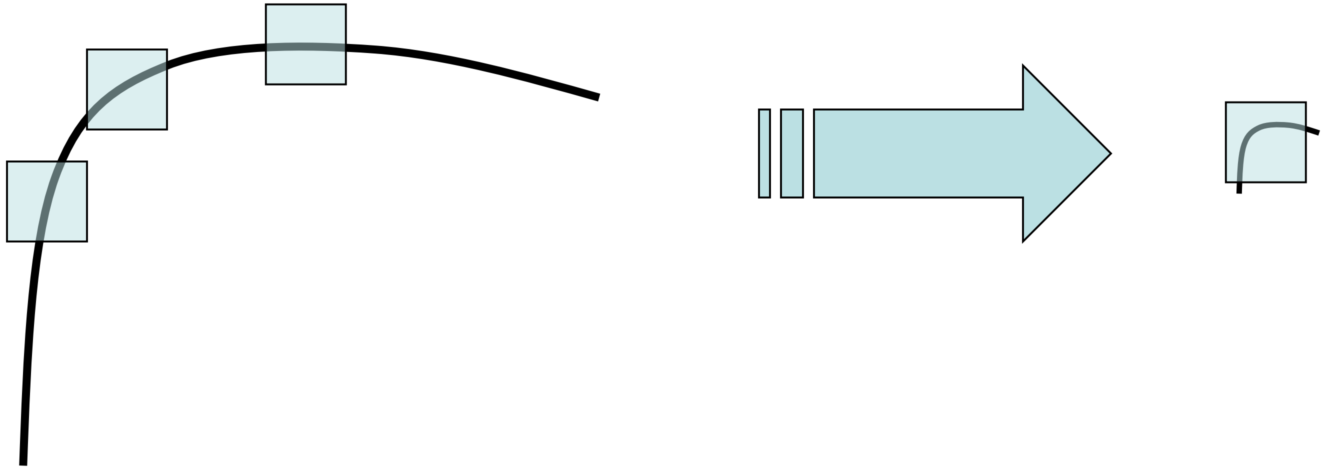
  ✓ Intensity scale: $I \rightarrow a\, I$





Harris Detector: Some Properties

- But: non-invariant to *image scale*!



All points will be classified as edges

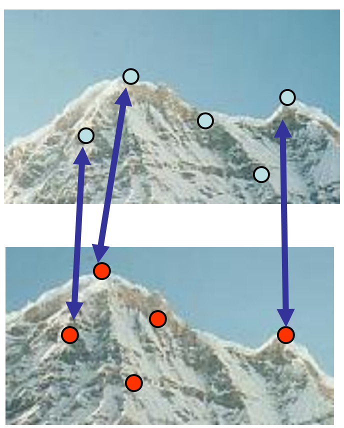
Corner !

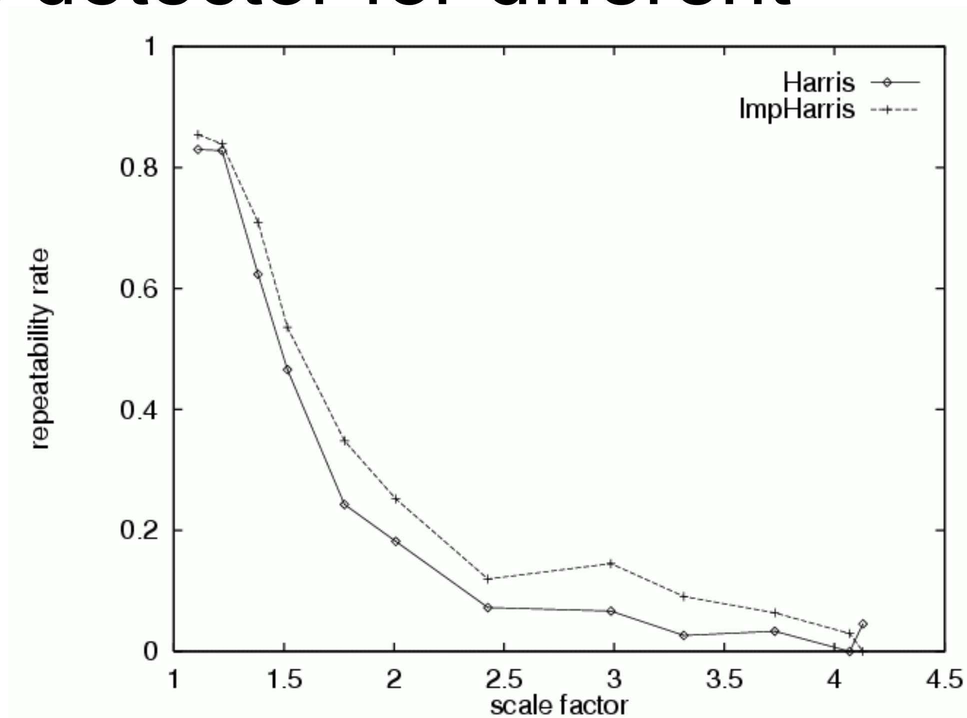
Harris Detector: Some Properties

- Quality of Harris detector for different scale changes

Repeatability rate:

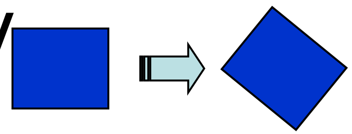
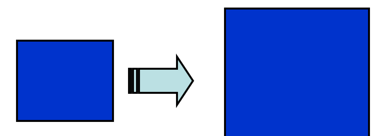
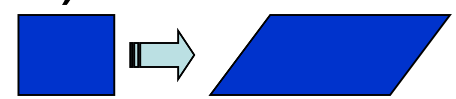
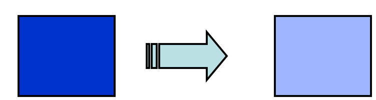
$$\frac{\#\ \text{correspondences}}{\#\ \text{possible correspondences}}$$





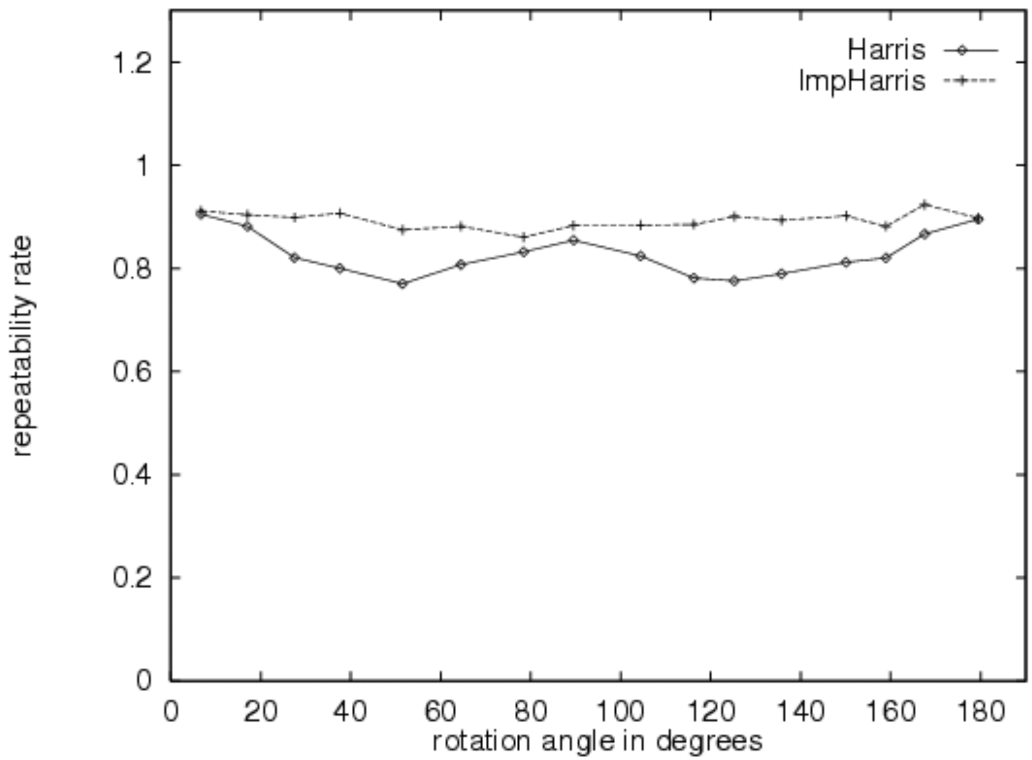
C.Schmid et.al. "Evaluation of Interest Point Detectors". IJCV 2000

Models of Image Change

- Geometry
  
  - Rotation
  
  - Similarity (rotation + uniform scale)
  
  - Affine (scale dependent on direction) valid for: orthographic camera, locally planar object
- Photometry
  
  - Affine intensity change ($I \rightarrow a\, I + b$)

Rotation Invariant Detection

- Harris Corner Detector



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