Turing Machine

In the following, B is the blank symbol. Initially, all the tape's cells are filled with blank symbols, except those used by the input string. All tapes are supposed to be **bi-infinite**.

Exercise 1 *Deterministic Turing Machine*

Let M a Turing, a bi-infinite tape Turing machine, be defined by:

$$(Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, B\}, \delta, q_0, F = \{q_2\}, B)$$

δ is defined as follows:

	Symbol		
State	0	1	B
q_0	$q_0, 0, R$	$q_0, 1, R$	q_1, B, L
q_1	$q_2, 1, L$	$q_1, 0, L$	$q_2, 1, L$
q_2	—	—	—

Question 1 *Give some details about the elements defining the machine. Draw the transition diagram corresponding to the definition of M. Simulate the computation of M with the following initial configurations: $w = q_0 10$, $w' = q_0 11$, $w'' = q_0 1001$? In each case, provide the values of time and space complexity. What does this machine compute ?*

Exercise 2 *From Two-way to One-way infinite tape*

Let us assume some steps of a Two-way infinite tape Turing machine are as follows:

$$cbbaq_0aba \rightarrow cbbaaq_1ba \rightarrow cbbaq_2aba \rightarrow cbbq_2aaba \rightarrow cbq_2baaba \rightarrow cq_3bbaaba$$

where in the initial configuration (state q_0) the head is positionned onto cell numbered 0 which contains a symbol a.

Question 2 *Represent how these steps could be simulated on a One-way infinite tape.*

Exercise 3 *Non-Deterministic Turing Machine*

Let M a Turing machine be defined by:

$$(Q = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{a, b, c\}, \Gamma = \{a, b, c, B\}, \delta, q_0, F = \{q_4\}, B)$$

Transition function δ is defined as follows:

State	Symbol a	b	c	B
q_0	$q_0, a, \triangleright$	$q_0, b, \triangleright$	$q_0, c, \triangleright$	—
	$q_1, a, \triangleright$	$q_2, b, \triangleright$	$q_3, c, \triangleright$	—
q_1	$q_4, a, \triangleright$	$q_1, b, \triangleright$	$q_1, c, \triangleright$	—
q_2	$q_2, a, \triangleright$	$q_4, b, \triangleright$	$q_2, c, \triangleright$	—
q_3	$q_3, a, \triangleright$	$q_3, b, \triangleright$	$q_4, c, \triangleright$	—
q_4	—	—	—	—

Question 3 *Draw the transition diagram corresponding to the definition of M. Simulate the computation of M with the following initial configurations: $w = abcca$ (draw the whole computation tree). If it exists, write the accepting computation with the input word $w' = acbc$.*

Question 4 *In the proof of the equivalence between non-deterministic and deterministic Turing machine provided in the course:*

- *we defined $r = \max_{q \in Q, a \in \Gamma} |\{(q, a, q', z, Z) \in \delta\}|$. What is the value of r in the case of the Turing Machine M ?*
- *paths in the computation tree are encoded into strings. What are the first ten values of these strings. Choose one string y with $|y| \geq 4$ which encodes an existing path in the computation tree and draw this path. Indicate one string which does not encode an existing path in the computation tree.*

Question 5 *What is the value of $t_M(abcca)$? Assuming that the input string is accepted by M when the computation halts in state q_4, is the input string cbabca accepted ? Same question with input string cab (computations are not required here). What is the language accepted by this machine ?*

Exercise 4 *Multi-tape Turing machine*

Let M a 2-tape Turing machine be defined by:

$$(Q = \{q_0, q_1, q_2, q_3\}, \Sigma = \{a, b\}, \Gamma = \{a, b, O, B\}, \delta, q_0, F = \{q_3\}, B)$$

with the set of transitions δ defined as follows:

$$\begin{aligned}\delta = \{&(q_0, B, B, q_3, B, B, R, R), (q_0, a, B, q_1, a, O, R, R), (q_0, b, B, q_2, b, O, R, R),\\ &(q_1, a, O, q_1, a, O, R, R), (q_1, a, B, q_1, a, B, R, R), (q_1, b, B, q_1, b, B, R, L),\\ &(q_1, B, O, q_3, B, B, R, R), (q_1, b, O, q_2, b, O, R, R), (q_2, a, B, q_2, a, B, R, L),\\ &(q_2, b, B, q_2, b, B, R, R), (q_2, b, O, q_2, b, O, R, R), (q_2, a, O, q_1, a, O, R, R), (q_2, B, O, q_3, B, B, R, R), \}\end{aligned}$$

Convention: $(q_0, a, B, q_1, a, O, R, R)$ encodes a transition in which machine is state q_0, reads a symbol a on the first tape, a symbol B on the second tape, and then switches to state q_1, writes a symbol a on the first tape and moves the corresponding head to the right (first R), writes a symbol O on the second tape and moves the corresponding head to the right (second R). This constitutes one step of a computation of this machine.

Question 6 *Simulate the computation of M if the content of the first tape is $w = abba$, the first head positioned onto the first a on the left, all others tapes containing only blank symbols. Same question with the input word $w' = aba$.*

Question 7 *Describe briefly one method to simulate the computation of this 2-tapes Turing machine by a one-tape Turing machine. What would be the content of the tape of this one-tape Turing machine before and after the first step of the computation with input word $w = abba$? Same question for the input word $w' = aba$.*