Exercise Session 6

1 Probabilistic logic circuits

Consider the logic circuit given in Figure 1 as an AND-OR tree.

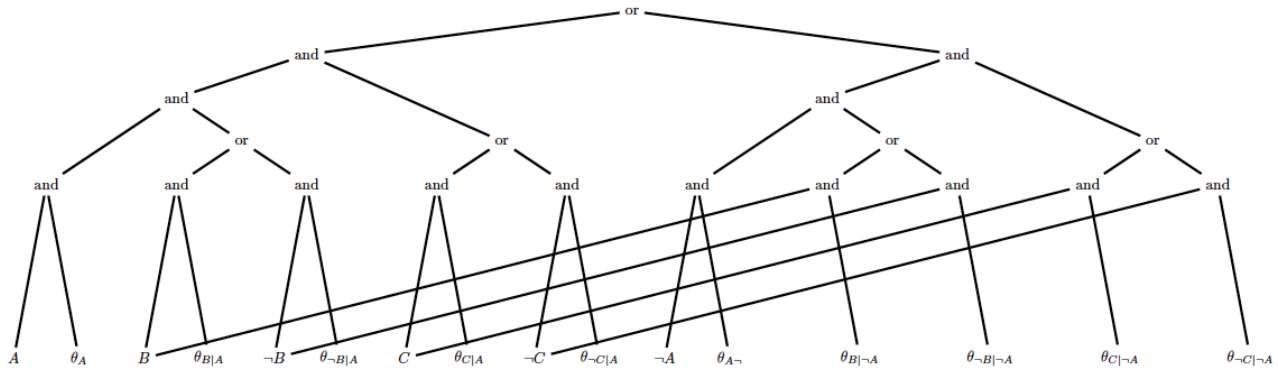

Figure 1: A logic theory compiled into the NNF representation.

1. Which of the following properties are satisfied? Explain clearly and give your reasoning.
   (a) is it decomposable ?
   (b) is it deterministic ?
   (c) is it smooth ?
2. Given the following CPTs compute the MAP assignment for $p(A, B, C)$ correctly transforming the AND-OR tree. (The variables are binary.)

$$\begin{aligned} \theta_A &= 0.3 \\ \theta_{B|A} = 0.5 \quad \theta_{B|\neg A} &= 0.2 \\ \theta_{C|A} = 0.3 \quad \theta_{C|\neg A} &= 0.4 \end{aligned} \tag{1}$$

2 Inference in a Bayesian network using WMC

Consider the Bayesian network in Figure 2 and the conditional probability tables from Table 1.

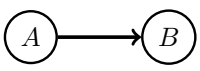

Figure 2: Bayesian network

1. Calculate $p(A = 1)$ using WMC.
2. Determine MAP estimate for B given $A = 1$ using WMC.

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A	$p(A)$
1	$\theta_A = 0.5$
0	$\theta_{\neg A} = 0.5$

(a) $p(A)$

B	A	$p(B\|A)$
1	1	$\theta_{B\|A} = 0.2$
0	1	$\theta_{\neg B\|A} = 0.8$
1	0	$\theta_{B\|\neg A} = 0.7$
0	0	$\theta_{\neg B\|\neg A} = 0.3$

(b) $p(B|A)$

Table 1: Conditional probability tables

3 More inference in a Bayesian network using WMC

Consider the Bayesian network in Figure 3 and the conditional probability tables from Table 2.

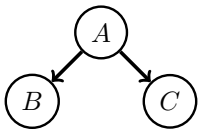

Figure 3: Bayesian network

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A	$p(A)$
1	$\theta_A = 0.5$
0	$\theta_{\neg A} = 0.5$

(a) $p(A)$

B	A	$p(B\|A)$
1	1	$\theta_{B\|A} = 1$
0	1	$\theta_{\neg B\|A} = 0$
1	0	$\theta_{B\|\neg A} = 0$
0	0	$\theta_{\neg B\|\neg A} = 1$

(b) $p(B|A)$

C	A	$p(C\|A)$
1	1	$\theta_{C\|A} = 0.8$
0	1	$\theta_{\neg C\|A} = 0.2$
1	0	$\theta_{C\|\neg A} = 0.2$
0	0	$\theta_{\neg C\|\neg A} = 0.8$

(c) $p(C|A)$

Table 2: Conditional probability tables

1. Calculate $p(A = 1, C = 0)$ using WMC.
2. Determine MAP estimate for B given $A = 1$ and $C = 0$ using WMC.

4 Bayesian Networks in ProbLog

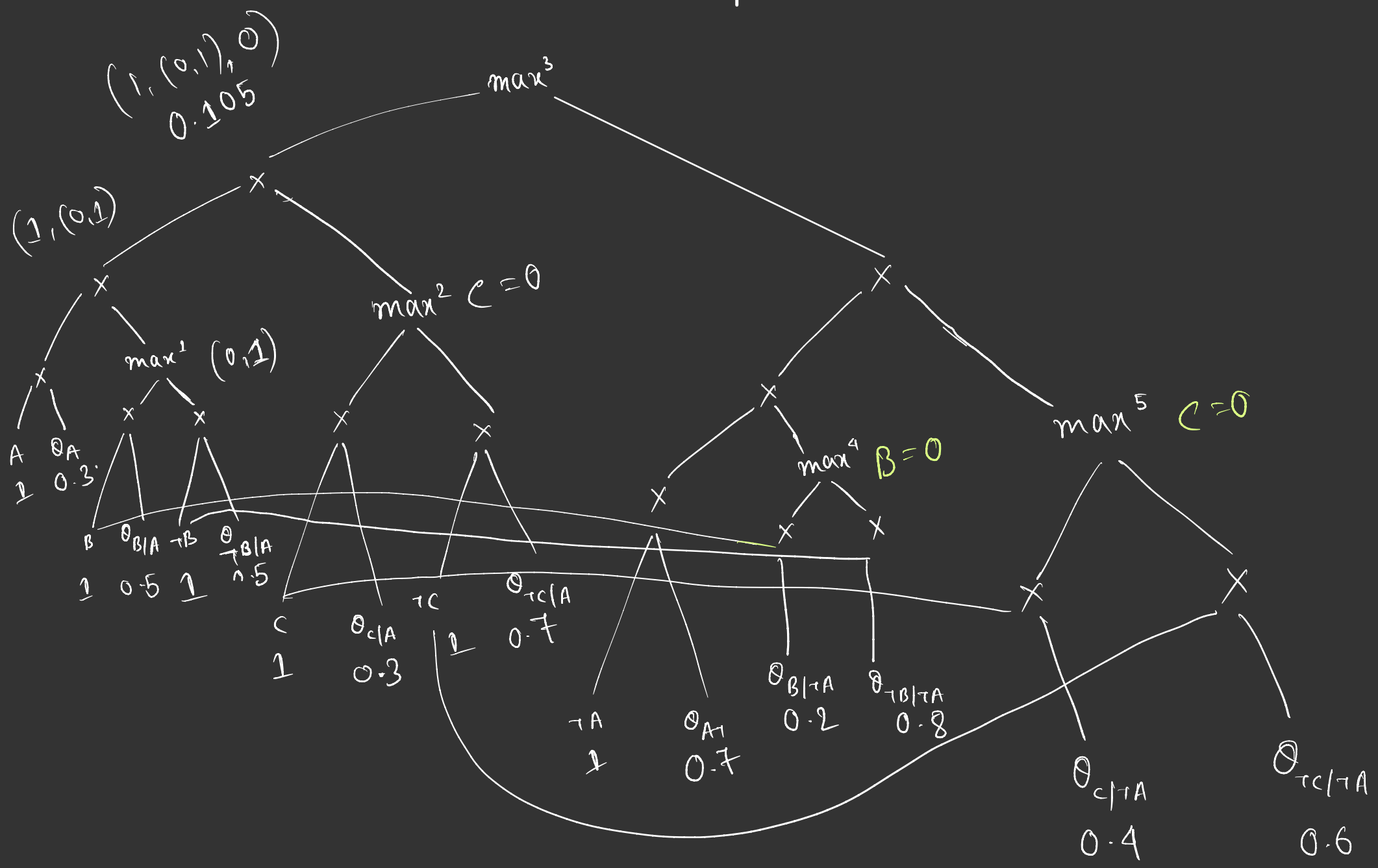
The Rock lives in a very bad neighbourhood in a city in California. Suppose there is a burglary in his house with 70% probability and an earthquake with 20% probability. Most of his money goes into protein and new t-shirts, so he had to buy an alarm from AliExpress. Whether the alarm will ring and he gets to save the day depends on both there being burglary and/or an earthquake.

- if there is a burglary and an earthquake, the alarm rings with probability 0.9;
- if there is only a burglary, it rings with probability 0.8;
- if there is only an earthquake, it rings with probability 0.1;
- if there is neither a burglary nor an earthquake, the alarm doesn't ring.

His default sidekick is busy and you have to step in to help him decide whether it is worth it to interrupt his training.

1. Model this scenario as a Problog program.
2. Write down the query to find the probability of burglary given that the alarm rang.
3. Write down the query to find the probability of earthquake given that the alarm rang.

① OR has different values of variable



A=

$\therefore 0.3 \times 0.5 \times 0.7 = 0.105$

A=0, B=0, C=0

$\therefore 0.7 \times 0.8 \times 0.6$

$= 0.336$

2) A → B

A	P(A)
1	$\theta_A = 0.5$
0	$\theta_{\neg A} = 0.5$

$P(A=1)$

$P(A,B) = P(B|A)\,P(A)$

$(A \wedge \theta_A) \wedge \{ (\theta_{B|A} \wedge B) \vee (\theta_{\neg B|A} \wedge \neg B) \} \vee$

$B = 0$

$0.5(0.2 + 0.8)$

3) $P(A)\,P(B|A)\,P(C|A)$

$(A \wedge \theta_A) \wedge \{ ((\theta_{B|A} \wedge B) \vee (\theta_{\neg B|A} \wedge \neg B)) \wedge (\theta_{\neg C|A} \wedge \neg C) \}$

$= 0.5 \times (1 \times 0.2) = 0.1$

4) 0.7 :: burglary.
0.2 :: earthquake.
0.9 :: alarm :- burglary, earthquake.
0.8 :: alarm :- burglary, \+ earthquake.
0.1 :: alarm :- earthquake, \+ burglary.
evidence(alarm, true)
query(burglary)
query(earthquake)