
Inductive Logic Programming. Part 2

Based partially on Luc De Raedt's slides [http://
www.cs.kuleuven.be/~lucdr/lrl.html](http://www.cs.kuleuven.be/~lucdr/lrl.html)

Specialisation and generalisation

A formula G is a **specialisation** of a formula F

iff F entails from G

(If G is true, F is true)

$$G \models F$$

= each model of G is also a model of F .

Specialisation operator

assign a formula a set of all its specialisations

Generalisation = the other direction

$$\text{dog}(x) \models \text{mammal}(x)$$

$$G \models F$$

F follows *deductively* from G

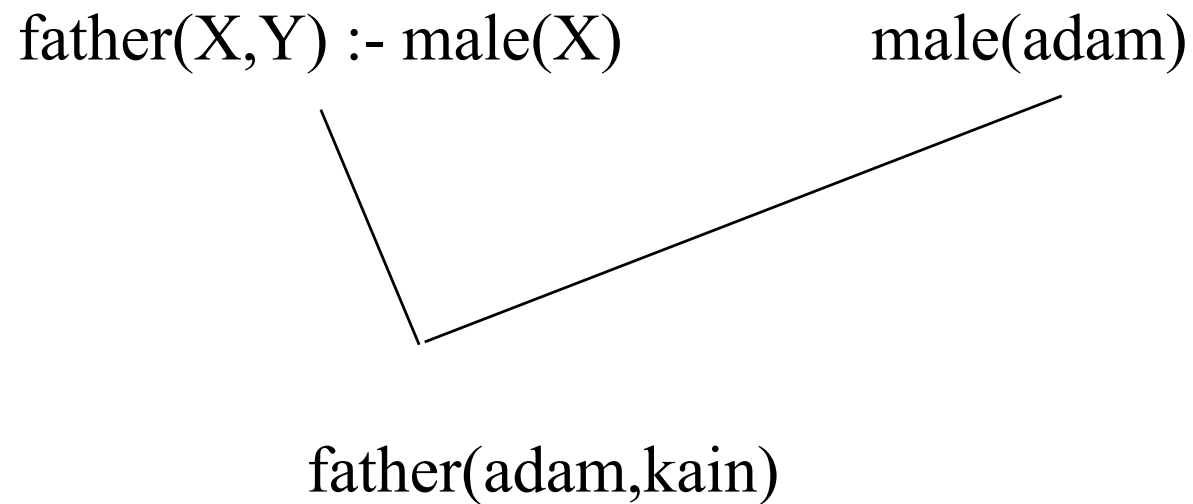
G follows *inductively* from F

therefore induction is the *inverse* of deduction

this is an operational point of view because there are
many deductive operators \vdash that implement \models

take any deductive operator and invert it and one
obtains an inductive operator

Resolution



Inverse resolution

Example: Learn a relation `father/2` given domain knowledge `parent/2` and `male/2`:

`male(adam).` `male(kain).` `male(abdullah).` `male(muhammad).` `male(moses).`
`parent(adam,kain).` `parent(eve,kain).` `parent(abdullah,muhammad),` and
an example `father(adam,kain).`

Inverse resolution

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an example `father(adam,kain).`

`father(adam,kain)`

Inverse resolution

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an example `father(adam,kain)`

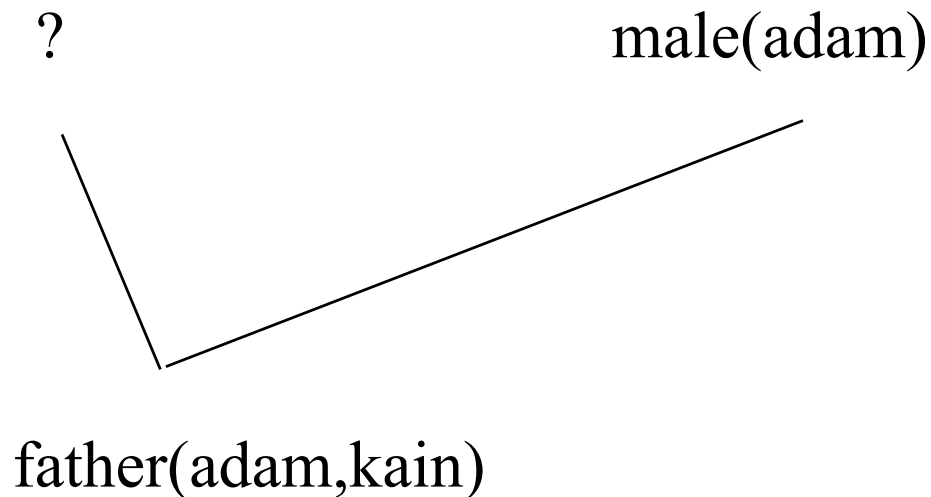
`male(adam)`

`father(adam,kain)`

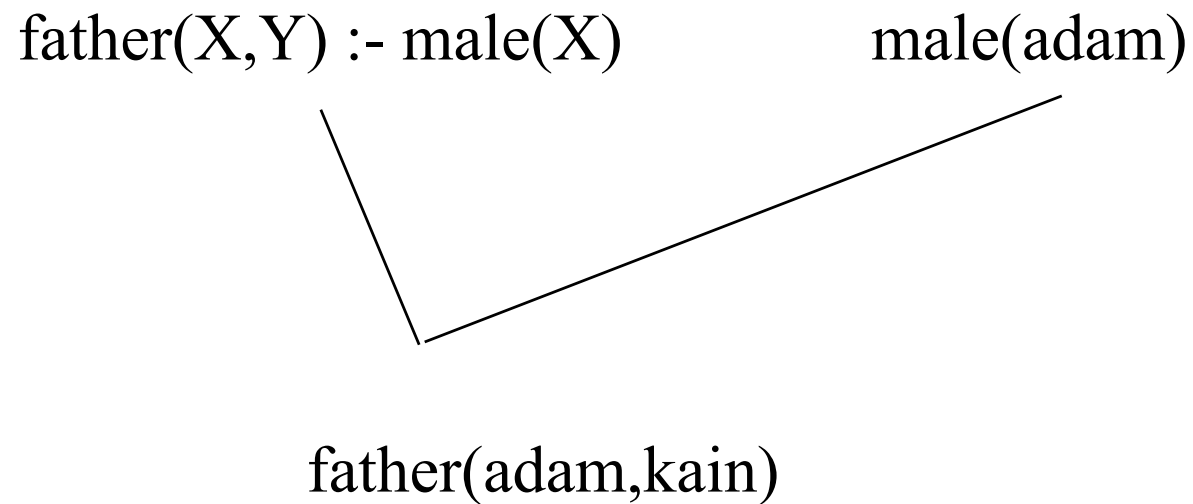
Inverse resolution

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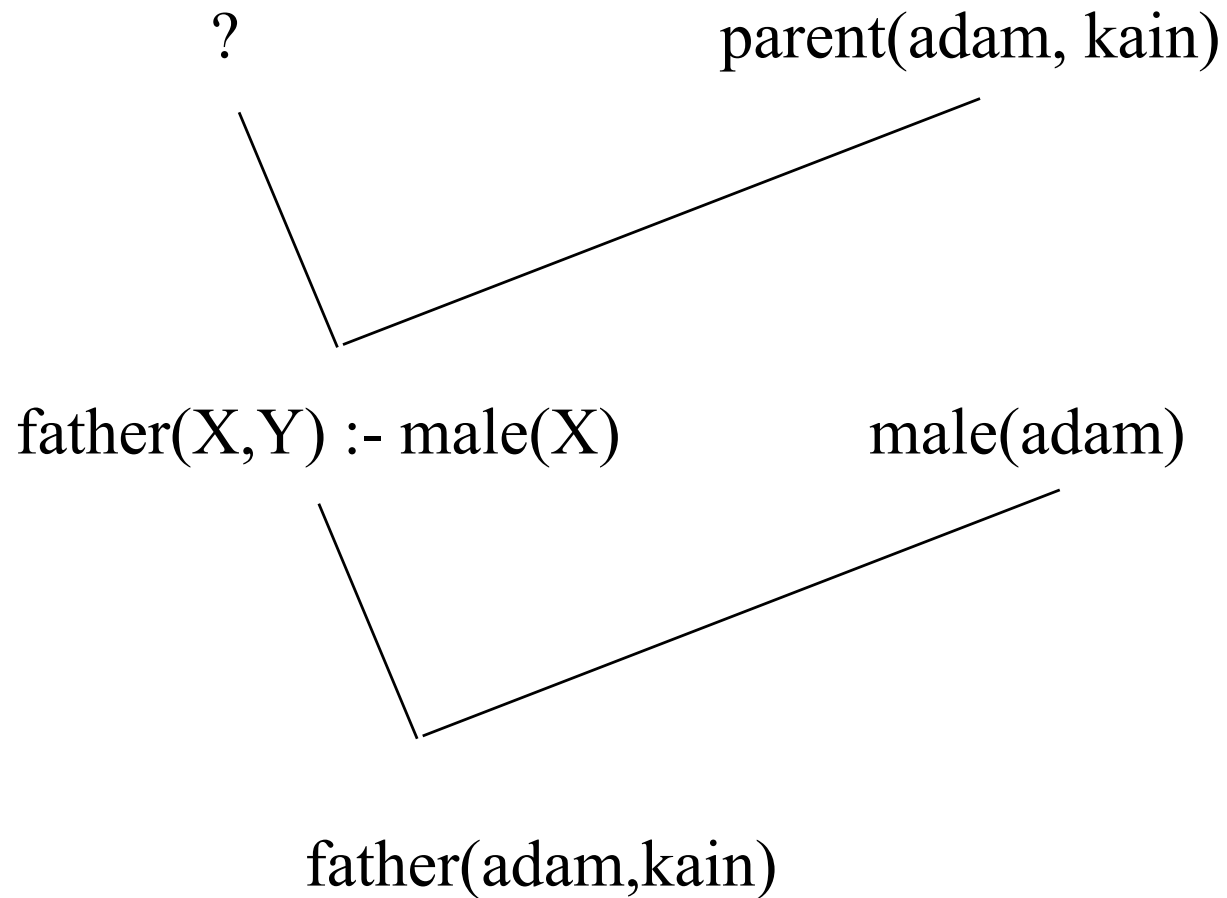
`male(adam)`. `male(kain)`. `male(abdullah)`. `male(muhammad)`. `male(moses)`.
`parent(adam,kain)`. `parent(eve,kain)`. `parent(abdullah,muhammad)`, and
an example `father(adam,kain)`



Inverse resolution



Inverse resolution



Inverse resolution

father(X,Y) :- male(X),parent(X,Y) parent(adam, kain)

father(X,Y) :- male(X) male(adam)

father(adam,kain)

Inverse resolution

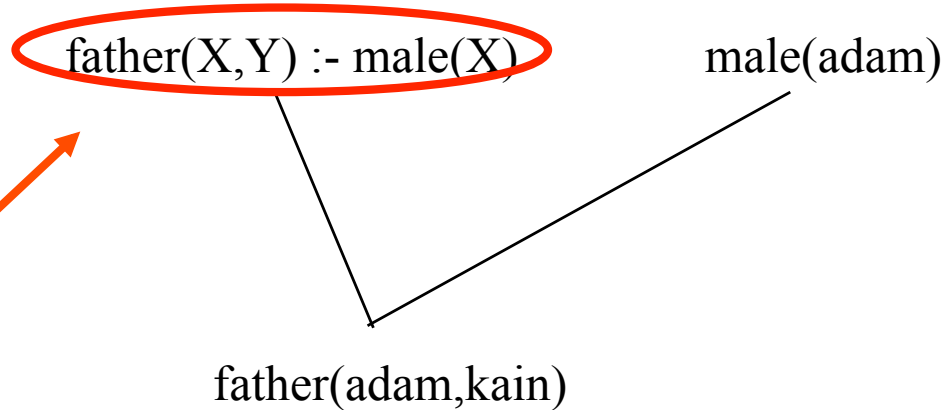
Given C_1 which is of the form $A \vee B$, and resolvent which is of the form $B \vee C$, the aim is to find C_2 .

In propositional logic:

1. Find a literal L that appears in C_1 but not in the resolvent.
2. Then C_2 is given by either
$$(\text{Resolvent} - (\text{Resolvent} \cap C_1)) \cup \{\neg L\}$$
or by
$$(\text{Resolvent} - (C_1 - \{L\})) \cup \{\neg L\}$$

Inverse resolution

In predicate logic:



1. Find a literal L_1 in C_1 that is not in the resolvent.
Then in C_2 there must be L_2 that $L_1 \Theta = L_2 \Theta$.
2. Assume $\Theta = \Theta_1 \Theta_2$ such that $L_1 \Theta_1 = L_2 \Theta_2$. Then $L_2 = \neg L_1 \Theta_1 \Theta_2^{-1}$
3. Then $C_2 = (\text{Resolvent} - (C_1 - \{L_1\} \Theta_1)) \Theta_2^{-1} \cup \neg L_1 \Theta_1 \Theta_2^{-1}$
4. C_1 is ground $\Rightarrow \Theta_1 = \{\}$
 $C_2 = (\text{Resolvent} - (C_1 - \{L_1\})) \Theta_2^{-1} \cup \neg L_1 \Theta_2^{-1}$

Inverse resolution

Main drawback

nondeterminism

father(X,Y) :- male(X)

father(X,kain) :- male(X)

father(adam,kain) :- male(adam)

male(adam)

father(adam,kain)

Subsumption and Θ -subsumption

Clause G **subsumes** clause F if and only if $G \models F$ or, equivalently $G \subseteq F$

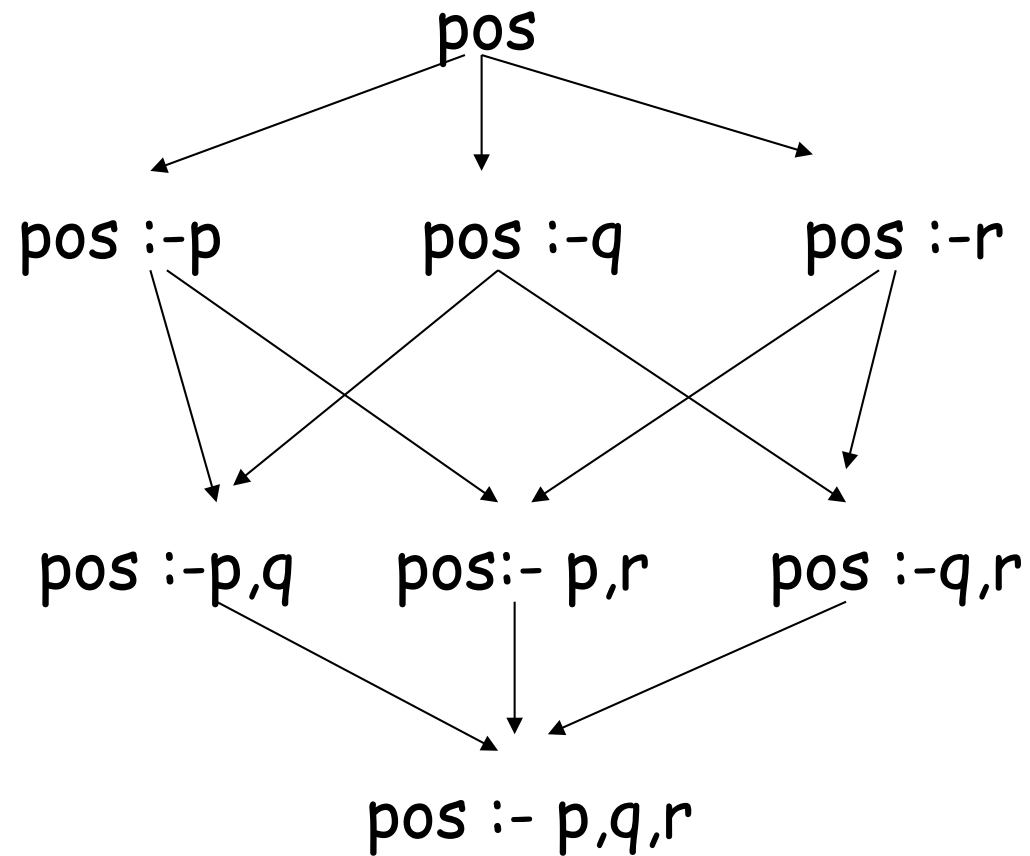
Example - propositional logic

$$\text{pos} :- p, q, r \models \text{pos} :- p, q, r, s, t$$

because

$$\{\text{pos}, \neg p, \neg q, \neg r\} \subseteq \{\text{pos}, \neg p, \neg q, \neg r, \neg s, \neg t\}$$

Subsumption in propositional logic



Subsumption in propositional logic

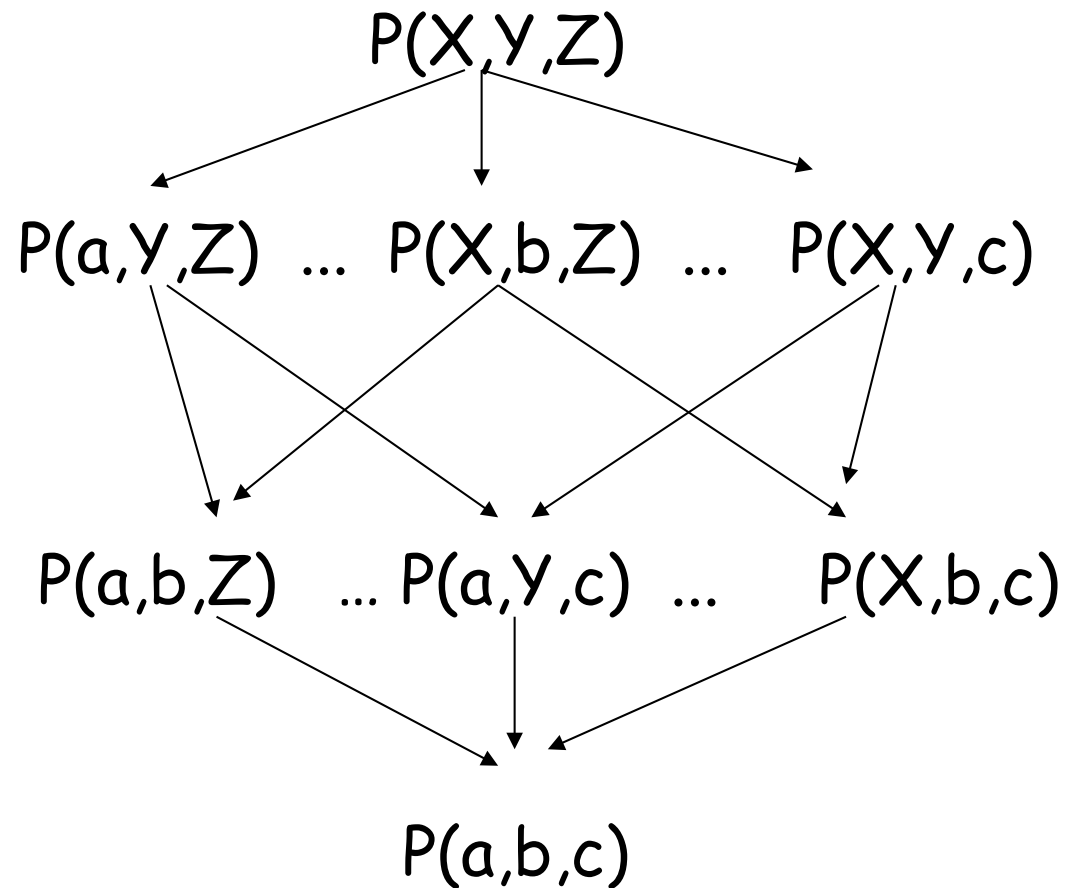
- Perfect structure
- Complete lattice
 - any two clauses have unique
 - least upper bound (least general generalization)
 - greatest lower bound
- No syntactic variants
- Easy specialization, generalization

Subsumption in predicate logic

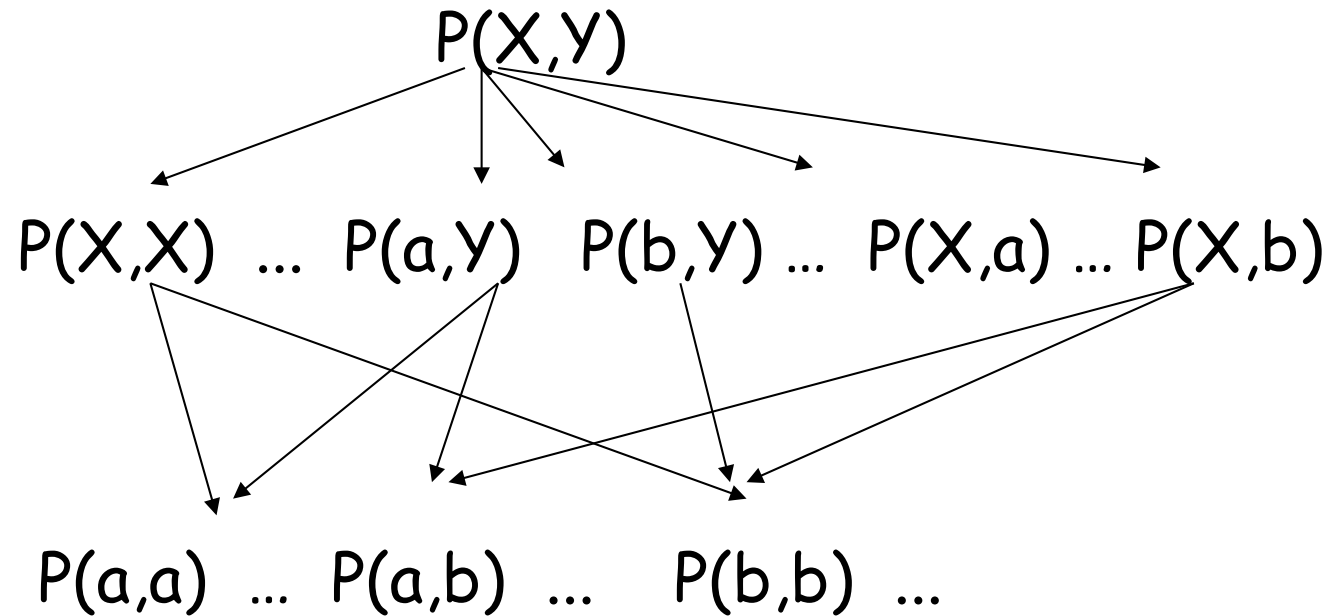
Subsumption in logical atoms

- g subsumes s if and only if there is a substitution θ such that $g\theta = s$
- e.g. $p(X, Y, X)$ subsumes $p(a, Y, a)$
- e.g. $p(f(X), Y)$ subsumes $p(f(a), Y)$

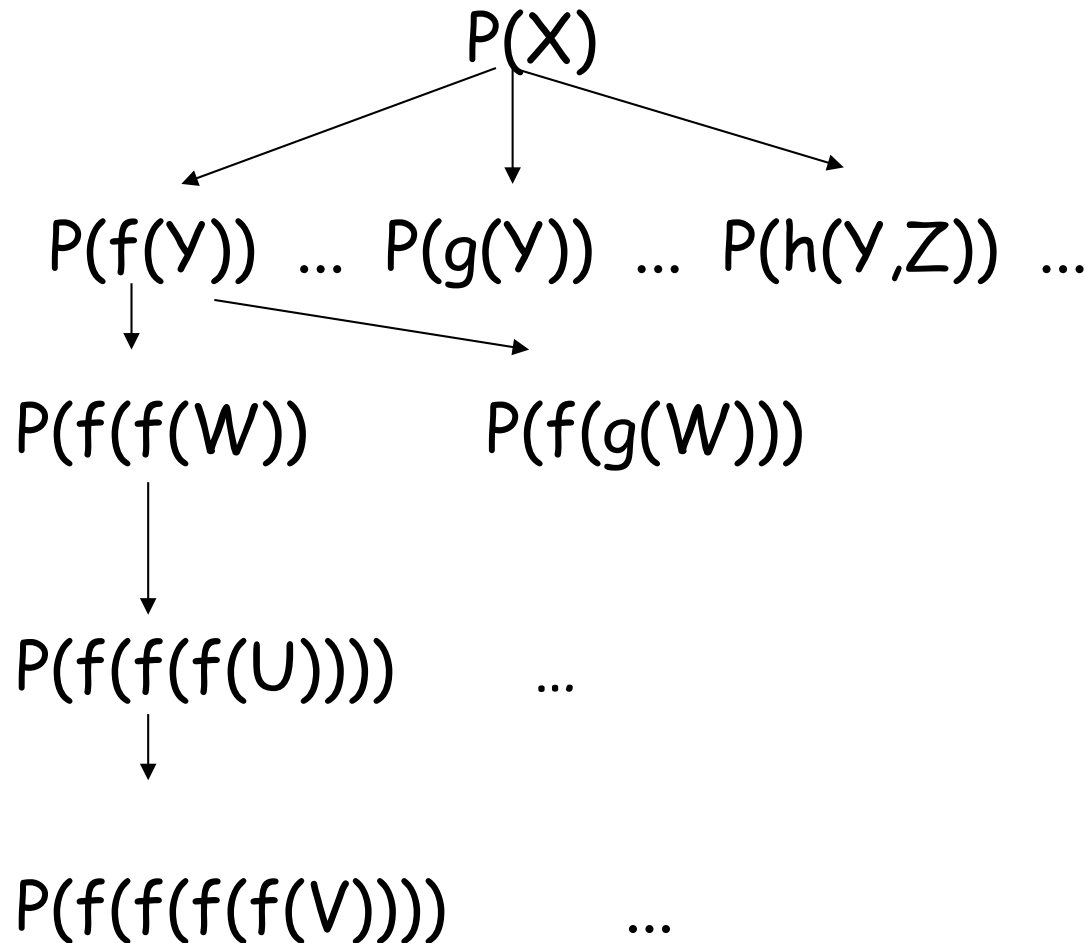
Subsumption in simple logical atoms



Subsumption in simple logical atoms



Subsumption in logical atoms



Subsumption in logical atoms

G subsumes F iff there is a substitution θ such that $G\theta = F$

- Still nice properties and complete lattice up to variable renaming
 - $p(X,a)$ and $p(U,a)$
 - greatest lower bound = unification
 - unification $p(X,a)$ and $p(b,U)$ gives $p(b,a)$
 - least upper bound = anti-unification = lgg
 - $\text{lgg } p(X,a,b)$ and $p(c,a,d) = p(X,a,Y)$
 - $\text{lgg } p(X,f(X,c))$ and $p(a,f(a,Y))$ gives $p(U,f(U,T))$

Ideal Specialization Operator

- Ideal Specialization operator :
 - apply a substitution $\{ X / Y \}$ where X, Y already appear in atom
 - apply a substitution $\{ X / f(Y_1, \dots, Y_n) \}$ where Y_i new variables
 - apply a substitution $\{ X / c \}$ where c is a constant
- Ideal Generalization operator :
 - apply an inverse substitution
 - Inverse substitution substitutes terms at specified places by variables
 - Invert one of the specialization steps above
 - Replace some (but not all) occurrences of a variable X by a different variable Y
 - Replace all terms $f(Y_1, \dots, Y_n)$ where Y_i are distinct by a new variable X
 - Replace some occurrences of a constant by a new variable

Ideal Specialization Operator

Properties

Ideal specialisation operator must be

- locally complete
- globally complete
- proper

Ideal Specialization Operator

Let A be an atom. Then

$$\rho_{s,a,i}(A) = \{A\theta \mid \theta \text{ is an elementary substitution}\} \quad (5.4)$$

where an elementary substitution θ is of the form

$$\theta = \begin{cases} \{X/f(X_1, \dots, X_n)\} & \text{with } f \text{ a functor of arity } n \text{ and} \\ & \text{the } X_i \text{ are variables not occurring in } A \\ \{X/c\} & \text{with } c \text{ a constant} \\ \{X/Y\} & \text{with } X \text{ and } Y \text{ are variables occurring in } A \end{cases} \quad (5.5)$$

It is relatively easy to see that $\rho_{s,a,i}$ is an ideal operator for atoms.

Optimal Specialization Operator

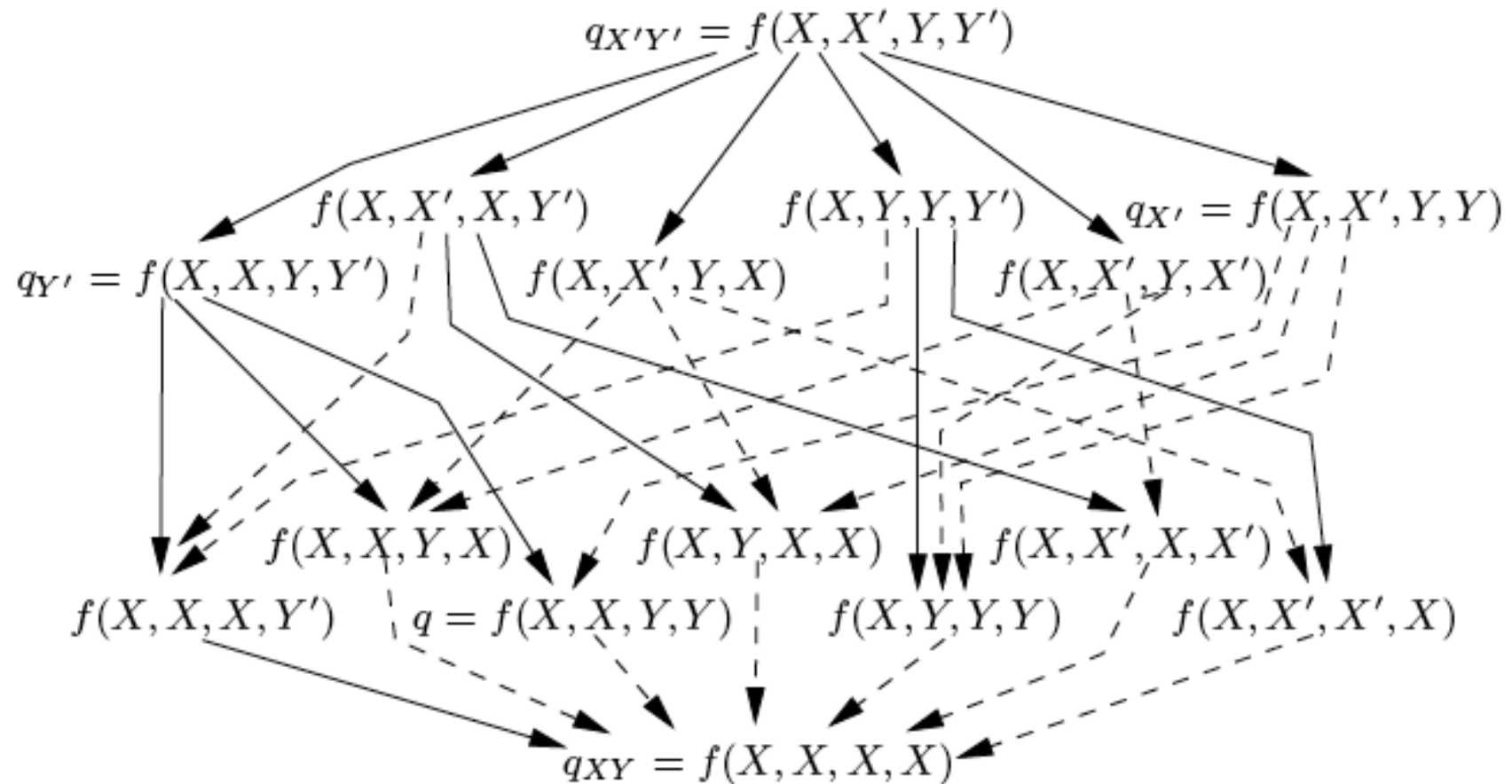


Fig. 5.6. Example of duplicate avoidance for Unification

Optimal Specialization Operator

Let A be an atom. Then

$$\rho_{s,a,o}(A) = \{A\theta \mid \theta \text{ is an optimal elementary substitution}\} \quad (5.6)$$

where an elementary substitution θ is of the form θ is an *optimal elementary substitution* for an atom A iff it is of the form

$$\theta = \begin{cases} \{X/f(X_1, \dots, X_n)\} & \text{with } f \text{ a functor of arity } n \text{ and} \\ & \text{the } X_i \text{ variables not occurring in } A \\ \{X/c\} & \text{with } c \text{ a constant} \\ \{X/Y\} & \text{where } X \text{ and } Y \text{ are variables occurring in } A \\ & X \text{ occurs once, and all variables to the right of} \\ & X \text{ occur only once in } A \end{cases} \quad (5.7)$$

Theta-subsumption (Plotkin 70)

- Most important framework for inductive logic programming. Used by all major ILP systems.
- F and G are single clauses
- Combines propositional subsumption and subsumption on logical atoms
- $c1$ theta-subsumes $c2$ if and only if there is a substitution θ such that $c1 \theta \subseteq c2$
- $c1 : \text{father}(X,Y) \text{ :- } \text{parent}(X,Y), \text{male}(X)$
- $c2 : \text{father}(\text{adam}, \text{kain}) \text{ :- } \text{parent}(\text{adam}, \text{kain}), \text{parent}(\text{adam}, \text{an}), \text{male}(\text{adam}), \text{female}(\text{an})$
- $\theta = \{ X / \text{adam}, Y / \text{kain} \}$

Example

- $d1 : p(X,Y) :- q(X,Y), q(Y,X)$
- $d2 : p(Z,Z) :- q(Z,Z)$
- $d3 : p(a,a) :- q(a,a)$
- $\text{theta}(1,2) : \{X / Z, Y / Z\}$
- $\text{theta}(2,3) : \{Z/a\}$
- $d1$ is a generalization of $d3$
- Mapping several literals onto one leads (sometimes) to combinatorial problems

Properties

- Soundness : if $c1$ theta-subsumes $c2$ then $c1 \models c2$
- Incompleteness (but only for self-recursive clauses) wrt logical entailment
 - $c1 : p(f(X)) :- p(X)$
 - $c2 : p(f(f(Y))) :- p(Y)$
- Decidable (but NP-complete)
- transitive and reflexive but not anti-symmetric

Specialisation operations

binding of two distinct variables

$\text{path}(X,Y)$. . . *There is a path between nodes X and Y in a graph*

$\text{edge}(X,Y)$. . . *There is an edge between X and Y*

$\text{spec}(\text{path}(X, Y)) = \text{path}(X, X)$

adding a most general atom into a clause body

arguments are distinct and so far unused variables

$\text{spec}(\text{path}(X,Y)) = (\text{path}(X,Y) \text{ :- } \text{edge}(U,V))$

= a minimal set of specialisation operations for logic programs without function symbols:

Specialisation operations

Logic programs with functions:

A minimal set extended with

Substitution a variable with a most general term

arguments are distinct and so far unused variables

$\text{spec}(\text{number}(X)) = \text{number}(0)$

$\text{spec}(\text{number}(X)) = \text{number}(s(Y))$.

Specialisation and generalisation

Domain-dependent operations - examples

$\text{triangle} \leq \text{n-angle} \leq \text{plannar object}$

$\text{town} \leq \text{district} \leq \text{region} \leq \text{country} \leq \text{continent}$

$[0,1) \leq [0,11) \leq [0,111) \leq [0,\text{inf})$