

# Exercise Session 1

## 1 Joint Probability Distribution

- a) Normalize such that the table represents the joint probability table of  $P(a, b)$  (with  $a$  a binary variable with domain  $\{1, 2\}$  and  $b$  a discrete variable with domain  $\{true, false, maybe\}$ )?

$P(a, b)$		$b$		
		true	false	maybe
$a$	1	0.1	0.5	0.3
	2	0.5	0.2	0.3

- b) Normalize such that the table represents the joint probability table of  $P(a, b, c)$  (with  $a, b$  and  $c$  binary variables with domain  $\{true, false\}$ )?

$P(a, b, c = true)$			$b$	
			true	false
$a$	true		0.3	0.5
	false		0.2	0.3

$P(a, b, c = false)$			$b$	
			true	false
$a$	true		0.2	0.1
	false		0.2	0.1

## 2 Continuous Probability Distribution

- a) Draw a rectangular continuous probability distribution.  
b) Draw a triangular continuous probability distribution.

## 3 Conditional Probability Distribution

- a) Normalize such that the table represents the conditional probability table of  $P(a|b)$  (with  $a$  a binary variable with domain  $\{1, 2\}$  and  $b$  a discrete variable with domain  $\{true, false, maybe\}$ )?

$P(a b)$		$b$		
		true	false	maybe
$a$	1	0.1	0.5	0.3
	2	0.5	0.2	0.3

- b) Normalize such that the table represents the conditional probability table of  $P(a, b|c)$  (with  $a, b$  and  $c$  binary variables with domain  $\{true, false\}$ )?

$P(a, b c = true)$			$b$	
			true	false
$a$	true		0.3	0.5
	false		0.2	0.3

$P(a, b c = false)$			$b$	
			true	false
$a$	true		0.2	0.1
	false		0.2	0.1

## 4 Marginal Probability Distribution from Joint Probability Distribution

- a) Calculate the marginal probability distribution  $p(b)$  from the normalized probability distribution calculated in question 1.a.

- b) Calculate the marginal probability distribution  $p(a, c)$  from the normalized probability distribution calculated in question 1.b.

## 5 Conditional Probability Distribution from Joint Probability Distribution

- a) Calculate the conditional probability distribution  $p(a|b)$  from the normalized probability distribution calculated in question 1.a.
- b) Calculate the conditional probability distribution  $p(b|a, c)$  from the normalized probability distribution calculated in question 1.b.

## 6 Independence

- a) Fill in the question marks such that  $a$  and  $b$  are independent in the following probability distribution table.

$P(a, b)$		$b$		
		true	false	maybe
	1	0.2	0.1	0.3
$a$	2	0,133	?	?

- b) Fill in the question marks such that  $a$  and  $b$  are independent in the following probability distribution table.

$P(a, b)$		$b$		
		true	false	maybe
	1	0.05	0.1	0.15
$a$	2	?	?	?

## 7 Conditional Independence

- a) Fill in the question marks such that  $a$  and  $b$  are conditionally independent given  $c$  in the following probability distribution table.

$P(a, b, c = true)$		$b$	
		true	false
	x	0.056	0.084
$a$	y	0.008	?
	z	0.016	?

$P(a, b, c = false)$		$b$	
		true	false
	x	0.072	0.008
$a$	y	0.432	?
	z	0.216	?

- b) Fill in the question marks such that  $a$  and  $b$  are conditionally independent given  $c$  in the following probability distribution table.

$P(a, b, c = true)$		$b$	
		true	false
	x	0.056	0.084
$a$	y	0.008	0.012
	z	0.016	0.024

$P(a, b, c = false)$		$b$	
		true	false
	x	0.072	?
$a$	y	0.432	?
	z	0.216	?