Master MLDM/DSC/CPS2 - First year
Introduction to Artificial Intelligence
Exam on Propositional and First Order Logics
October 28, 2021

Time allocated: 3h
No documents authorized. Your copy must be written in English. Grading will depend on the cleanliness of your copy and the clarity of your explanations.
TAKE CARE: any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

1 Truth table (2 points)

The Sheffer stroke, written "$\uparrow$", denotes a logical operation that is equivalent to the negation of the conjunction operation. Hence, $\Phi \uparrow \Psi$ is logically equivalent to $\neg(\Phi \wedge \Psi)$. **Using the truth table method**, show that:

1. $\neg p$ is logically equivalent to $p \uparrow p$

p	∼	p	∼ (	p	&	p)
T	F	T	F	T	T	T
F	T	F	T	F	F	F

2. $p \Rightarrow q$ is logically equivalent to $p \uparrow (q \uparrow q)$

p	q	p	→	q	∼ (	p	&	∼ (	q	&	q))
T	T	T	T	T	T	T	F	F	T	T	T
T	F	T	F	F	F	T	T	T	F	F	F
F	T	F	T	T	T	F	F	F	T	T	T
F	F	F	T	F	T	F	F	T	F	F	F

3. $p \Rightarrow q$ is logically equivalent to $p \uparrow (p \uparrow q)$

p	q	p	→	q	∼ (	p	&	∼ (	p	&	q))
T	T	T	T	T	T	T	F	F	T	T	T
T	F	T	F	F	F	T	T	T	T	F	F
F	T	F	T	T	T	F	F	T	F	F	T
F	F	F	T	F	T	F	F	T	F	F	F

4. $p \wedge q$ is logically equivalent to $(p \uparrow q) \uparrow (p \uparrow q)$

p	q	p	&	q	∼ (	∼ (	p	&	q)	&	∼ (	p	&	q))
T	T	T	T	T	T	F	T	T	T	F	F	T	T	T
T	F	T	F	F	F	T	T	F	F	T	T	T	F	F
F	T	F	F	T	F	T	F	F	T	T	T	F	F	T
F	F	F	F	F	F	T	F	F	F	T	T	F	F	F

5. $p \vee q$ is logically equivalent to $(p \uparrow p) \uparrow (q \uparrow q)$

p	q	p	∨	q	∼ (	∼ (	p	&	p)	&	∼ (	q	&	q))
T	T	T	T	T	T	F	T	T	T	F	F	T	T	T
T	F	T	T	F	T	F	T	T	T	F	T	F	F	F
F	T	F	T	T	T	T	F	F	F	F	F	T	T	T
F	F	F	F	F	F	T	F	F	F	T	T	F	F	F

2 Validity, unsatisfiability, contingency (5 points)

Using resolution reasoning in proposition logic or first order logic, and the methodology we saw during the course, say whether each sentence below is valid, unsatisfiable or contingent:

1. $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$
   Let's try to prove this sentence is unsatisfiable. To do so, convert it into CNF and try to prove the empty set from the clauses obtained.
   $\neg(\neg p \vee q) \vee (\neg\neg q \vee \neg p)$
   $(p \wedge \neg q) \vee (q \vee \neg p)$

$(p \vee q \vee \neg p) \wedge (\neg q \vee q \vee \neg p)$
So we get two clauses:
$\{p, q, \neg p\}$
and
$\{\neg q, q, \neg p\}$
and we cannot generate the empty clause with that, so **the sentence is not unsatisfiable**.
So now let us consider the negation of the sentence and convert it into CNF.
$\neg((p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p))$
$\neg(\neg(\neg p \vee q) \vee (\neg\neg q \vee \neg p))$
$\neg((p \wedge \neg q) \vee (q \vee \neg p))$
$\neg(p \wedge \neg q) \wedge \neg(q \vee \neg p))$
$(\neg p \vee q) \wedge \neg q \wedge p$
So we get three clauses:
(1) $\{\neg p, q\}$
(2) $\{\neg q\}$
(3) $\{p\}$
and now we can use the resolution principle several time to get the empty clause:
1+2: $\{\neg p\}$ (4)
4+3: $\{\}$
So the negation of the sentence is unsatisfiable, that means **the sentence is valid**.

2. $(((p \vee \neg q) \wedge (r \vee q)) \Rightarrow (p \vee r))$
Let's try to prove this sentence is unsatisfiable. To do so, convert it into CNF and try to prove the empty set from the clauses obtained.
$(((p \vee \neg q) \wedge (r \vee q)) \Rightarrow (p \vee r))$
$\neg((p \vee \neg q) \wedge (r \vee q)) \vee (p \vee r)$
$(\neg(p \vee \neg q) \vee \neg(r \vee q)) \vee (p \vee r)$
$(\neg p \wedge q) \vee (\neg r \wedge \neg q)) \vee (p \vee r)$
$(\neg p \vee \neg r \vee p \vee r) \wedge (\neg p \vee \neg q \vee p \vee r) \wedge (q \vee \neg r \vee p \vee r) \wedge (q \vee \neg q \vee p \vee r)$
So we get four clauses:
(1) $\{\neg p, \neg r, p, r\}$
(2) $\{\neg p, \neg q, p, r\}$
(3) $\{q, \neg r, p, r\}$
(4) $\{q, \neg q, p, r)$
and we cannot generate the empty clause with that, so **the sentence is not unsatisfiable**.
So now let us consider the negation of the sentence and convert it into CNF.
$\neg(((p \vee \neg q) \wedge (r \vee q)) \Rightarrow (p \vee r))$
$\neg(\neg((p \vee \neg q) \wedge (r \vee q)) \vee (p \vee r))$
$\neg\neg((p \vee \neg q) \wedge (r \vee q)) \wedge \neg(p \vee r))$
$(p \vee \neg q) \wedge (r \vee q) \wedge \neg p \wedge \neg r)$
So we get four clauses:
(1) $\{p, \neg q\}$
(2) $\{r, q\}$
(3) $\{\neg p\}$
(4) $\{\neg r\}$
and now we can use the resolution principle several time to get the empty clause:
1+3: $\{\neg q\}$ (4)
4+2: $\{r\}$ (5)
5+4: $\{\}$
So the negation of the sentence is unsatisfiable, that means **the sentence is valid**.

3. $\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
Let's try to prove this sentence is unsatisfiable. To do so, convert it into CNF and try to prove the empty set from the clauses obtained.
$\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
$\forall X.(\neg p(X) \vee q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
$\forall X.(\neg p(X) \vee q(X)) \wedge \exists Y.(p(Y) \wedge \neg q(Y))$

$\forall X.(\neg p(X) \vee q(X)) \wedge p(a) \wedge \neg q(a)$
$(\neg p(X) \vee q(X)) \wedge p(a) \wedge \neg q(a)$
So we get three clauses:
(1) $\{\neg p(X), q(X)\}$
(2) $\{p(a)\}$
(3) $\{\neg q(a)\}$
and now we can use the resolution principle several time to get the empty clause:
1+2 $\{X \leftarrow a\}$: $\{q(a)\}$ (4)
4+3: $\{\}$
This means **the sentence is unsatisfiable**.

4. $\forall X.p(X) \Rightarrow \exists X.p(X)$
Let's try to prove this sentence is unsatisfiable. To do so, convert it into CNF and try to prove the empty set from the clauses obtained.
$\forall X.p(X) \Rightarrow \exists X.p(X)$
$\neg\forall X.p(X) \vee \exists X.p(X)$
$\exists X.\neg p(X) \vee \exists X.p(X)$
$\exists X.\neg p(X) \vee \exists Y.p(Y)$
$\neg p(a) \vee p(b)$
So we get one clause: (1) $\{\neg p(a), p(b)\}$
and we cannot generate the empty clause with that, so **the sentence is not unsatisfiable**.
So now let us consider the negation of the sentence and convert it into CNF.
$\neg(\forall X.p(X) \Rightarrow \exists X.p(X))$
$\neg(\neg\forall X.p(X) \vee \exists X.p(X))$
$\forall X.p(X) \wedge \neg\exists X.p(X)$
$\forall X.p(X) \wedge \forall X.\neg p(X)$
$\forall X.p(X) \wedge \forall Y.\neg p(Y)$
$\forall X.p(X) \wedge \forall Y.\neg p(Y)$
$p(X) \wedge \neg p(Y)$
So we get two clauses:
(1) $\{p(X)\}$
(2) $\{\neg p(Y)\}$
and now we can use the resolution principle several time to get the empty clause:
1+2 $\{X \leftarrow Y\}$: $\{\}$
This means the negation of the sentence is unsatisfiable so, **the sentence is valid**.

5. $\forall X.\forall Y.((lt(X,Y) \Rightarrow \neg lt(Y,X)) \vee \exists Z.\forall X.(lt(X,Y) \Rightarrow \forall X.lt(Z,X)))$
Let's try to prove this sentence is unsatisfiable. To do so, convert it into CNF and try to prove the empty set from the clauses obtained.
$\forall X.\forall Y.((lt(X,Y) \Rightarrow \neg lt(Y,X)) \vee \exists Z.\forall X.(lt(X,Y) \Rightarrow \forall X.lt(Z,X)))$
$\forall X.\forall Y.((\neg lt(X,Y) \vee \neg lt(Y,X)) \vee \exists Z.\forall X.(\neg lt(X,Y) \vee \forall X.lt(Z,X)))$
$\forall X.\forall Y.((\neg lt(X,Y) \vee \neg lt(Y,X)) \vee \exists Z.\forall A.(\neg lt(A,Y) \vee \forall B.lt(Z,B)))$
$\forall X.\forall Y.((\neg lt(X,Y) \vee \neg lt(Y,X)) \vee \forall A.(\neg lt(A,Y) \vee \forall B.lt(f(X,Y),B)))$
$\neg lt(X,Y) \vee \neg lt(Y,X) \vee \neg lt(A,Y) \vee lt(f(X,Y),B)$
So we get one clause:
(1) $\{\neg lt(X,Y) \vee \neg lt(Y,X) \vee \neg lt(A,Y) \vee lt(f(X,Y),B)\}$
and we cannot generate the empty clause with that, so **the sentence is not unsatisfiable**.
So now let us consider the negation of the sentence and convert it into CNF.
$\neg(\forall X.\forall Y.((lt(X,Y) \Rightarrow \neg lt(Y,X)) \vee \exists Z.\forall X.(lt(X,Y) \Rightarrow \forall X.lt(Z,X))))$
$\exists X.\exists Y.\neg((lt(X,Y) \Rightarrow \neg lt(Y,X)) \vee \exists Z.\forall X.(lt(X,Y) \Rightarrow \forall X.lt(Z,X)))$
$\exists X.\exists Y.\neg((\neg lt(X,Y) \vee \neg lt(Y,X)) \vee \exists Z.\forall X.(\neg lt(X,Y) \vee \forall X.lt(Z,X)))$
$\exists X.\exists Y.(\neg(\neg lt(X,Y) \vee \neg lt(Y,X)) \wedge \neg(\exists Z.\forall X.(\neg lt(X,Y) \vee \forall X.lt(Z,X))))$
$\exists X.\exists Y.(lt(X,Y) \wedge lt(Y,X)) \wedge (\forall Z.\exists X.\neg(\neg lt(X,Y) \vee \forall X.lt(Z,X))))$
$\exists X.\exists Y.(lt(X,Y) \wedge lt(Y,X)) \wedge (\forall Z.\exists X.(lt(X,Y) \wedge \exists X.\neg lt(Z,X))))$
$\exists X.\exists Y.(lt(X,Y) \wedge lt(Y,X)) \wedge (\forall Z.\exists A.(lt(A,Y) \wedge \exists B.\neg lt(Z,B))))$
$\exists X.\exists Y.(lt(X,Y) \wedge lt(Y,X)) \wedge (\forall Z.(lt(f(Z),Y) \wedge \neg lt(Z,g(Z)))))$
$(lt(a,b) \wedge lt(b,a)) \wedge (lt(f(Z),b) \wedge \neg lt(Z,g(Z)))$
So we get four clauses:
(1) $\{lt(a,b)\}$

(2) $\{lt(b, a)\}$
(3) $\{lt(f(Z), b)\}$
(4) $\{\neg lt(Z, g(Z))\}$
and we cannot generate the empty clause with that, so the negation of the sentence is not unsatisfiable and thus **the sentence is not valid**.
The conclusion is that **the sentence is contingent**.

3 Resolution principle (3 points)

3.1 Subject

For each pair of clauses below, say whether the resolution principle can be applied. If yes give the resolvent, if no, explain why.

1. $\{r(X), q(X, a, f(X)\}$ and $\{\neg q(b, Y, f(b), s(Y)\}$
2. $\{\neg p(X, f(Y), k), \neg p(g(a, b), f(c), W), \neg p(W)\}$ and $\{p(A, f(B), C), p(A, B)\}$
3. $\{p(X, a, Y), s(Y)\}$ and $\{r(X), \neg p(b, X, Z)\}$
4. $\{p(a, f(X, g(Y), Z), b, T), r(X, Y), p(X, f(a, T, c), Y, g(Y))\}$ and $\{\neg p(A, f(C, D, E), b, F), s(A, F)\}$
5. $\{p(A), \neg q(b)\}$ and $\{\neg p(a), q(B)\}$
6. $\{r(A), p(A, B), s(B), p(a, C), t(C), p(A, b)\}$ and $\{\neg p(X, b), \neg p(a, Y), r(X), w(Y)\}$

3.2 Correction

First of all, let us recall the resolution principle in first order logic.

$$\begin{array}{c} \Phi \\ \Psi \\ \hline ((\Phi' - \{\phi\}) \cup (\Psi' - \{\neg\psi\}))\sigma \\ \text{where } \tau \text{ is a variable renaming on } \Phi \\ \text{where } \Phi' \text{ is a factor of } \Phi\tau \text{ and } \phi \in \Phi' \\ \text{where } \Psi' \text{ is a factor of } \Psi \text{ and } \neg\psi \in \Psi' \\ \text{where } \sigma = mgu(\phi, \psi) \end{array}$$

Using this rule we can then solve the six questions.

1. $\{r(X), q(X, a, f(X))\}$ and $\{\neg q(b, Y, f(b)), s(Y)\}$
   The resolution principle can be applied and the resolvent is: $\{r(b), s(a)\}$
2. $\{\neg p(X, f(Y), k), \neg p(g(a, b), f(c), W), \neg p(W)\}$ and $\{p(A, f(B), C), p(A, B)\}$
   The resolution principle can be applied and the resolvent is: $\{\neg p(k), p(g(a, b), c)\}$
3. $\{p(X, a, Y), s(Y)\}$ and $\{r(X), \neg p(b, X, Z)\}$
   The resolution principle can be applied and the resolvent is: $\{r(a), s(Z)\}$
4. $\{p(a, f(X, g(Y), Z), b, T), r(X, Y), p(X, f(a, T, c), Y, g(Y))\}$ and $\{\neg p(A, f(C, D, E), b, F), s(A, F)\}$
   The resolution principle can be applied and the resolvent is: $\{r(a, b), s(a, g(b))\}$
5. $\{p(A), \neg q(b)\}$ and $\{\neg p(a), q(B)\}$
   The resolution principle can be applied and the resolvent is: $\{p(A), \neg p(a)\}$ or $\{q(B), \neg q(b)\}$
6. $\{r(A), p(A, B), s(B), p(a, C), t(C), p(A, b)\}$ and $\{\neg p(X, b), \neg p(a, Y), r(X), w(Y)\}$
   The resolution principle can be applied and the resolvent is: $\{r(a), s(b), t(b), w(b)\}$

4 Problem modeling and solving I (5 points)

4.1 Subject

Below is information about a simple world.

If Mary has taken the course on database and the course on algorithms then she masters the fundamental units and can continue in computer science. If John has taken the course on algorithms or the course on maths then he can continue in maths. If Mary can continue in computer science then she can have a good job and earn money. Mary has taken the courses on algorithms and databases. John has taken the course on maths. If John can continue in maths then he is happy and wants to dance with Mary. If Mary can have a good job or sees John is happy then she wants to dance with John. If John wants to dance with Mary and Mary wants to dance with John, then life is beautiful.

Model this universe using **propositional logic**, and then provide a resolution proof of: *Life is beautiful.*

Take care: Modelling this using first-order logic would lead to a score of 0/5 for this exercise.

4.2 Correction

4.2.1 Define the proposition constants

mdb: Mary has taken the course on database.
mal: Mary has taken the course on algorithms.
mmf: Mary masters the fundamental units.
mcc: Mary can continue computer science.
jal: John has taken the course on algorithms.
jma: John has taken the course on maths.
jcm: John can continue in maths.
mgj: Mary can have a good job.
mem: Mary can earn money.
jih: John is happy.
jdm: John wants to dance with Mary.
mjh: Mary sees John is happy.
mdj: Mary wants to dance with John.
lib: Life is beautiful.

4.2.2 Convert the English sentences into PL

$mdb \wedge mal \Rightarrow mmf \wedge mcc$
$jal \vee jma \Rightarrow jcm$
$mcc \Rightarrow mgj \wedge mem$
mal
mdb
jma
$jcm \Rightarrow jih \wedge jdm$
$mgj \vee mjh \Rightarrow mdj$
$jdm \wedge mdj \Rightarrow lib$
We also have to negate the conclusion we want to prove: $\neg lib$

4.2.3 Convert the PL sentences into CNF

(1) $\{\neg mdb, \neg mal, mmf\}$
(2) $\{\neg mdb, \neg mal, mcc\}$
(3) $\{\neg jal, jcm\}$
(4) $\{\neg jma, jcm\}$
(5) $\{\neg mcc, mgj\}$
(6) $\{\neg mcc, mem\}$
(7) $\{mal\}$

(8) $\{mdb\}$
(9) $\{jma\}$
(10) $\{\neg jcm, jih\}$
(11) $\{\neg jcm, jdm\}$
(12) $\{\neg mgj, mdj\}$
(13) $\{\neg mjh, mdj\}$
(14) $\{\neg jdm, \neg mdj, lib\}$
(15) $\{\neg lib\}$

4.2.4 Prove the empty set from the set of clauses

(15)+(14): $\{\neg jdm, \neg mdj\}$ (16)
(16)+(11):$\{\neg jcm, \neg mdj\}$ (17)
(17)+(4): $\{\neg jma, \neg mdj\}$ (18)
(18)+(9): $\{\neg mdj\}$ (19)
(19)+(12): $\{\neg mgj\}$ (20)
(20)+(5): $\{\neg mcc\}$ (21)
(21)+(2): $\{\neg mdb, \neg mal\}$ (22)
(22)+(8): $\{\neg mal\}$ (23)
(23)+(7): $\{\}$

So **the conclusion is logically entailed by the set of sentences describing the world**.

5 Problem modeling and solving II (5 points)

5.1 Subject

Below is information about a simple world, for any two people P1 and P2.

- *If a person P1 is richer than another person P2, and they live in the same place then P1 pays more taxes than P2.*
- *If a person P1 is richer than another person P2, and P1 lives in a town and P2 in a countryside, then P1 is smarter than P2.*
- *If a person P1 is richer than a person P3 that is richer than a person P2, then P1 is richer than P2 (the relationship "richer than" is transitive).*
- *The relationship "younger than" is also transitive.*
- *If a person P1 is smarter than a person P2, or if P1 is younger than P2 and P1 is a student and P2 is a teacher, then P1 is happier than P2.*
- *If P1 pays more taxes than P2 then P1 is jealous of P2.*
- *If P1 is happier than P2 or P1 is jealous of P2, then P1 has feelings.*
- *Bess and Dana live in the same place.*
- *John is a student.*
- *John is younger than Mary.*
- *Suzy is a teacher.*
- *Cody lives in town.*
- *Cody is richer than Dana.*
- *Dana lives in the countryside.*
- *Bess is richer than Cody.*
- *Mary is younger than Suzy.*

Using resolution reasoning in first order logic, **find the three persons that have feelings**.

5.2 Correction

5.2.1 Convert the English sentences into FOL

$\forall P1.\forall P2.(richer(P1, P2) \wedge samePlace(P1, P2) \Rightarrow paysMoreTaxes(P1, P2))$
$\forall P1.\forall P2.(richer(P1, P2) \wedge livesInTown(P1) \wedge livesInCountrySide(P2) \Rightarrow smarter(P1, P2))$
$\forall P1.\forall P2.\forall P3(richer(P1, P3) \wedge richer(P3, P2) \Rightarrow richer(P1, P2))$
$\forall P1.\forall P2.\forall P3(younger(P1, P3) \wedge younger(P3, P2) \Rightarrow younger(P1, P2))$
$\forall P1.\forall P2.((smarter(P1, P2) \vee (younger(P1, P2) \wedge student(P1) \wedge teacher(P2))) \Rightarrow happier(P1, P2))$
$\forall P1.\forall P2.(paysMoreTaxes(P1, P2) \Rightarrow jealous(P1, P2))$
$\forall P1.\forall P2.((happier(P1, P2) \vee jealous(P1, P2)) \Rightarrow hasFeelings(P1))$
$samePlace(bess, dana)$
$student(john)$
$younger(john, mary)$
$teacher(suzy)$
$livesInTown(cody)$
$richer(cody, dana)$
$livesInCountrySide(dana)$
$richer(bess, cody)$
$younger(mary, suzy)$

The conclusion we need to prove to find the persons that have feelings is : $\exists P.hasFeelings(P)$

5.2.2 Convert the English sentences into CNF

(1) $\{\neg richer(P1, P2), \neg samePlace(P1, P2), paysMoreTaxes(P1, P2)\}$
(2) $\{\neg richer(P1, P2), \neg livesInTown(P1), \neg livesInCountrySide(P2), smarter(P1, P2)\}$
(3) $\{\neg richer(P1, P3), \neg richer(P3, P2), richer(P1, P2)\}$
(4) $\{\neg younger(P1, P3), \neg younger(P3, P2), younger(P1, P2)\}$
(5) $\{\neg smarter(P1, P2), happier(P1, P2)\}$
(6) $\{\neg younger(P1, P2), \neg student(P1), \neg teacher(P2), happier(P1, P2)\}$
(7) $\{\neg paysMoreTaxes(P1, P2), jealous(P1, P2)\}$
(8) $\{\neg happier(P1, P2), hasFeelings(P1, P2)\}$
(9) $\{\neg jealous(P1, P2), hasFeelings(P1, P2)\}$
(10) $\{samePlace(bess, dana)\}$
(11) $\{student(john)\}$
(12) $\{younger(john, mary)\}$
(13) $\{teacher(suzy)\}$
(14) $\{livesInTown(cody)\}$
(15) $\{richer(cody, dana)\}$
(16) $\{livesInCountrySide(dana)\}$
(17) $\{richer(bess, cody)\}$
(18) $\{younger(mary, suzy)\}$

Finally, we need to consider the negation of the conclusion, that is the clause:
(19): $\{\neg hasFeelings(P)\}$

5.2.3 Prove the empty set from the set of clauses

19+9: $\{\neg jealous(P, P2)\}$ (20)
20+7: $\{\neg paysMoreTaxes(P, P2)\}$ (21)
21+1: $\{\neg richer(P, P2), \neg samePlace(P, P2)\}$ (22)
22+3: $\{\neg richer(P, P3), \neg richer(P3, P2), \neg samePlace(P, P2)\}$ (23)
23+17 $\{P \leftarrow bess\} : \{\neg richer(P3, P2), \neg samePlace(bess, P2)\}$ (24)
24+15 $\{P3 \leftarrow cody, P2 \leftarrow dana\} : \{\neg samePlace(bess, dana)\}$ (25)
25+10: $\{\}$
So a first solution is **P=bess**

Another way to prove the empty set is:
19+8: $\{\neg happier(P, P2)\}$ (26)
26+6: $\{\neg younger(P, P2), \neg student(P), \neg teacher(P2)\}$ (27)
27+4: $\{\neg younger(P, P3), \neg younger(P3, P2), \neg student(P), \neg teacher(P2)\}$ (28)
28+12 $\{P \leftarrow john, P3 \leftarrow mary\} : \{\neg younger(mary, P2), \neg student(john), \neg teacher(P2)\}$ (29)
29+18 $\{P2 \leftarrow suzy\} : \{\neg student(john), \neg teacher(suzy)\}$ (30)
30+11: $\{\neg teacher(suzy)\}$ (31)
31+13: $\{\}$
So a second solution is **P=john**

Another way to prove the empty set is:
19+8: $\{\neg happier(P, P2)\}$ (26)
26+5: $\{\neg smarter(P, P2)\}$ (32)
32+2: $\{\neg richer(P, P2), \neg livesInTown(P), \neg livesInCountrySide(P2)\}$ (33)
33+15 $\{P \leftarrow cody, P2 \leftarrow dana\} : \{\neg livesInTown(cody), \neg livesInCountrySide(dana)\}$ (34)
34+14: $\{\neg livesInCountrySide(dana)\}$ (35)
35+16: $\{\}$
So a third solution is **P=cody**