Exercise Session 3

1 Hidden Markov Models

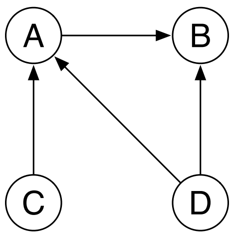
The weather in Belgium can be a little tricky. We can have rainy and sunny days and the initial probability of having a sunny day is 25%. When the previous day was rainy, the probability of having a new rainy day is 80%. When the previous day was sunny, the probability of having a new sunny day is 50%. Each morning we look outside and try to predict the weather but since the weather can change quickly in Belgium we are often wrong. Observing a sunny day in the morning is only in 75% of the cases correct. Observing rain in the morning is only in 80% of the cases correct.

a) Model the example above as an HMM.

b) Calculate the probability of a rainy day on day 5 given that day 1 was rainy, day 2 was sunny, day 3 was rainy day 4 was sunny and we've seen sunny weather in the morning.

2 Maps

1. Consider the Bayesian network:

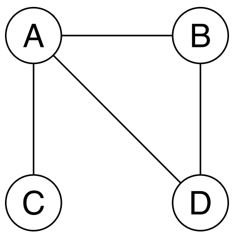


Given that $\mathcal{L}_p = \{C \perp\!\!\!\perp A, A \perp\!\!\!\perp D, C \perp\!\!\!\perp B | A\}$.

Is the given graphical representation a dependence-map, an independence-map or a perfect-map?

Justify your answer.

2. Consider the Markov network:

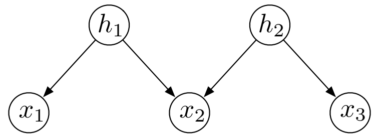


Given that $\mathcal{L}_p = \{C \perp\!\!\!\perp D | A, C \perp\!\!\!\perp B | A\}$.

Is the given graphical representation a dependence-map, an independence-map or a perfect-map?

Justify your answer.

3. Consider the belief network:



a) Write down a Markov network of $p(x_1, x_2, x_3)$.

b) Is your Markov network a perfect map of $p(x_1, x_2, x_3)$

3 Factorization in Markov Networks

(a) Consider the following pairwise Markov network:

$$p(x_1, x_2, x_3, x_4) = \phi(x_1, x_2)\phi(x_2, x_3)\phi(x_3, x_4)\phi(x_4, x_1) \tag{1}$$

- Draw a graphical representation.
- Express the following conditional probabilities in terms of ϕ and simplify as much as possible:

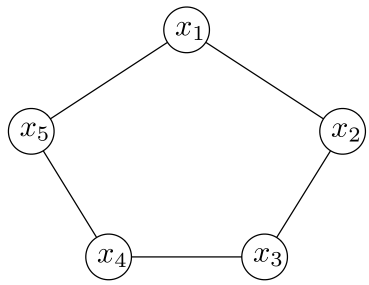
  a) $p(x_1|x_2, x_4)$

  b) $p(x_2|x_1, x_3)$

  c) $p(x_3|x_2, x_4)$

  d) $p(x_4|x_1, x_3)$

(b) Consider now the following undirected graph:



It represents a Markov network with nodes x_1, x_2, x_3, x_4, x_5 and pairwise potentials $\phi(x_i, x_j)$.

Show that the joint distribution can be written as

$$p(x_1, x_2, x_3, x_4, x_5) = \frac{p(x_1, x_2, x_5)p(x_2, x_4, x_5)p(x_2, x_3, x_4)}{p(x_2, x_5)p(x_2, x_4)} \tag{2}$$

4 Independence and factorization

You are given that:

$$x \perp\!\!\!\perp y | (z, u), u \perp\!\!\!\perp z | \emptyset \tag{3}$$

Derive the most general form of probability distribution $p(x, y, z, u)$ consistent with these statements. Does this distribution have a simple graphical model?

5 Bucket elimination

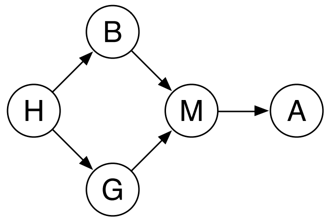


Figure 1: Bayesian network representing Edsger's finances

When Edsger is (H)ungry he is quite likely to go to a (G)rocery shop. And because he is a very sweet guy he also starts (B)aking himself a pie with some probability. Both these activities cost (M)oney (he has to pay for his groceries and the electricity his oven consumes). When Edsger spends money his (A)ccount gets charged. These causal relations are encoded in the Bayes network in Figure 5

Use bucket elimination to compute $P(B)$. Solve numerically and show the symbolic computation that you did. Choose as variable elimination order A, M, G, H with A the highest and H the lowest bucket.

$$\begin{array}{rclrcl}
p(H = true) & = & 0.8 & & & \\
p(B = true|H = true) & = & \pi/4 & p(B = true|H = false) & = & \frac{2}{3\pi} \\
p(G = true|H = true) & = & 0.9 & p(G = true|H = false) & = & 0.2 \\
p(M = true|G = true, B = true) & = & 0.15 & p(M = true|G = true, B = false) & = & 0.25 \\
p(M = true|G = false, B = true) & = & 0.66 & p(M = true|G = false, B = false) & = & 0.33 \\
p(A = true|M = true) & = & 0.99 & p(A = true|M = false) & = & 0.01
\end{array}$$