Master MLDM/DSC/CPS2 - First year
Introduction to Artificial Intelligence
Exam on Propositional and First Order Logics

Time allocated: 3h
No documents authorized. Your copy must be written in English. Scoring will depend on the cleanliness of your copy and the clarity of your explanations.
TAKE CARE: any cheating will be severely punished and will lead to a formal complaint to the disciplinary council of the university.

1 Truth table (2 points)

The Pierce's arrow, written "$\downarrow$", also known as the NOR operator, denotes a logical operation that is equivalent to the negation of the disjunction operation. Hence, $\Phi \downarrow \Psi$ is logically equivalent to $\neg(\Phi \vee \Psi)$. Using the truth table method, show that:

1. $\neg p$ is logically equivalent to $p \downarrow p$
2. $p \Rightarrow q$ is logically equivalent to $((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$
3. $p \wedge q$ is logically equivalent to $(p \downarrow p) \downarrow (q \downarrow q)$
4. $p \vee q$ is logically equivalent to $(p \downarrow q) \downarrow (p \downarrow q)$

2 Validity, unsatisfiability, contingency (4 points)

Using resolution reasoning in proposition logic or first order logic, and the methodology we saw during the course, say whether each sentence below is valid, unsatisfiable or contingent:

1. $(p \Rightarrow q) \Rightarrow (\neg p \Rightarrow \neg q)$
2. $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \wedge q) \Rightarrow r)$
3. $((p \vee q) \Rightarrow (r \vee t)) \Rightarrow ((p \Rightarrow q) \Rightarrow (q \Rightarrow t))$
4. $\forall X.(p(X) \Rightarrow q(X)) \wedge \exists X.(p(X) \wedge \neg q(X))$
5. $\forall X.p(X) \Rightarrow \exists X.p(X)$

3 Unification (2 points)

For each pair of logical sentences below, say whether they are unifiable or not. In case they are unifiable give their most general unifier, in case they are not unifiable, explain why.

1. p(a,b,Z) and p(X,Y,c)
2. p(Z,f(X),c) and p(X,Z,Y)
3. p(X,f(Y),Z,W) and p(g(a,b),f(c),k)
4. p(A,f(B),C) and q(a,C,f(D))
5. p(a,f(X,g(Y),Z),b,T) and p(X,f(a,T,c),Y,f(Y)).
6. p(a,f(X,g(Y),Z),b,T) and p(X,f(a,T,c),Y,g(Y)).
7. p(a,f(X,g(Y),Z),b,T) and p(X,f(a,T,c),Y,g(T)).
8. p(a,f(X,g(Y,a),Z),b,T) and p(X,f(a,T,a),Y,g(b,Z)).

4 Conjonctive Normal Form (2 points)

Convert to CNF the sentences below:

1. $\forall X.\forall Y.((lt(X,Y) \Rightarrow \neg lt(Y,X)) \vee \exists Z.\forall X.(lt(X,Y) \Rightarrow \forall X.lt(Z,X)))$
2. $\forall Y.\exists X.p(X,Y) \Leftrightarrow \forall Z.\forall X.p(Z,X)$

5 Problem modeling and solving I (5 points)

Here are some informations about a simple world:

When Mary isn't sick, she sings, she dances with John, and Harry is jealous. When Lucy is sick and wants to run outside, John is afraid. When Mary is not happy, she cannot eat. When Mary is dancing with John or Harry, Lucy is sick. When John or Harry is jealous, Lucy is sick. Mary isn't sick. When John is afraid or Harry is jealous, Mary is not happy. When Mary sings, Lucy wants to run outside.

Model this universe using propositional logic, and then provide a resolution proof of: *Harry is jealous and Mary cannot eat.*

6 Problem modeling and solving II (5 points)

Here are some informations about a simple world:

- *The antelope Emma is a herbivorous animal.*
- *The lion Harry is a ferocious animal.*
- *A ferocious animal is a carnivore.*
- *A carnivorous animal eats meat.*
- *A herbivorous animal eats grass.*
- *All animals drink water.*
- *Any carnivore can eat any herbivore.*
- *Any animal consumes what it drinks or eats.*

Using resolution reasoning in first order logic, answer these questions :*"Is there a ferocious animal in this world, and what does it consume?"*