

# Dynamic Time Warping

- How similar are EXERCISE and EXIRSAIS ?

E X E R C I S E

E X I R S A I S

d = 5 ?

- 'exercise' vs. 'zexercises' ; 'page' vs. 'ages'

E X E R C I S E

Z E X E R C I S E S

d = ???

P A G E

A G E S

d = ???

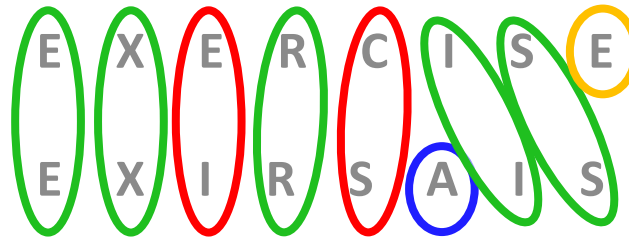
# EDIT DISTANCE

## Exercise vs. Exirsais

EDIT DISTANCE = A *matching* paradigm for *sequences*

REFERENCE

OBSERVATION

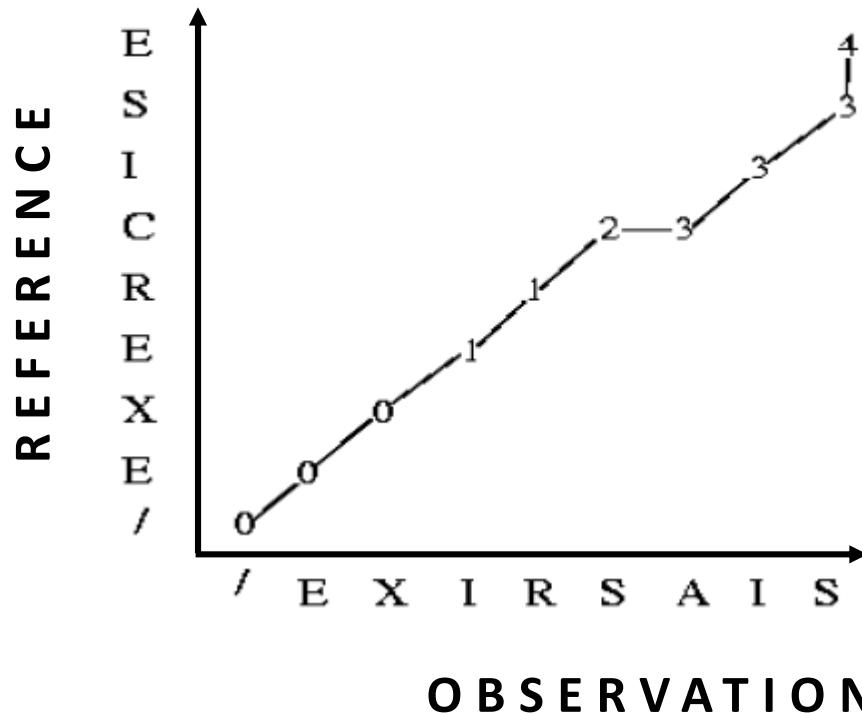


Match

Substitution

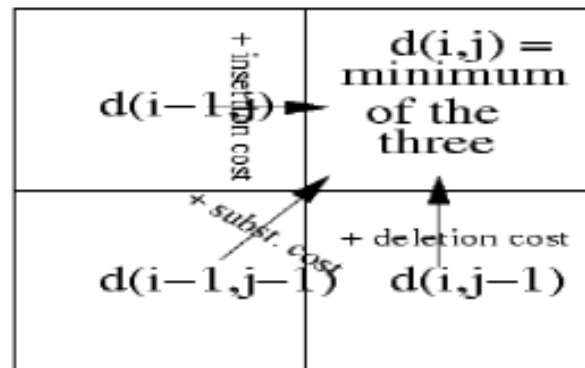
Insertion

Deletion



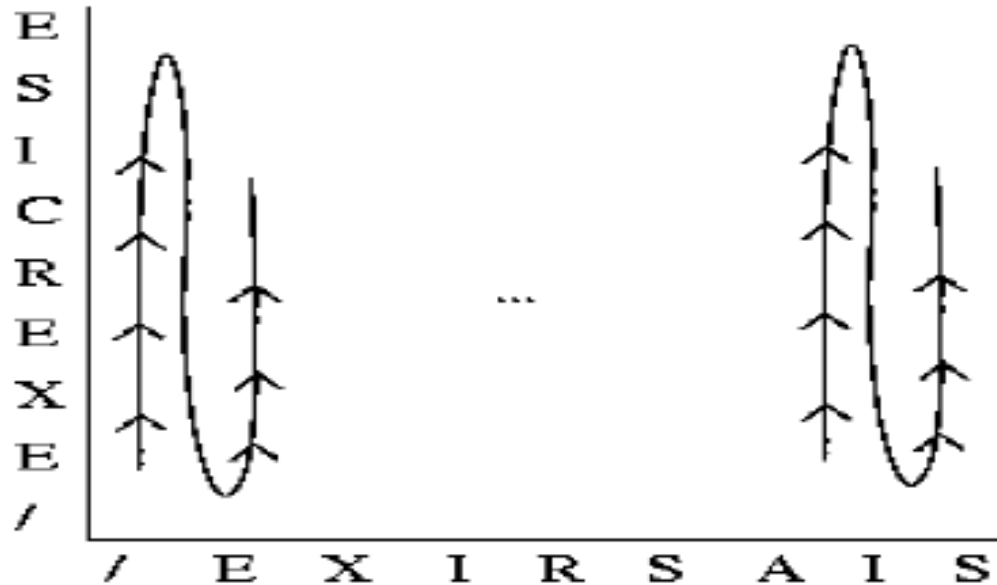
# Minimum Edit Distance

- **MINIMUM EDIT DISTANCE (MED):** minimum number of edits that is required to transform REFERENCE to OBSERVATION
- The MED can be incrementally computed
  - the MED for a partial string edit (reference up to 'j', observation up 'i')  
= the minimum of {MED for a shorter match + EDIT[  $y_j \rightarrow x_i$  ]}
  - Incremental Single EDITS
    - Substitution (cost=0 for matching characters in observation and reference)
    - Deletion (1 character in reference string, no character in observation string)
    - Insertion (no character in reference string, 1 character in observation string)



# Trellis Computation

- Initialization
  - $d(0,0) = 0$
  - time needs to move forward for reference and observation
    - compute column by column
    - compute each column bottom to top



# Trellis Computations

## From local to accumulated distances

E	1	0	1	1	1	1	1	1	1
S	1	1	1	1	1	0	1	1	0
I	1	1	1	0	1	1	1	0	1
C	1	1	1	1	1	1	1	1	1
R	1	1	1	1	0	1	1	1	1
E	1	0	1	1	1	1	1	1	1
X	1	1	0	1	1	1	1	1	1
E	1	0	1	1	1	1	1	1	1
/	0	1	1	1	1	1	1	1	1
	/	E	X	I	R	S	A	I	S

local distances  
(substitution costs)

E	8								
S	7								
I	6								
C	5								
R	4	3	?						
E	3	2							
X	2	1							
E	1	0							
/	0	1							
	/	E	X	I	R	S	A	I	S

accumulated distances

4  $+ D_{\text{INS}}$   $\min$   
 $\{4+1, 3+1, 2+1\}$   
 $= 3$

$+ D_{\text{SUB}}$   
 $+ D_{\text{DEL}}$   
2

# Trellis Computations

## Backpointers, Backtracking and Alignment

local distances

E	1	0	1	1	1	1	1	1	1
S	1	1	1	1	1	0	1	1	0
I	1	1	1	0	1	1	1	0	1
C	1	1	1	1	1	1	1	1	1
R	1	1	1	1	0	1	1	1	1
E	1	0	1	1	1	1	1	1	1
X	1	1	0	1	1	1	1	1	1
E	1	0	1	1	1	1	1	1	1
/	0	1	1	1	1	1	1	1	1
	/	E	X	I	R	S	A	I	S

accumulated distances

E	8	7	6	5	5	4	4	5	4
S	7	6	5	4	4	3	4	4	3
I	6	5	4	3	3	3	3	3	4
C	5	4	3	3	2	2	3	4	5
R	4	3	2	2	1	2	3	4	5
E	3	2	1	1	2	3	4	4	6
X	2	1	0	1	2	3	4	4	6
E	1	0	1	2	3	4	5	6	7
/	0	1	2	3	4	5	6	7	8
	/	E	X	I	R	S	A	I	S

Ref	E	X	E	R	C	-	I	S	E
Obs	E	X	I	R	S	A	I	S	-

ALIGNMENT: is found by **backtracking**, i.e. following **backpointers** from finishing cell (top+right) to starting cell (bottom+left)

# DTW – Questions1

## DTW-Q1.1

- Compute the minimum EDIT distance between
  - word (ref) vs. words (obs) (1)
  - word (ref) vs. wods (obs) (2)
  - words (ref) vs. word (obs) (1)

## DTW-Q1.2

- Same as DTW-Q1.1, but assume a DELETION cost of 2 for consonants (all other costs are assumed to be '1')
- If someone types 'wors', did he/she intend to type 'worse' or 'words' according to your system ?

# DTW – Questions1

## DTW-Q1.3

Compute the error rate of a speech recognition system for which you are given the result of a very small test set (Ref = Reference; Hyp = Recognizer output)

S1-Ref: fauchelevent limped along behind the horse in a very contented frame of mind  
S1-Hyp: locheven limped along behind the heard in very contented frame of mind

$$\frac{3}{13}$$

S2-Ref: he would have loved to be king in such a non nonsense paradise  
S2-Hyp: he had loved the king in a no sense paradigm

$$\frac{8}{13}$$

S3-Ref: do you know the names of the seven dwarfs in Disney's Snow White movie ?  
S3-Hyp: do you know the names of the seven warfs in the sneeze now white movie ?

$$\frac{5}{15}$$

$$ERR = \frac{16}{41}$$



1.1

d	4	3	2	1	0	1
r	3	2	1	0	1	2
o	2	1	0	1	2	3
w	1	0	1	2	3	4
#	0	1	2	3	4	5
#	w	o	r	d	s	

word<sup>s</sup>  
word s

d	4	3	2	1	2
r	3	2	1	1	2
o	2	1	0	1	2
w	1	0	1	2	3
#	0	1	2	3	4
#	w	o	d	s	

s	5				1
d	4				0
r	3			0	
o	2		0		
w	1	0			
#	0	1	2	3	4
#	w	o	r	d	

1.3

cross → merge

s	10	8	6	4	
d	8	6	4	2	3
r	6	4	2	0	1
o	4	2	0	1	2
w	2	0	1	2	3
#	0	1	2	3	4
#	w	o	r	d	

d	8	6	4	3	2
r	6	4	2	1	2
o	4	2	0	1	2
w	2	0	1	2	3
#	0	1	2	3	4
#	w	o	d	s	

s	10	8	6	4	2
d	8	6	4	2	0
r	6	4	2	0	1
o	4	2	0	1	2
w	2	0	1	2	3
#	0	1	2	3	4
#	w	o	r	d	