Q.1

max $F = 3x + 4y - 5z$

$$2x - 3y \le 5$$
$$z - 3y \ge -3$$
$$z + 3y = 3$$
$$x \ge 0,\ y \ge 0$$

We introduce slack variable $s_1 \leftarrow$ surplus

variable s_2. & $z = P - q$, $P, q \ge 0$

max $F = 3x + 4 - 5P + 5q$

$$2x - 3y + s_1 = 5$$
$$P - q - 3y + s_2 = -3$$
$$P - q + 3y = 3$$
$$x, y, s_1, s_2, P, q \ge 0$$

Q.2

$$x_3 = 100 - 8x_1 - 4x_2$$
$$x_4 = 120 - 4x_1 - 6x_2$$
$$x_5 = 34 - x_1$$
$$x_6 = 14$$

$$z = 0 + 10x_1 + 15x_2$$

-Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
							0
1	10	15	0	0	0	0	160
0	8	4	1	0	0	0	120
0	4	6	0	1	0	0	34
0	1	0	0	0	1	0	14
0	0	0	0	0	0	1	

Q.3

Choose x_1

$$\begin{aligned} 34 \\ 30 \\ 20 \end{aligned}$$

$z + 48 = x_1$
$x_8 = 14$ ∴ $x_1 = \Delta$
$x_1 + x_7 = 34$ ∴ $\Delta = 20$
$4x_1 + x_4 = 120$
$8x_1 + x_3 = 160$ ∴ $Z = -28$

∴ $x_7 = 34 - 20 = 14$
$x_4 = 120 - 4 \cdot 20 = 40$
$x_3 = 0$

		5	4	1	0	0	0	0	0	
B		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
x_1	20	1	0.5	0.125	0	0	0	0	0	x 9
x_4	40	0	4	-0.5	1	0	0	0	0	
x_7	14	0	-0.5	-0.125	0	-1	0	1	0	
x_8	14	0	1	0	0	0	-1	0	1	

$z + 28$ | 0 | 0.5 | -0.125 | 0 | -1 | -1 | 0 | 0

$x_1 + 0.5\Delta = 20$ $x_2 = $ 40
                                                   10
$4\Delta + x_4 = 40$ -28
                                                   14
$-0.5\Delta - x_7 = 14$
                          $\Delta = 10$

$z + 28 = 0.5\Delta$

$\to Z = 23$ $x_1 = 15$

$x_8 + 10 = 14$ $x_4 = 0$
$\to x_8 = 4$ $x_7 = 14 - 5 = 9$

Q.4

max $5000x + 3000y$

Subject to

$$25x + 15y \le 400$$
$$6x + 10y \le 100$$
$$x + y \le 30$$

max $z = 5000x + 3000y$

Subject to

$$25x + 15y + s_1 = 400$$
$$6x + 10y + s_2 = 100$$
$$x + y + s_3 = 30$$

-Z	x	y	s_1	s_2	s_3	RHS
0	25	15	1	0	0	400
0	6	10	0	1	0	100
0	1	1	0	0	1	30
1	5000	3000	0	0	0	0

$s_1 = 400$
$s_2 = 100$ $z = 0$
$s_3 = 30$

Q.5

1. maximize the profit:

$$36(0.8x_1 + 0.4x_2) + 24(0.05x_1 + 0.1x_2) + 21(0.1x_1 + 0.36x_2) + 10(0.05x_1 + 0.1x_2)$$

$$- 0.5x_1 - x_2$$
$$- 24x_1 - 15x_2$$

$$0.8x_1 + 0.44x_2 \le 24000$$

$$0.05x_1 + 0.1x_2 \le 2000$$

$$0.10x_1 + 0.36x_2 \le 6000$$

$$x_1, x_2 \ge 0$$

Q.6

$$\max_{x_1, x_2, x_3} \quad Z = 3x_1 + 4x_2 - 5x_3$$

$$x_1 + x_2 + x_3 \ge 4$$
$$x_1 - x_3 \le 1$$
$$x_2 + 2x_3 \le 2$$
$$x_1, x_2, x_3 \ge 0$$

-Z	x_1	x_2	x_3	Δ_1	Δ_2	Δ_3	RHS
1	3	4	-5	0	0	0	0
0	1	1	1	-1	0	0	4
0	1	0	-1	0	1	0	1
0	0	1	2	0	0	1	2

Q.7

$$\max \quad Z = 3x_1 - x_2$$

$$-x_1 + x_2 \ge -1$$
$$-2x_1 + x_2 \le 1$$
$$2x_1 - x_2 \le 5$$
$$x_1, x_2 \ge 0$$

-Z	x_1	x_2	Δ_1	Δ_2	Δ_3	RHS
1	3	-1	0	0	0	0