

Probability:

It's the measure of the likelihood of an event.

$$\text{Ex - Probability} = \frac{\text{no. of favorable outcome}}{\text{Total no of possible outcome}}$$

Probability for head or tail to come in a coin when tossed.

$$P(\text{Head}) = \frac{1}{2}$$

$$P(\text{tail}) = \frac{1}{2}$$

Mutual Exclusive event.

Two events are mutual exclusive event when both output are different or say both of them occurrence are different.

EX - Head and Tail can't occur once in coin at same time.

Non Mutual Exclusive event.

when both output occurs at same time it's Non Mutual Exclusive event.

Ex- win and loss happens at same time.

Addition rule for non-mutual exclusive event:

If A and B are non-mutual exclusive event then probability of getting A or B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cap B) = 0$ in case of mutual exclusive.

Multiplication rule:

- **Dependent events:** Two events are called **dependent** if they **affects each other**.

Ex- let bag has 10 balls (3 Red, 7 Green).

If we pick any one bag probability of other will change as total no.s of ball will decrease.

$$P(\text{Red}) = \frac{3}{10} \quad [\text{Initially 10 Balls}]$$

$$P(\text{Green}) = \frac{7}{9} \quad [\text{one red ball has taken so total left is 9}]$$

Independent events: when two events do not effect one another then it's Independent events.

Ex- Probability of getting 3, 5 from dice.

Probability of getting each face in dice is equal which is $\frac{1}{6}$

Now, Probability of getting 3, 5

$$= P(3) \times P(5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{3}$$

Permutation and combination:

Permutation: Different arrangements which can be made by taking same or all of given no.s of object at a time.

Ex- Permutation of three items a, b and c taken two at a time are ab, bc, ca, ba, cb, ac

ab and ba are considered separately

$$\text{formulat} = {}^n P_r = \frac{n!}{(n-r)!}$$

Combination: when same elements selected in different order are considered as same say 1 then it's combination.

$$\text{formulat} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

Ex - As earlier ab and ba were considered separately but in combination its same.

Covariance:

It's **statistical tool** used to determine the **relationship b/w** the **movements of two random variables**.

Positive covariance means **variable are proportional** to each other vs negative covariance means they are **inversely proportional** to each other.

$$\text{Covariance}(x, y) = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{N - 1}$$

X	Y
2	5
6	10
7	12
8	13
12	15

$$\bar{X} = \sum_{i=1}^n \frac{x_i}{n} = 7$$

$$\text{Similarly, } \bar{Y} = 11$$

$$\begin{aligned} \text{Cov}(X, Y) &= (2-7) \times (5-11) + (6-7) \times (10-11) + (7-7) \times (12-11) \\ &\quad + (8-7) \times (13-11) + (12-7) \times (15-11) \\ &= \frac{53}{4} \end{aligned}$$

$$= 13.25$$

It's positive covariance it means X increase then Y will also increase.

Pearson correlation coefficient.

It's linear correlation coefficient that return value b/w -1 to +1.

As the value of covariance can vary

extensionally and thus it's tough to work in huge range. So, restricting value within bounded range would be relevant.

Pearson correlation coefficient bound value b/w -1 to $+1$.

$$\text{Pearson carr. coeff} = \frac{\text{Cov}(X, Y)}{\sigma_X, \sigma_Y}$$

Considering previous value of X and Y .

$$\begin{array}{lcl} X & - & 2, 6, 7, 8, 12 \\ \hline Y & - & 5, 10, 12, 13, 15 \end{array} \quad \begin{array}{l} \bar{X} = 7 \\ \bar{Y} = 11 \end{array}$$

$$\sigma_X = \sqrt{\frac{(X_i - \bar{X})^2}{5-1}} = \frac{7.21}{2} = 3.605$$

$$\sigma_Y = \sqrt{\frac{(Y_i - \bar{Y})^2}{5-1}} = \frac{7.61}{2} = 3.805$$

$$\text{Cov}(X, Y) = 13.25 \left[\begin{array}{l} \text{from previous} \\ \text{data} \end{array} \right]$$

Pearson correlation coef

$$= r = \frac{\text{Cov}(X, Y)}{\sigma_X, \sigma_Y}$$

$$= \frac{13.25}{3.63 \times 3.81}$$

$$= 0.96$$

Thus, we have bounded value b/w -1 to +1 making it easy for us to compare.

Spearman's rank correlation.

It measures strength and direction of association b/w two ranked variables.

we use it for Non linear variables.

It gives measure of monotonicity of the relation b/w two variables.

$$r_s = \frac{\text{Cov}(R(X), R(Y))}{\sigma(R(X)) \times \sigma(R(Y))}$$

$R(X)$ and $R(Y)$ is simply natural no. assigned to X and Y when arranged in ascending order.

$$\text{Ex} - X = 2, 3, 6, 4, 7 \Rightarrow \text{arranged}_X = 2, 3, 4, 6, 7$$

$$X \rightarrow 2, 3, 4, 6, 7$$

$$R(X) \rightarrow 1, 2, 3, 4, 5$$

Considering previous value of X and Y

$$\begin{array}{c|c} X - 2, 6, 7, 8, 12 & Y - 5, 10, 12, 13, 15 \\ \hline R(X) - 1, 2, 3, 4, 5 & R(Y) - 1, 2, 3, 4, 5 \end{array}$$

calculating $\overline{R(X)}$ and $\overline{R(Y)}$

$$= 1 + 2 + 3 + 4 + 5 / 5 = 3$$

$$\text{Cov}(R(X), R(Y)) = 2.5$$

$$\sigma_{R(X)} = \sigma_{R(Y)} = 1.56$$

$$\text{calculating, } r_s = \frac{2.5}{1.56 \times 1.56} = \frac{2.5}{2.49} = 1.004$$

$$\approx 1$$

Thus, **monotonicity** is **proved** b/w X & Y .