



German Jordanian University

School of Applied Technical Sciences

Department of Mechatronics Engineering

ME548 - Control Systems 2

Done by:

Amro Habboush 20181102011

Rami Abu Al Nadi 20181102047

Hashem Altaha 20181102010

Supervised By:

Dr. Mutaz Ryalat

➤ Introduction

A control system is a set of mechanical or electronic devices that regulates other devices or systems by way of control loops. Typically, control systems are computerized. Control systems are a central part of industry and of automation.

- **What is a generator?**

An electric generator is a device that converts mechanical energy obtained from an external source into electrical energy as the output. It operates based on the electromagnetic induction principle, which is the creation of an electric current by moving a wire next to a magnet.

There are three main types of generators:

1. Portable
2. inverter
3. standby

What is a DC motor?

A DC motor is defined as a class of electrical motors that convert direct current electrical energy into mechanical energy. From the above definition, we can conclude that any electric motor that is operated using direct current or DC is called a DC motor, typical DC motors may operate on as few as 1.5 Volts or up to 100 Volts or more.

Main Types of DC Motors:

1. Permanent Magnet DC Motors
2. Series DC Motors
3. Shunt DC Motors. ...
4. Compound DC Motors.

Part i: block diagram for the whole system (fig1)

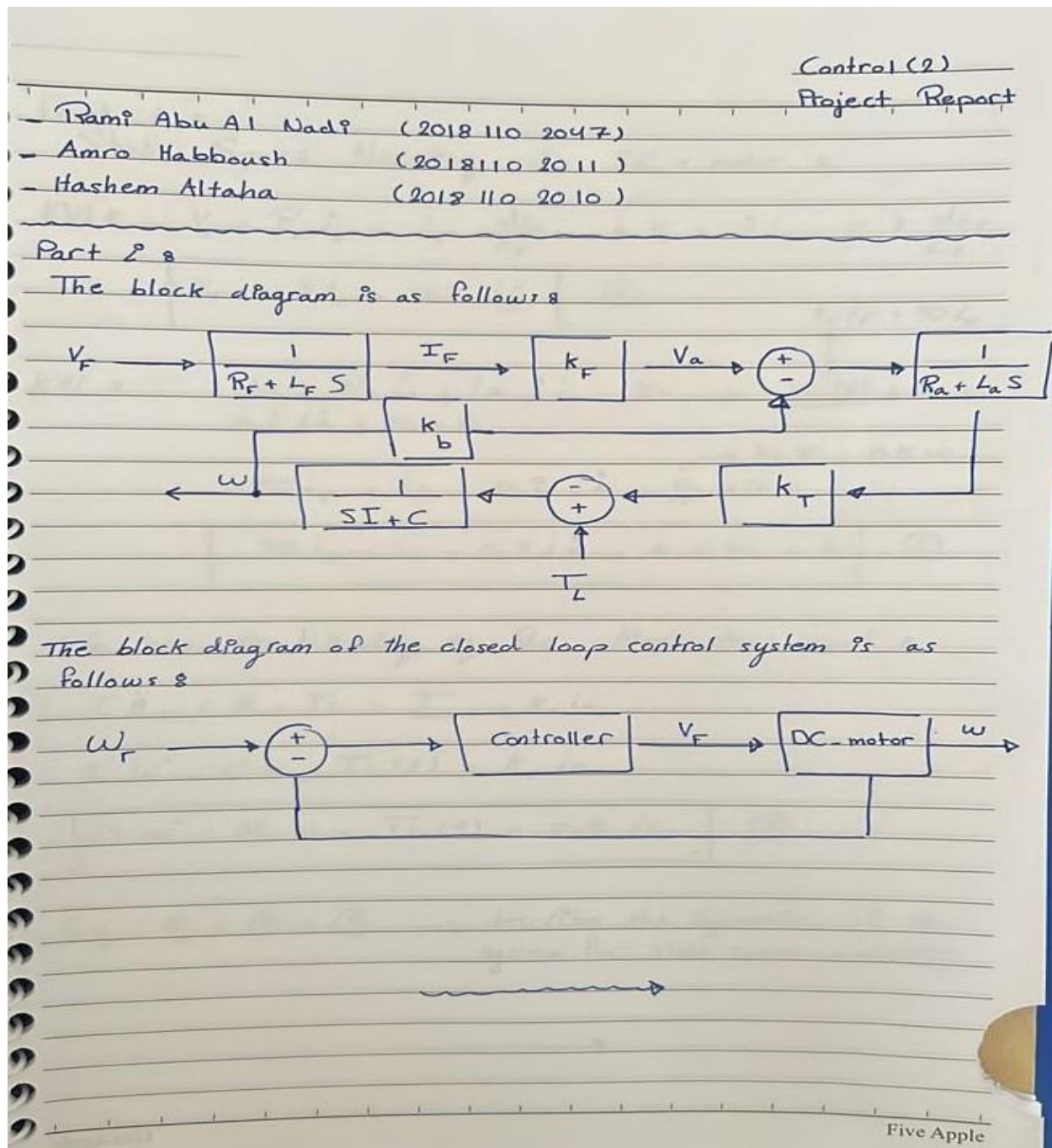


Fig 1 - block diagram

Part ii: differential equations - state-space representation (fig2)

Part ii :

State Space Modeling of DC-motor :

KVL : $V_F = R_F i_F + L_F \frac{di_F}{dt} \rightarrow V_F = 2 i_F + 0,2 \frac{di_F}{dt}$

$$\boxed{V_F = 2 i_F + 0,2 \dot{i}_F} \quad (1)$$

$$K_F i_F = 50 i_F$$

KVL : $-V_a + R_a i_a + L_a \dot{i}_a + V_b \rightarrow -V_a(t) + i_a + 0,2 \dot{i}_a + V_b = 0$

$$-50 i_F + i_a + 0,2 \dot{i}_a + \overset{K_b \omega = 0,5 \omega}{V_b} = 0$$

$$\boxed{-50 i_F + i_a + 0,2 \dot{i}_a + 0,5 \omega = 0} \quad (2)$$

We have the following eq. for Mechanical Load :

$$I \ddot{\theta} + C \dot{\theta} + T_L = T \rightarrow K_t i_a$$

$$I \omega' + C \omega + T_L(t) = K_t i_a$$

$$\boxed{10 \omega' + 20 \omega + T_L(t) = 0,5 i_a} \quad (3)$$

Eq. (1) & (2) & (3) \rightarrow describe the dynamics of the system for state space modeling

Five Apple

We consider the following state variables :

$$\begin{aligned} x_1(t) &= i_F(t) \\ x_2(t) &= i_a(t) \\ x_3(t) &= \omega(t) \end{aligned} \rightarrow \text{state vector } X = \begin{bmatrix} i_F \\ i_a \\ \omega \end{bmatrix}$$

Also we consider input $u(t) = V_F(t)$ and disturbance $T_L(t)$ and output $y(t) = \omega(t)$

Eq. (1) $\rightarrow \dot{i}_F = -10 i_F + 5 V_F(t)$
 $x_1'(t) = -10 x_1(t) + 5 u(t)$

Eq. (2) $\rightarrow \dot{i}_a(t) = 250 i_F - 5 i_a - 2,5 \omega$
 $x_2'(t) = 250 x_1 - 5 x_2 - 2,5 x_3$

Eq. (3) $\rightarrow \dot{\omega}(t) = 0,05 i_a - 2 \omega - 0,1 T_L(t)$
 $x_3'(t) = 0,05 x_2 - 2 x_3 - 0,1 T_L(t)$

The State Space Model of the System :

$$\begin{bmatrix} \dot{i}_F \\ \dot{i}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 \\ 250 & -5 & -2,5 \\ 0 & 0,05 & -2 \end{bmatrix} \begin{bmatrix} i_F \\ i_a \\ \omega \end{bmatrix}$$

$$+ \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} V_F(t) + \begin{bmatrix} 0 \\ 0 \\ -0,1 \end{bmatrix} T_L(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_F \\ i_a \\ \omega \end{bmatrix}$$

Fig 2 - differential equations and state space

Part iii: The Simulink model is as follow (Open loop system) (fig3)

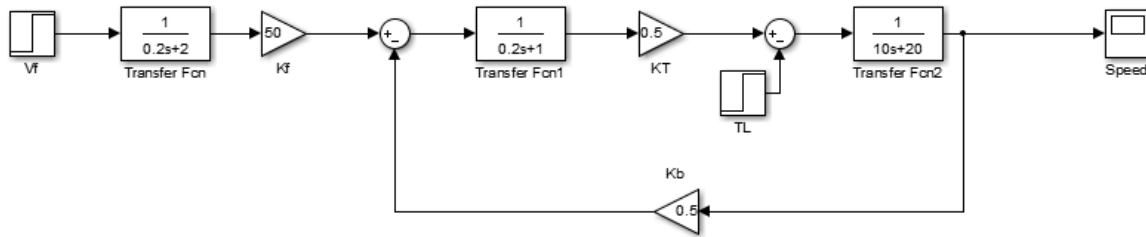


Fig 3 – open loop system

Part iv: The step response of the Open loop system is as follow (for zero disturbance $T_L(t)$) (fig4)

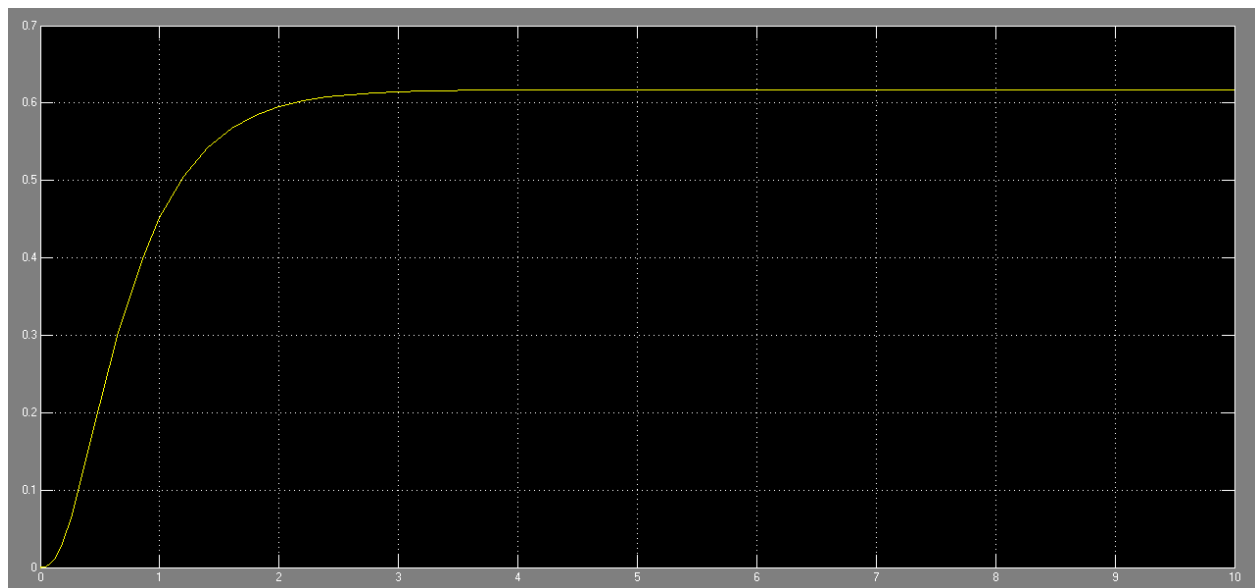


Fig4 - response of the Open loop

Part v: The step response (fig6) of the closed loop system (fig5)

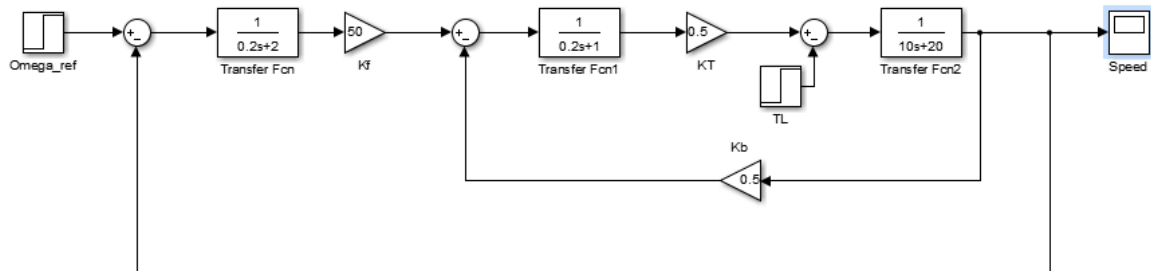


Fig5 - closed loop system

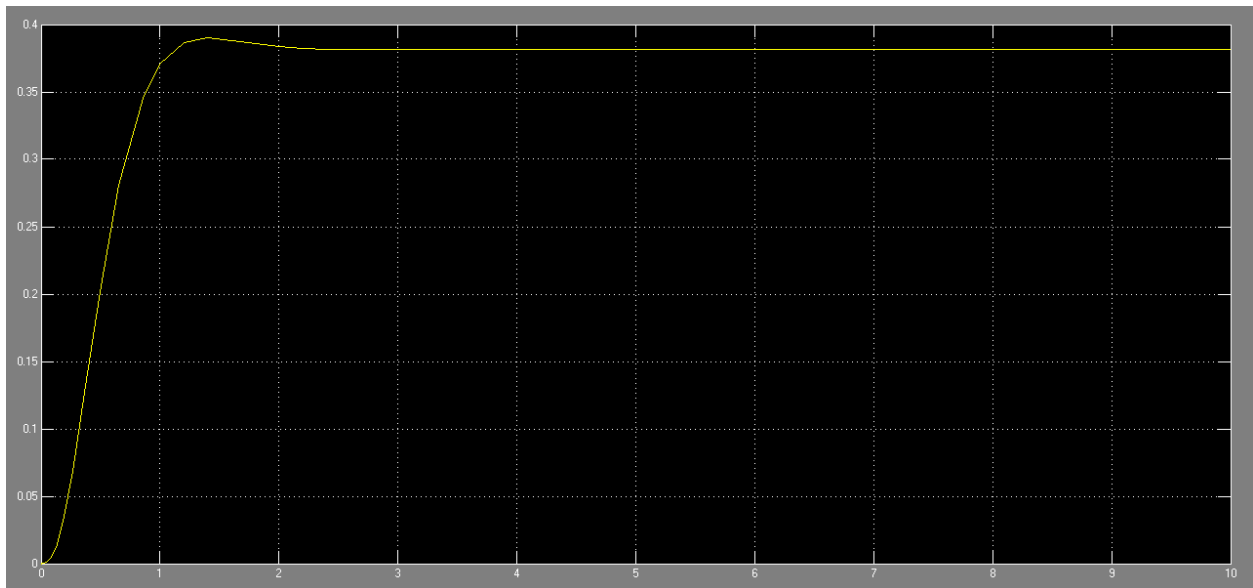


Fig6 - step response of the closed loop

Part vi: The characteristic of the open loop system and the closed loop system are compared as follow

	Stability	Performance		Steady-state Error
		Overshoot	Settling Time	
Open Loop System	Stable System	Zero	3 seconds	0.38
Closed Loop system		2.1 %	3 seconds	0.62

Both Open loop and closed loop systems are stable. But, the steady state error for both systems are unacceptable.

Part vii: State-Space Controller

The state space model of the system is as follow:

$$\frac{d}{dt} \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 \\ 250 & -5 & -2.5 \\ 0 & 0.05 & -2 \end{bmatrix} \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} v_f(t) + \begin{bmatrix} 0 \\ 0 \\ -0.1 \end{bmatrix} T_L(t)$$

$$y(t) = [0 \quad 0 \quad 1] \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix}$$

The eigenvalues of the uncontrolled system are located at -2.04, -4.95 and -10 which shows an stable system but with unacceptable performance.

We can see that the system is controllable as we have $\text{rank}([A \ AB \ A^2B]) = 3$.

Part vii -A: Pole-Placement Method:

Based on the given requirement on overshoot less than 10% and settling time less than 1 seconds, we consider the desired poles (eigenvalues) of the closed loop control system at $s = -6 \pm j8$ and $s = -60$. Then, we will tune the gain of the controller for steady-state error 2 %.

Now we consider state-feedback control $v_f(t) = -[k_1 \ k_2 \ k_3] \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} + k_4 \omega_{ref}$ and design state-feedback controller gains $[k_1 \ k_2 \ k_3]$ with pole-placement method such that locate the closed loop system poles (eigenvalues) at $s = -6 \pm j8$ and $s = -60$. We design the value of k_4 such that it results in steady-state error 2 %.

We use function place in MATLAB to design controller gains $[k_1 \ k_2 \ k_3]$ as follow

MATLAB Codes:	Results:
<pre> 1 - A=[-10 0 0;250 -5 -2.5; 0 0.05 -2]; 2 - eig(A) 3 - B=[5 0 0]'; 4 - rank(ctrb(A,B)) 5 - K=place(A,B,[-6+8*j -6-8*j -60]) </pre>	<pre> K = 11.0000 0.2839 74.1140 </pre>

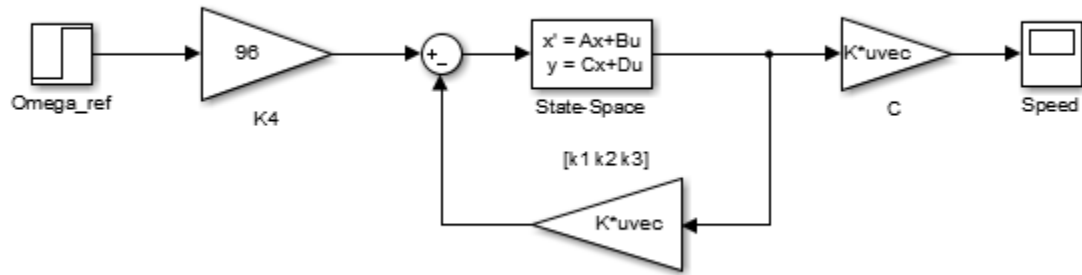


Fig7 - simulink model of the system (pole placement method)

First we set $k_4=1$. The steady-state value of the speed would be 0.0104. Now, we set $k_4 = \frac{1}{0.0104} \approx 96$. It results in almost zero steady state error for zero disturbance $T_L(t)$. The plot of the motor speed with

controller $v_f(t) = -[11 \ 0.2839 \ 74.140] \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} + 96 \omega_{ref}$.

It can be seen that the step response has steady state error less than 2%, Overshoot less than 10% and settling time less than 1 second. (fig8)

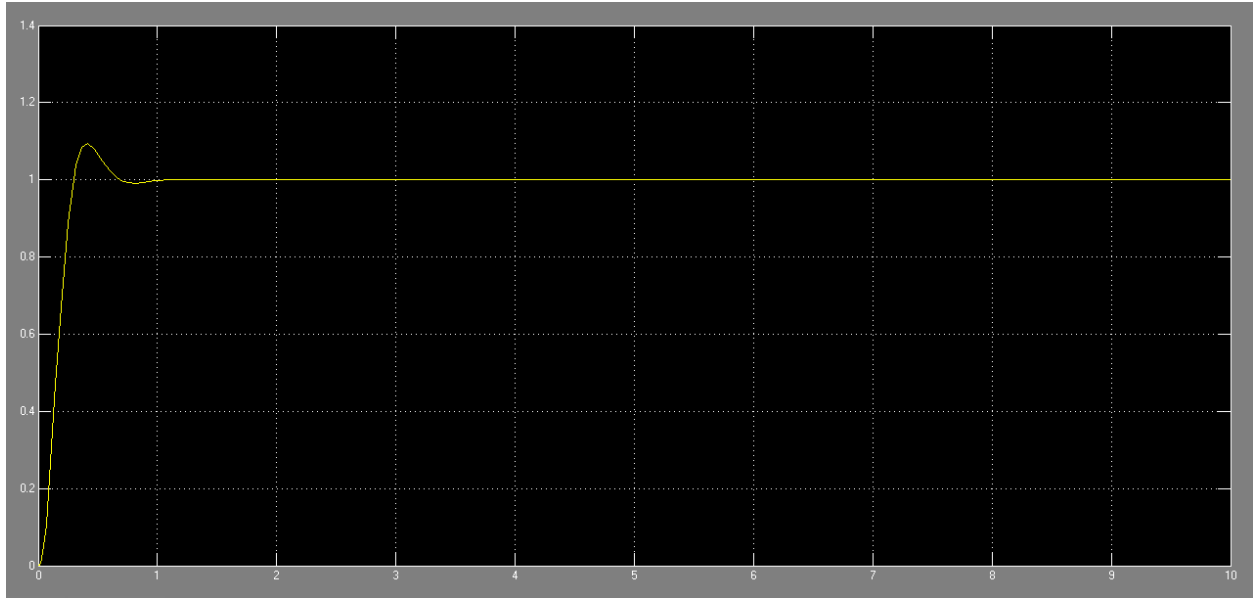


Fig8 - step response (Overshoot less than 10%)

Part vii -B: LQR Method:

In this section we design a LQR controller which minimize the cost function $J = \int (x^T Q x + u^T R u) dt$ where $x = \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix}$ and $u = v_f(t)$. We consider matrix $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10000 \end{bmatrix}$ and $R=1$. We consider state-feedback control $u(t) = -K_{lqr}x + k_4\omega_{ref}$ and design control gain K_{lqr} using lqr function in MATLAB as follow:

<pre>R=1; Q=[1 0 0;0 1 0;0 0 10000]; K_lqr=lqr(A,B,Q,R)</pre>	<pre>K_lqr = 7.9237 0.9348 62.2881</pre>
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First we set $k_4 = 1$. The steady-state value of the speed would be 0.009. Now, we set $k_4 = \frac{1}{0.0093} \approx 107$. It results in almost zero steady state error for zero disturbance $T_L(t)$. The plot of the motor speed with controller $v_f(t) = -[7.9237 \ 0.9348 \ 62.2881] \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} + 107 \omega_{ref}$

It can be seen that the step response has steady state error less than 2%, zero overshoot and settling time less than 1 second (fig9).



Fig9 - step response (Overshoot = 0)

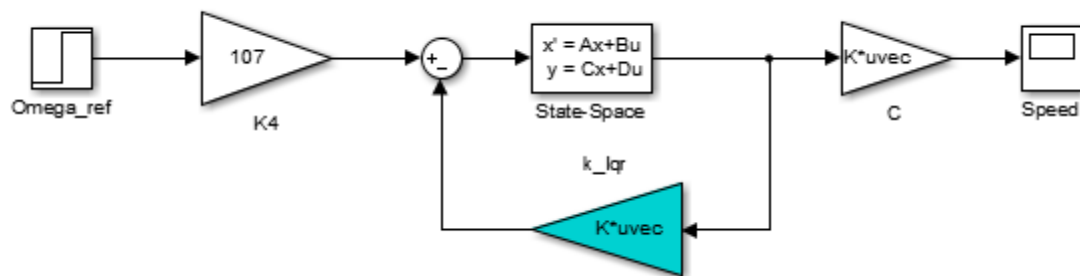


Fig10 - simulink model of the system (LQR method)

Part viii : PID Controller

To design a PID controller we use the following Simulink model (fig11). We set parameters of the PID controller as $K_p = 10$; $K_i = 10$; and $K_d = 2$.

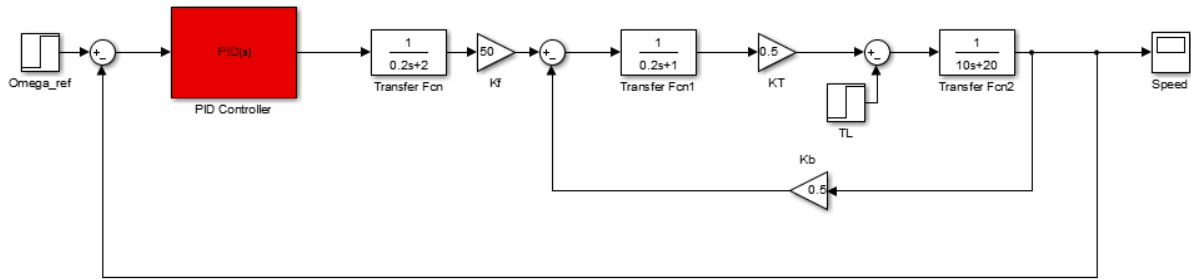


Fig 11 - simulink model of the system with PID controller

The step response of the closed loop system with PID controller shows zero steady state error. The overshoot is less than 10% and the settling time is less than 2 seconds (fig12)

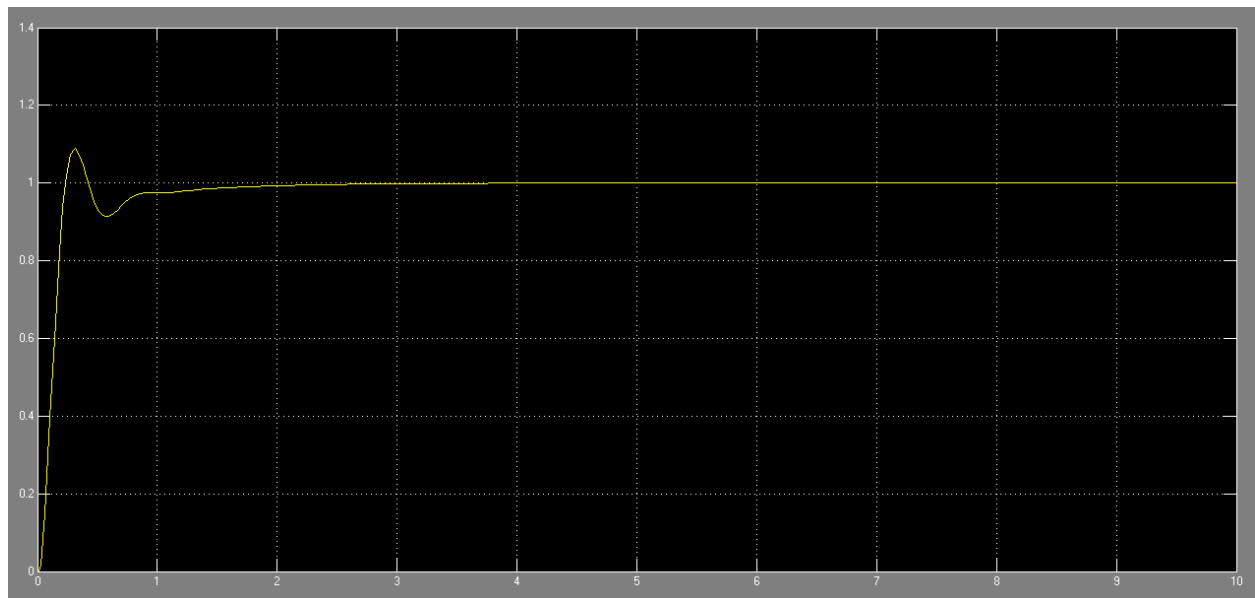


Fig 12 - step response with PID

Part ix : PI Controller

To design a PI controller $C(s) = Kp + \frac{Ki}{s}$ we use the following Simulink model (fig13). We set parameters of the PI controller as $Kp = 0.2$ and $Ki = 1.2$. The step response of the closed loop system with PI controller shows zero steady state error. The overshoot is less than 10% and the settling time is less than 6 seconds (fig14) (If we need a faster response we have to use derivative term)

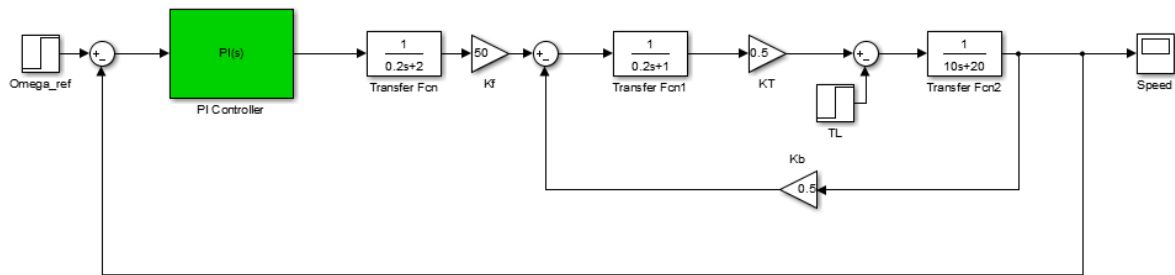


Fig 13 - simulink model of the system with PI controller

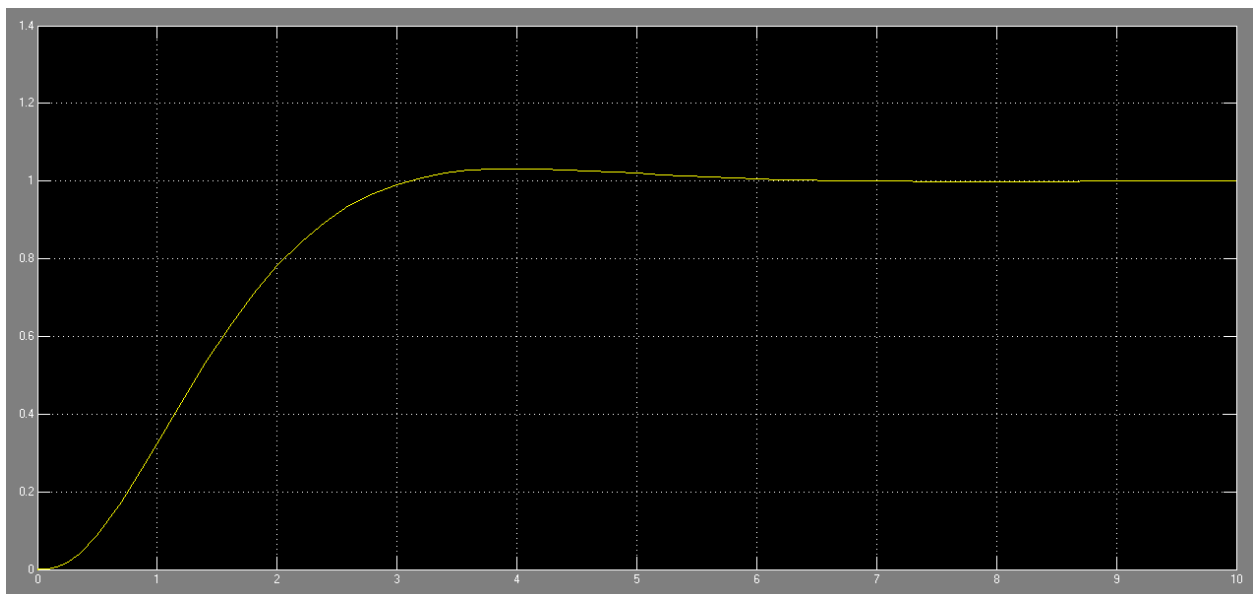


Fig 13 - step response with PI

➤ Conclusion

We conclude that using a state feedback controller we can place the poles of the closed loop system at desired location. However, it works just for controllable systems. Also, state feedback controller needs the values of all state at each time instants. It means that we have to use a sensor to measure each state in practice.

Moreover, large displacement in the poles leads to large control input efforts. We can use LQR control method to minimize a predefined cost function. By this methodology we can manage the convergence rate of the states as well as the control effort signals. However, it needs full states data, too.

Using a PID controller we just need sensors for some outputs to implement control systems. In this example, PID controller needs just a sensor to measure speed of the motor. While, State-feedback controller need three sensors for I_f , I_A and Speed.